Conventional Magnets for Accelerators Lecture 1

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Course Philosophy

An overview of magnet technology in particle accelerators, for **room temperature, static** (dc) **electromagnets**, and basic concepts on the use of **permanent magnets** (PMs).

Not covered: superconducting magnet technology.

Contents – lectures 1 and 2

- **DC Magnets: design and construction**
- **Introduction**
  - Nomenclature
  - Dipole, quadrupole and sextupole magnets
  - 'Higher order' magnets
- **Magnetostatics in free space (no ferromagnetic materials or currents)**
  - Maxwell's 2 magnetostatic equations
  - Solutions in two dimensions with scalar potential (no currents)
  - Cylindrical harmonic in two dimensions (trigonometric formulation)
  - Field lines and potential for dipole, quadrupole, sextupole
  - Significance of vector potential in 2D

Contents – lectures 1 and 2

- **Introducing ferromagnetic poles**
  - Ideal pole shapes for dipole, quad and sextupole
  - Field harmonics-symmetry constraints and significance
  - 'Forbidden' harmonics resulting from assembly asymmetries
- **The introduction of currents**
  - Ampere-turns in dipole, quad and sextupole
  - Coil economic optimisation-capital/running costs
- **Summary of the use of permanent magnets (PMs)**
  - Remnant fields and coercivity
  - Behaviour and application of PMs

Contents – lectures 1 and 2

- **The magnetic circuit**
  - Steel requirements: permeability and coercivity
  - Backleg and coil geometry: 'C', 'H' and 'window frame' designs
  - Classical solution to end and side geometries – the Rogowsky rolloff
- **Magnet design using FEA software**
  - FEA techniques and codes – Opera 2D, Opera 3D
  - Judgement of magnet suitability in design
  - Magnet ends – computation and design
- **Some examples of magnet engineering**

The Cockcroft Institute
of Accelerator Science and Technology

DC Magnets

INTRODUCTION

Units

- SI Units

Variable	Unit
Force, F	Newton (N)
Charge, q	Coulomb (C)
Flux density, B (commonly referred to as 'field')	Tesla (T) or Gauss (G) 1 T = 10,000 G
Magnetic field, H (magnetomotive force produced by electric currents)	Amp/metre (A/m)
Current, I	Ampere (A)
Energy, E	Joule (J)

- Permeability of free space

$$\mu_0 = 4\pi \times 10^{-7} \mathrm{T.m/A}$$

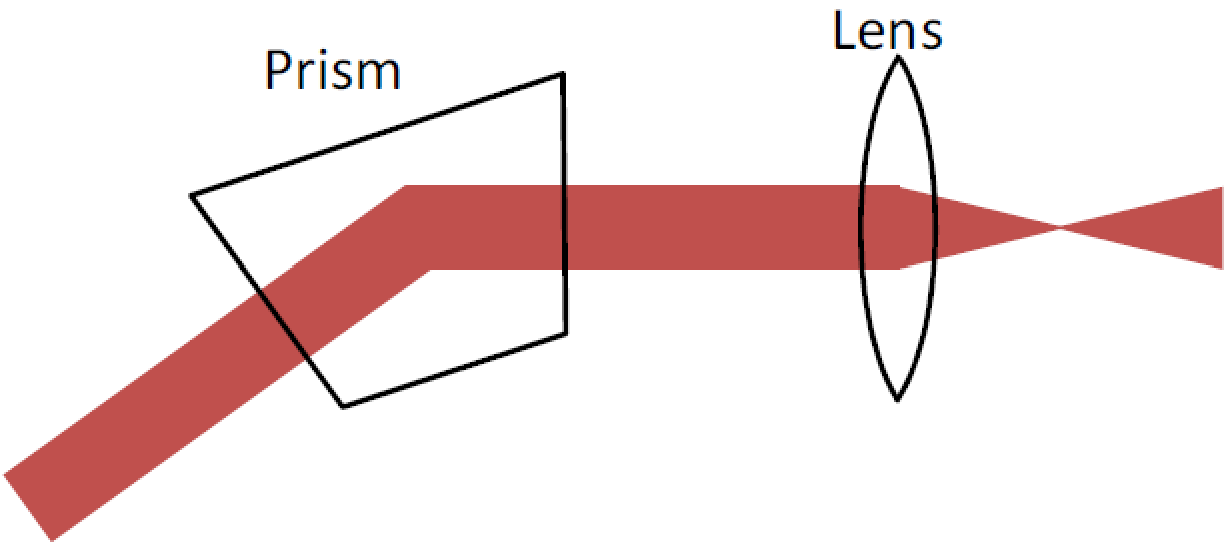
- Charge of 1 electron $e = -1.6 \times 10^{-19} \mathrm{C}$
  - 1 eV = $1.6 \mathrm{x} 10^{-19}$ J

Magnetic ~~Fields~~ Flux Density

Value	Item
0.1 - 1.0 pT	human brain magnetic field
24 μT	strength of magnetic tape near tape head
31-58 μT	strength of Earth's magnetic field at 0° latitude (on the equator)
0.5 mT	the suggested exposure limit for cardiac pacemakers by American Conference of Governmental Industrial Hygienists (ACGIH)
5 mT	the strength of a typical refrigerator magnet
0.15 T	the magnetic field strength of a sunspot
1 T to 2.4 T	coil gap of a typical loudspeaker magnet
1.25 T	strength of a modern neodymium-iron-boron ($Nd_2Fe_{14}B$) rare earth magnet.
1.5 T to 3 T	strength of medical magnetic resonance imaging systems in practice, experimentally up to 8 T
9.4 T	modern high resolution research magnetic resonance imaging system
11.7 T	field strength of a 500 MHz NMR spectrometer
16 T	strength used to levitate a frog
36.2 T	strongest continuous magnetic field produced by non-superconductive resistive magnet
45 T	strongest continuous magnetic field yet produced in a laboratory (Florida State University's National High Magnetic Field Laboratory in Tallahassee, USA)
100.75 T	strongest (pulsed) magnetic field yet obtained non-destructively in a laboratory (National High Magnetic Field Laboratory, Los Alamos National Laboratory, USA)
730 T	strongest pulsed magnetic field yet obtained in a laboratory, destroying the used equipment, but not the laboratory itself (Institute for Solid State Physics, Tokyo)
2.8 kT	strongest (pulsed) magnetic field ever obtained (with explosives) in a laboratory (VNIIEF in Sarov, Russia, 1998)
1 to 100 MT	strength of a neutron star
0.1 to 100 GT	strength of a magnetar

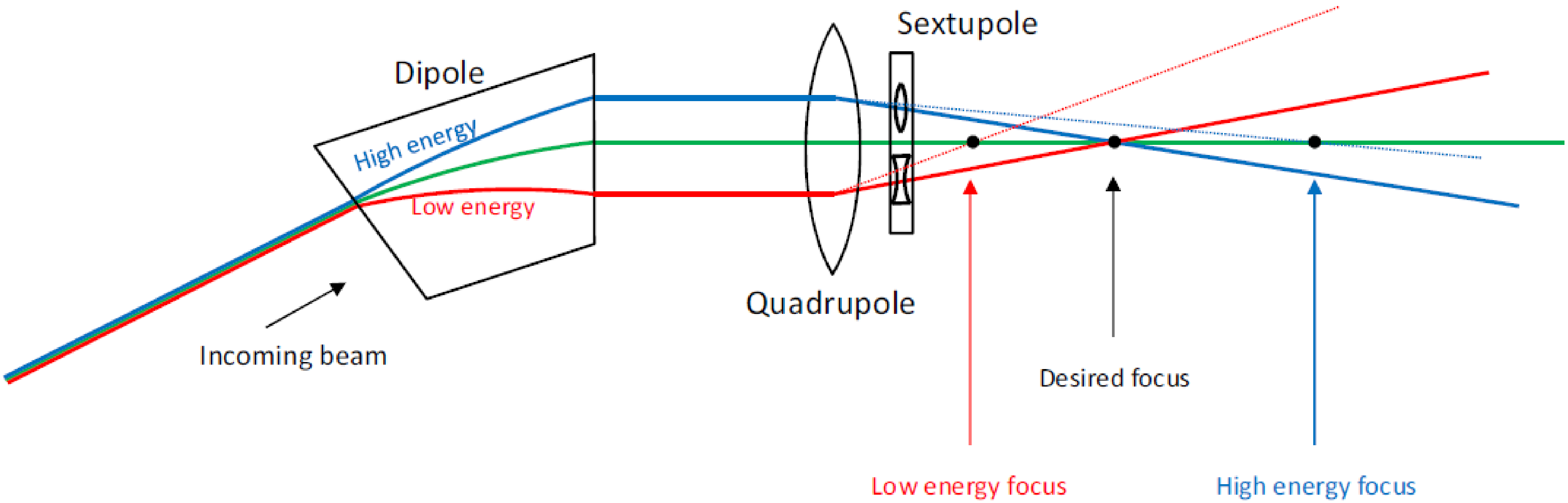
Why magnets?

Analogy with optics



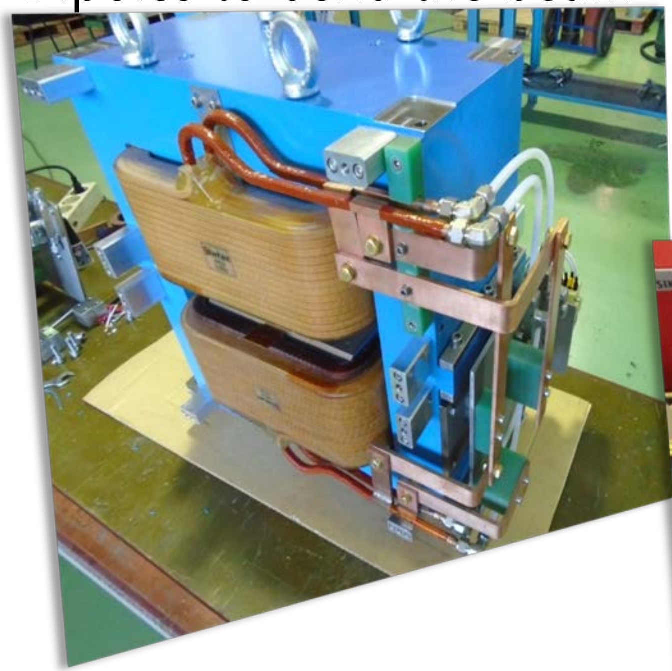
Magnets are like lenses…

… sort of



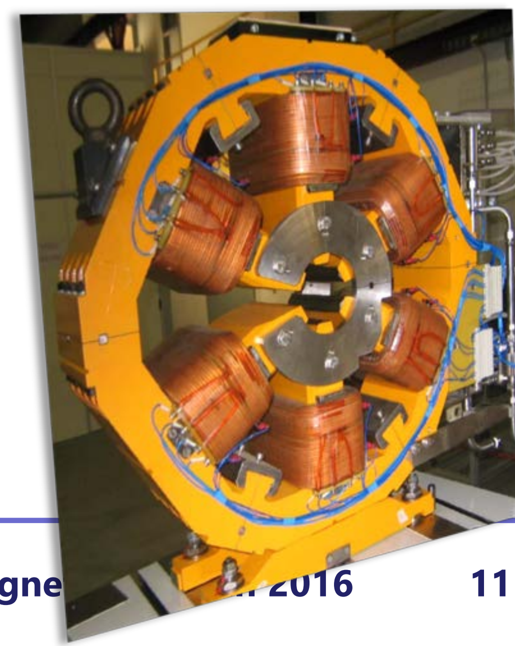
Magnet types

Dipoles to bend the beam



Quadrupoles to focus it

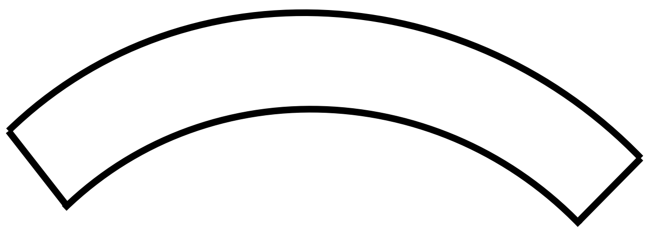
Sextupoles to correct chromaticity



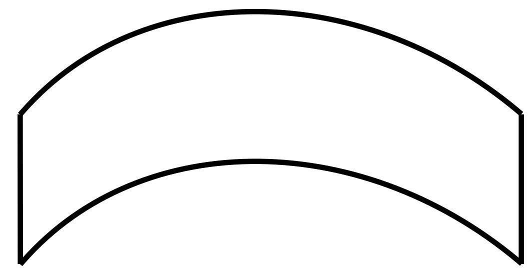
Magnets - dipoles

To bend the beam uniformly, dipoles need to produce a field that is constant across the aperture.

But at the ends they can be either:



Sector dipole

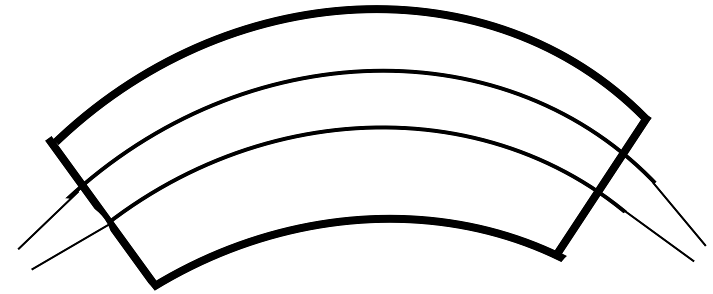


Parallel-ended dipole

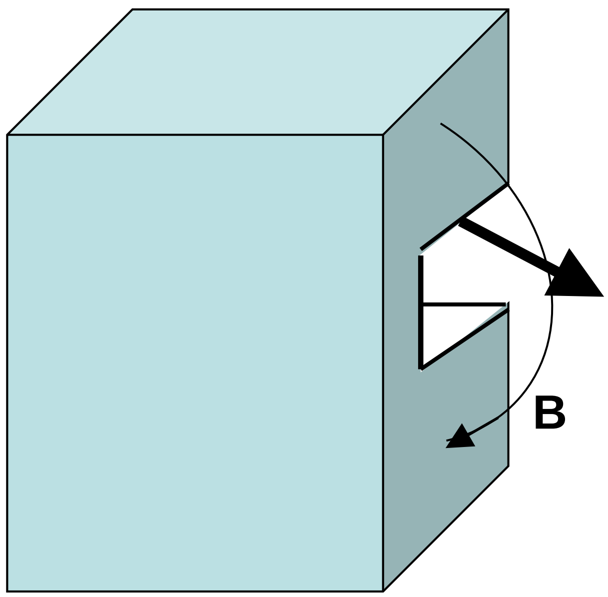
They have different focusing effect on the beam;
(their curved nature is to save material and has no effect on beam focusing).

Dipole end focusing

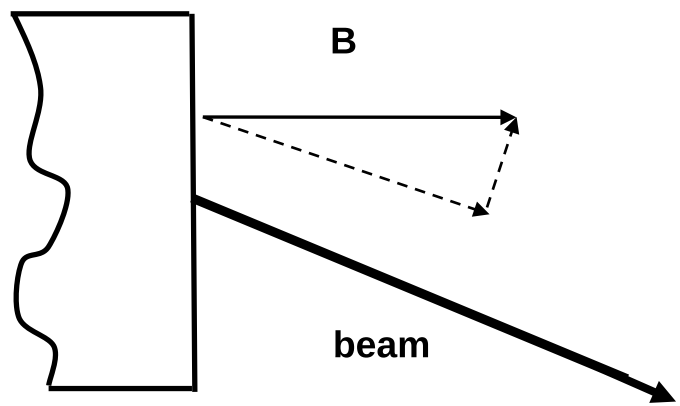
Sector dipoles focus horizontally →



The end field in a parallel ended dipole focuses vertically



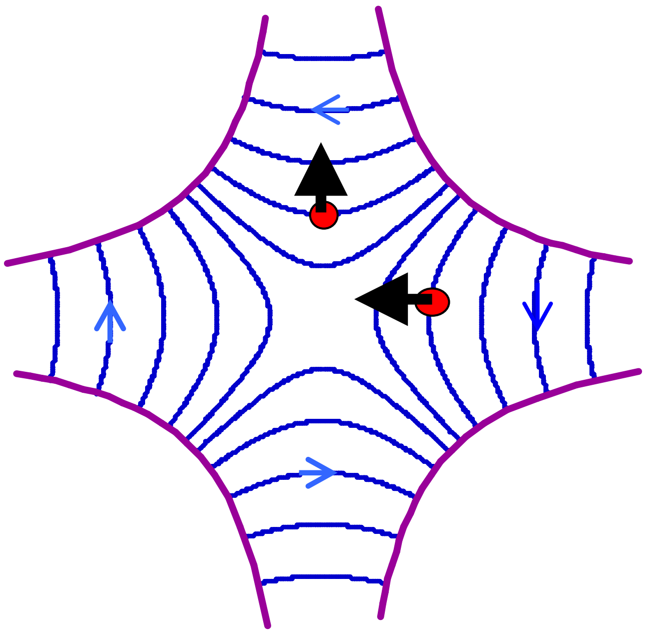
Off the vertical centre line, the field component normal to the beam direction produces a vertical focusing force.



Magnets - quadrupoles

Quadrupoles produce a linear field variation across the beam.

Field is **zero** at the 'magnetic centre' so that 'on-axis' beam is not bent.



Note: beam that is horizontally focused is vertically defocused.

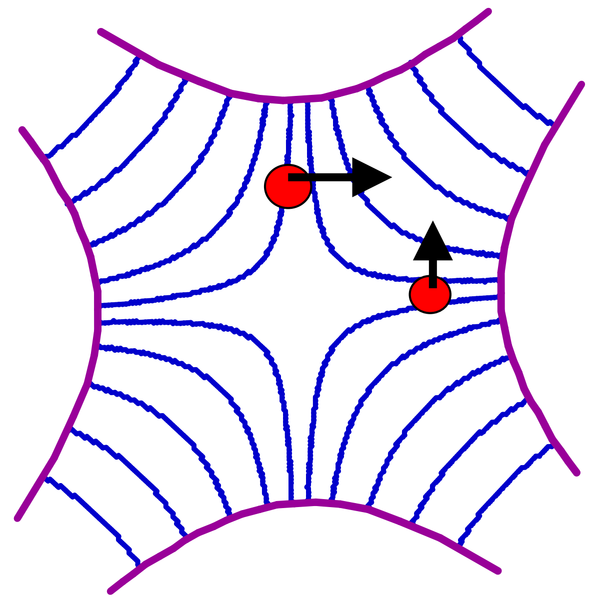
These are 'upright' quadrupoles.

Skew Quadrupoles

Beam that has **horizontal** displacement (but **not** vertical) is deflected **vertically**.

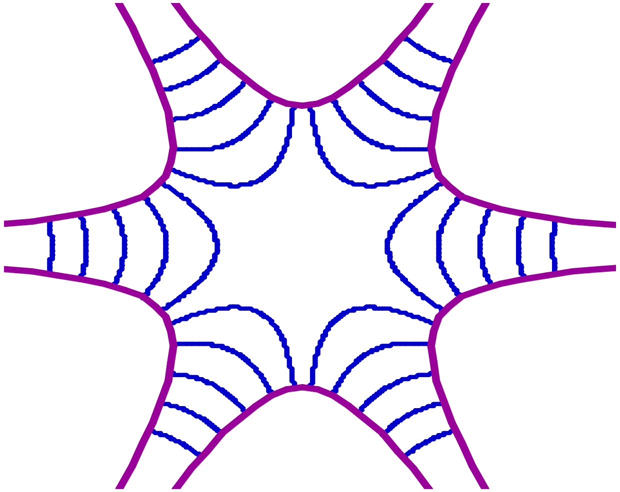
Horizontally centred beam with **vertical** displacement is deflected **horizontally**.

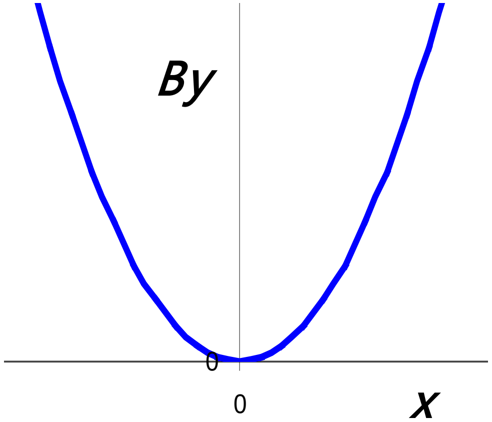
So skew quadrupoles **couple** horizontal and vertical transverse oscillations.



Sextupoles

In a **sextupole**, the field varies as the **square** of the displacement.





- Off-momentum particles are incorrectly focused in quadrupoles (e.g., high momentum particles with greater rigidity are under-focused), so transverse oscillation frequencies are modified – **chromaticity**.
- But off momentum particles circulate with a horizontal displacement (high momentum particles at larger *x*)
- So positive sextupole field corrects this effect – can reduce chromaticity to 0.

Higher order magnets

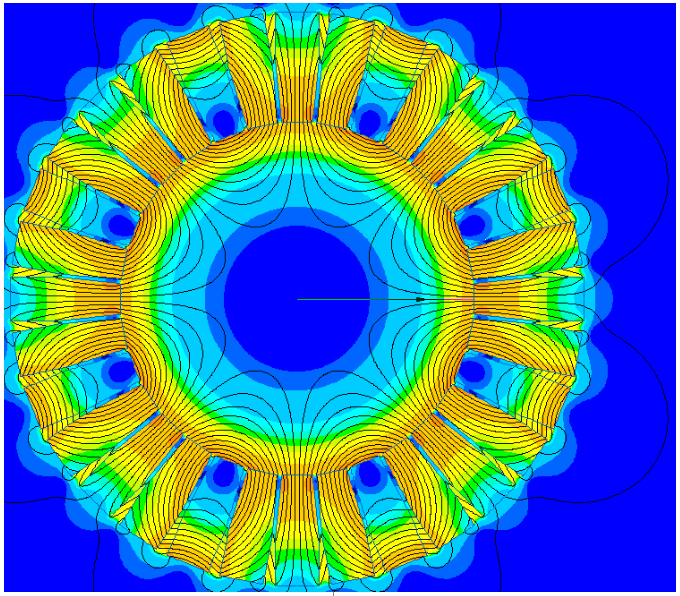
e.g. – Octupoles:

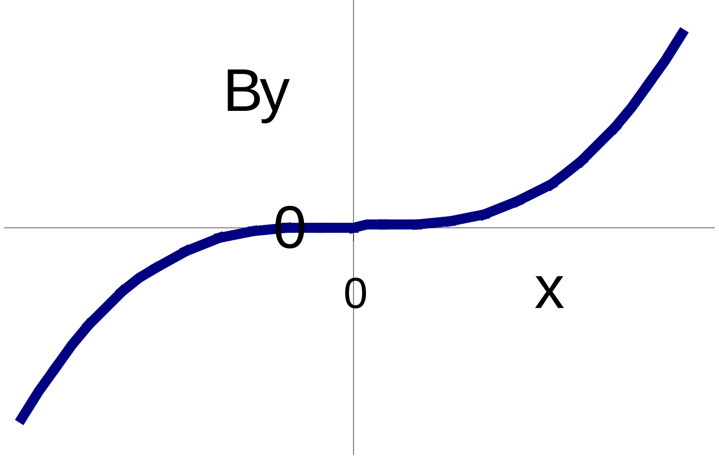
Effect?

$$B_y \propto x^3$$

Octupole field induces **Landau damping**:

- Introduces tune-spread as a function of oscillation amplitude
- De-coheres the oscillations
- Reduces coupling





Describing the field

MAGNETOSTATICS IN FREE SPACE

No currents, no steel - Maxwell's static equations in free space

$$\boldsymbol{\nabla} . \boldsymbol{B} = 0$$
$$\boldsymbol{\nabla} \times \boldsymbol{H} = \boldsymbol{j}$$

In the absence of currents: $\boldsymbol{j} = 0$

Then we can put: $\boldsymbol{B} = -\boldsymbol{\nabla}\phi$

So that: $\boldsymbol{\nabla}^{2}\phi = 0$ (Laplace's equation)

Taking the two dimensional case (i.e. constant in the *z* direction) and solving for polar coordinates (*r*, *θ*):

$$\phi = (E + F\,\theta)(G + H \ln r) + \sum_{n=1}^{\infty} J_n r^n \cos n\theta + K_n r^n \sin n\theta + L_n r^{-n} \cos n\theta + M_n r^{-n} \sin n\theta$$

In practical situations

The scalar potential simplifies to:

$$\Phi = \sum_{n=1}^{\infty} J_n r^n \cos n\theta + K_n r^n \sin n\theta$$

with n integral and J_n, K_n a function of geometry.

Giving components of flux density:

$$B_r = -\sum_{n=1}^{\infty} nJ_n r^{n-1} \cos n\theta + nK_n r^{n-1} \sin n\theta$$

$$B_\theta = -\sum_{n=1}^{\infty} -nJ_n r^{n-1} \sin n\theta + nK_n r^{n-1} \cos n\theta$$

Physical significance

This is an infinite series of cylindrical harmonics; they define the allowed distributions of ***B*** in 2 dimensions in the absence of currents within the domain of (r,θ).

Distributions not given by above are not physically realisable.

Coefficients J_n, K_n are determined by geometry (remote iron boundaries and current sources).

In Cartesian Coordinates

To obtain these equations in Cartesian coordinates, expand the equations for ϕ and differentiate to obtain flux densities

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 3\theta = \cos^3\theta - 3\cos\theta\sin^2\theta$$
$$\sin 3\theta = 3\sin\theta\cos^2\theta - \sin^3\theta$$

$$\cos 4\theta = \cos^4\theta + \sin^4\theta - 6\cos^2\theta\sin^2\theta$$
$$\sin 4\theta = 4\sin\theta\cos^3\theta - 4\sin^3\theta\cos\theta$$

etc (messy!);

$$x = r\cos\theta \qquad y = r\sin\theta$$
$$B_x = -\frac{\partial\phi}{\partial x} \qquad B_y = -\frac{\partial\phi}{\partial y}$$

n = 1: Dipole field

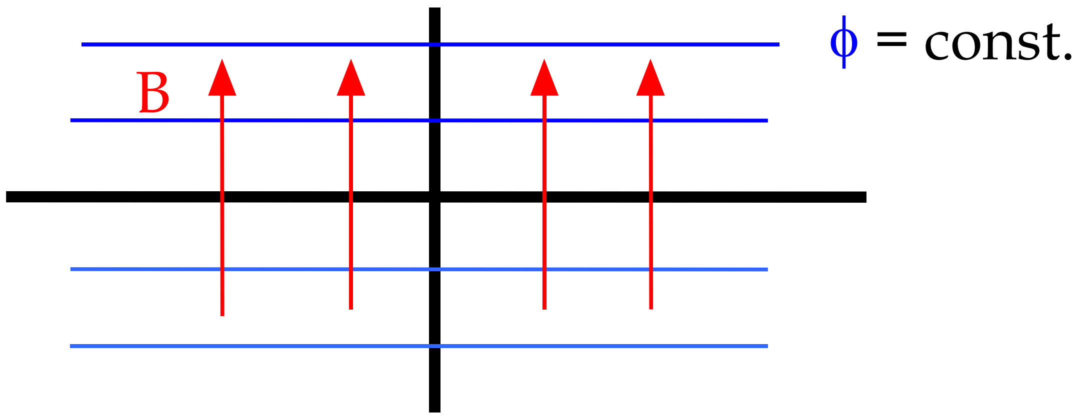
Cylindrical:

$$B_r = J_1 \cos\theta + K_1 \sin\theta$$
$$B_\theta = -J_1 \sin\theta + K_1 \cos\theta$$
$$\phi = J_1 r \cos\theta + K_1 r \sin\theta$$

Cartesian:

$$B_x = J_1$$
$$B_y = K_1$$
$$\phi = J_1 x + K_1 y$$

So, J_1 = 0 gives vertical dipole field:



K_1 =0 gives horizontal dipole field.

n = 2: Quadrupole field

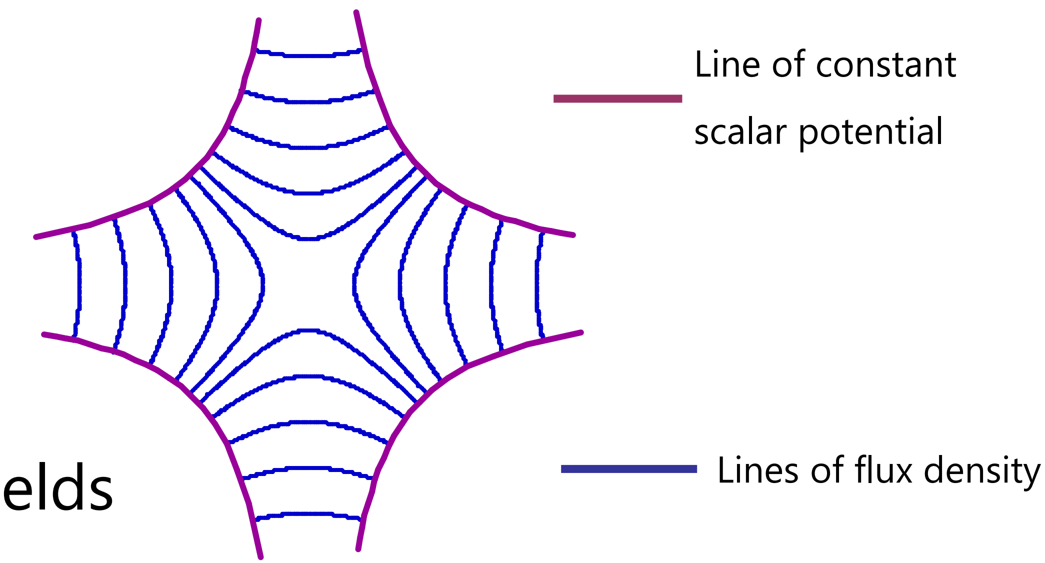
Cylindrical:

$$B_r = 2J_2 r\cos 2\theta + 2K_2 r\sin 2\theta$$
$$B_\theta = -2J_2 r\sin 2\theta + 2K_2 r\cos 2\theta$$
$$\phi = J_2 r^2\cos 2\theta + K_2 r^2\sin 2\theta$$

Cartesian:

$$B_x = 2(J_2 x + K_2 y)$$
$$B_y = 2(K_2 x - J_2 y)$$
$$\phi = J_2(x^2 - y^2) + 2K_2 xy$$

$J_2 = 0$ gives 'normal' or 'right' quadrupole field.



$K_2 = 0$ gives 'skew' quad fields (above rotated by $\pi/4$).

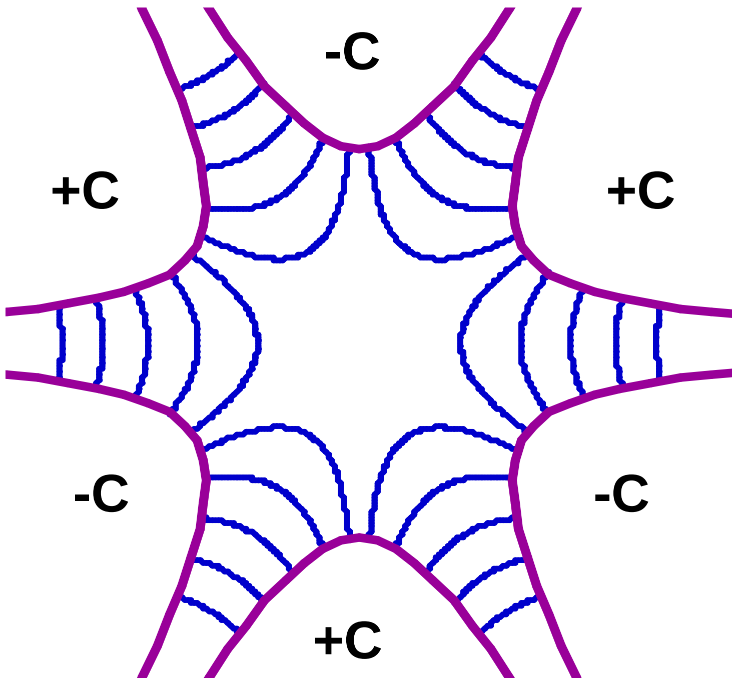
n = 3: Sextupole field

Cylindrical:

$$B_r = 3J_3 r^2 \cos 3\theta + 3K_3 r^2 \sin 3\theta$$
$$B_\theta = -3J_3 r^2 \sin 3\theta + 3K_3 r^2 \cos 3\theta$$
$$\phi = J_3 r^3 \cos 3\theta + K_3 r^3 \sin 3\theta$$

Cartesian:

$$B_x = 3(J_3(x^2 - y^2) + 2K_3 xy)$$
$$B_y = 3(K_2(x^2 - y^2) - 2J_3 xy)$$
$$\phi = J_3(x^3 - 3y^2 x) + K_3(3yx^2 - y^3)$$



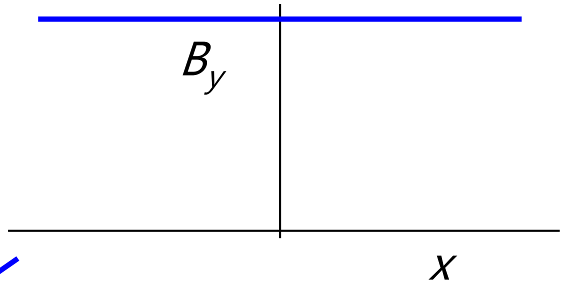
J_3 = 0 giving 'normal' or 'right' sextupole field.

Line of constant scalar potential

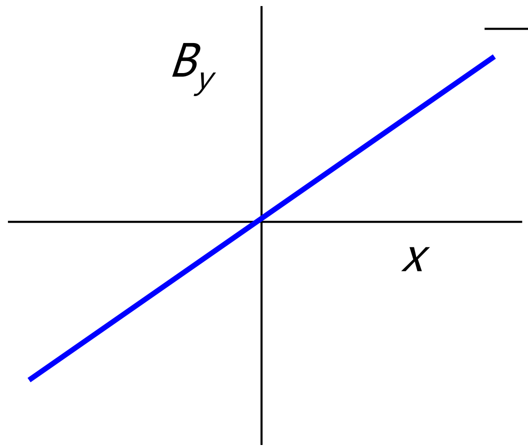
Lines of flux density

Summary: variation of B_y on x axis

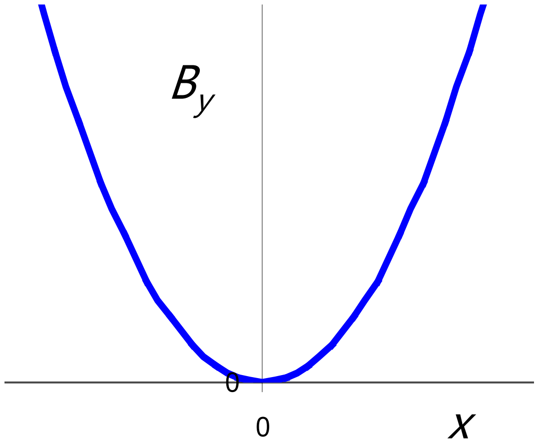
Dipole - constant field



Quad - linear variation



Sextupole - quadratic variation



Alternative notation (most lattice codes)

$$B(x) = B\rho \sum_{n=0}^{\infty} \frac{k_n x^n}{n!}$$

Magnet strengths are specified by the value of k_n (normalised to the beam rigidity)

order n of k is different to the 'standard' notation:

dipole is	$n = 0$
quad is	$n = 1$ etc.

k has units:

k_0 (dipole)	m^{-1}
k_1 (quadrupole)	m^{-2}

Significance of vector potential in 2D

We have: $$\boldsymbol{B} = \nabla \times \boldsymbol{A}$$ (**A** is vector potential)

and $$\nabla . \boldsymbol{A} = 0$$

Expanding: $$\boldsymbol{B} = \nabla \times \boldsymbol{A}$$

$$= \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\boldsymbol{i} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\boldsymbol{j} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\boldsymbol{k}$$

where ***i, j, k*** are unit vectors in *x, y, z.*

In 2 dimensions $B_z = 0$ and $\partial / \partial z = 0$

So $$A_x = A_y = 0$$

and $$\boldsymbol{B} = \frac{\partial A_z}{\partial y}\boldsymbol{i} - \frac{\partial A_z}{\partial x}\boldsymbol{j}$$

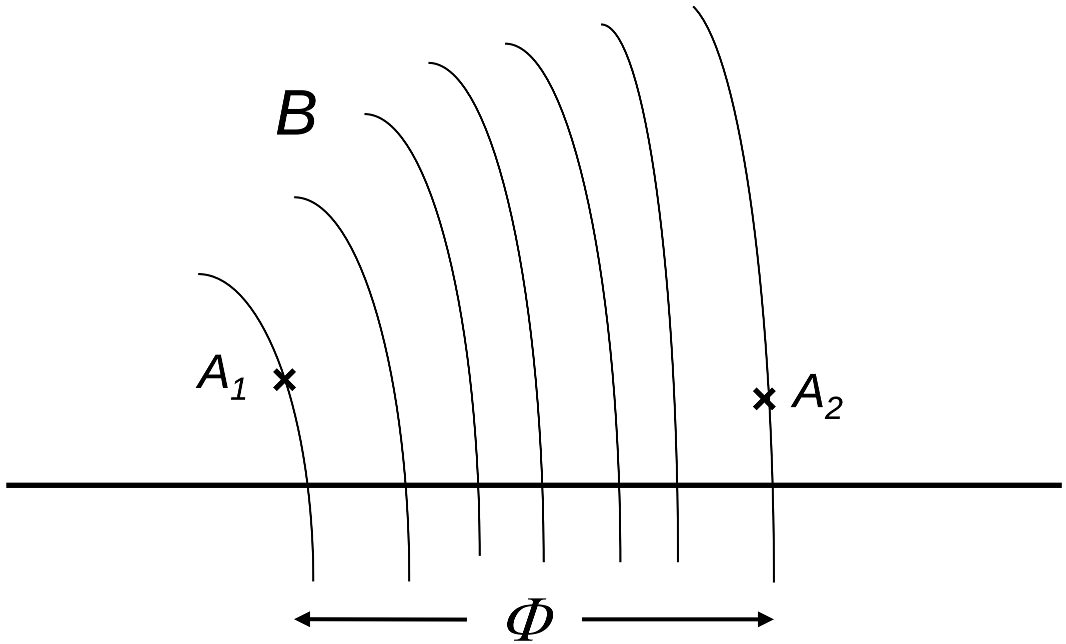
A is in the *z* direction, normal to the 2D problem.

Note: $$\nabla . \boldsymbol{B} = \frac{\partial^2 A_z}{\partial x \partial y} - \frac{\partial^2 A_z}{\partial x \partial y} = 0$$

Total flux between two points $\propto \Delta A$

In a two-dimensional problem the magnetic flux between two points is proportional to the difference between the vector potentials at those points.

$$\phi \propto (A_2 - A_1)$$



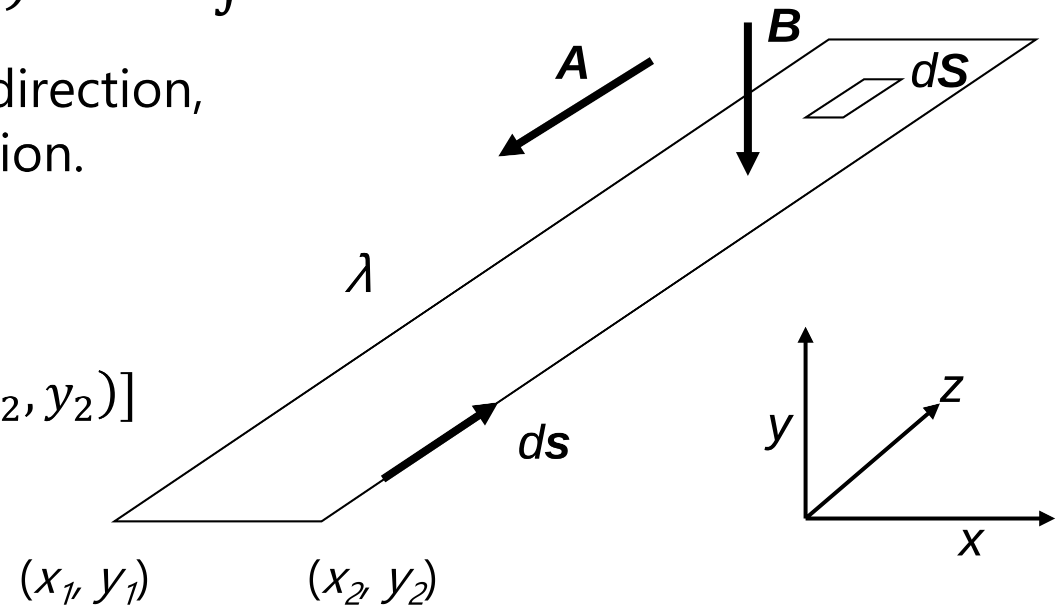
Proof on next slide.

Proof

Consider a rectangular closed path, length λ in z direction at (x_1, y_1) and (x_2, y_2); apply Stokes' theorem:

$$\phi = \iint \boldsymbol{B}.d\boldsymbol{S} = \iint (\boldsymbol{\nabla} \times \boldsymbol{A}).d\boldsymbol{S} = \oint \boldsymbol{A}.d\boldsymbol{s}$$

But $\boldsymbol{A}$ is exclusively in the z direction, and is constant in this direction.
So:



$$\int \boldsymbol{A}.d\boldsymbol{S} = \lambda[A(x_1, y_1) - A(x_2, y_2)]$$

$$\phi = \lambda[A(x_1, y_1) - A(x_2, y_2)]$$

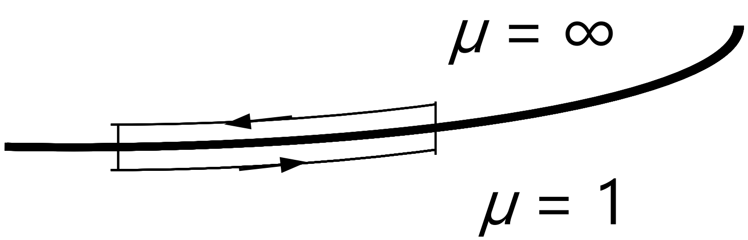
Going from fields to magnets

STEEL POLES AND YOKES

What is the perfect pole shape?

What is the ideal pole shape?

- Flux is normal to a ferromagnetic surface with infinite μ:



curl $H = 0$

therefore $\int H.ds = 0$

in steel $H = 0$

therefore parallel H air = 0

therefore B is normal to surface.

- Flux is normal to lines of scalar potential: $\boldsymbol{B} = -\nabla\phi$
- So the lines of scalar potential are the perfect pole shapes!

(but these are infinitely long!)

Equations for the ideal pole

Equations for Ideal (infinite) poles;

($J_n = 0$) for **normal** (ie not skew) fields:

Dipole:

$$y = \pm \frac{g}{2}$$

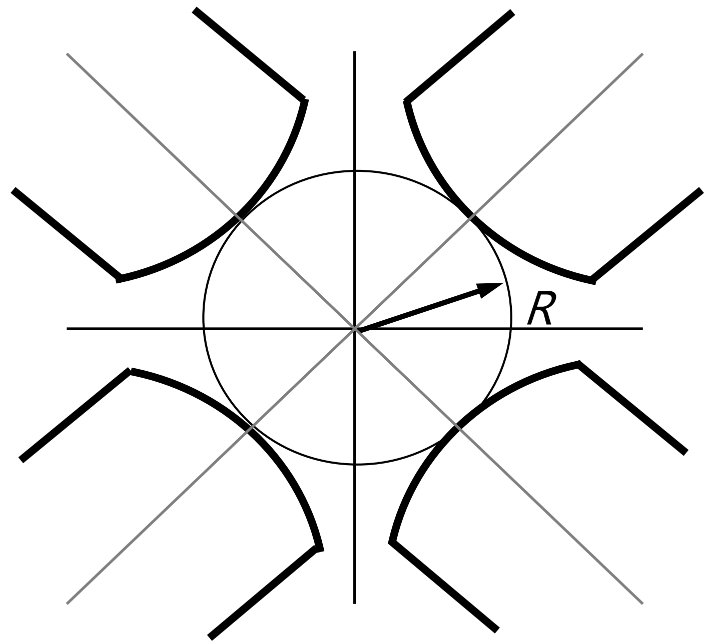
(g is interpole gap)

Quadrupole:

$$xy = \pm \frac{R^2}{2}$$

Sextupole:

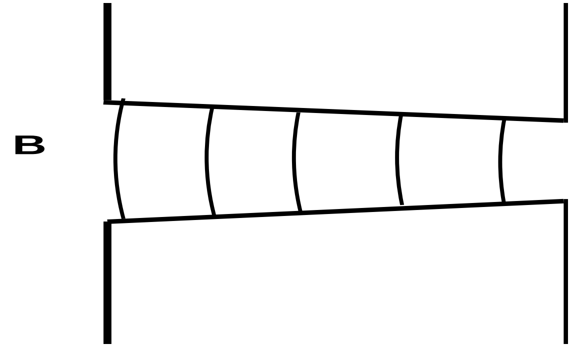
$$3x^2y - y^3 = \pm R^3$$



Combined function (CF) magnets

'Combined Function Magnets' - often dipole and quadrupole field combined (but see next-but-one slide):

A quadrupole magnet with physical centre shifted from magnetic centre.



Characterised by 'field index' *n*, positive or negative depending on direction of gradient - do not confuse with harmonic *n*!

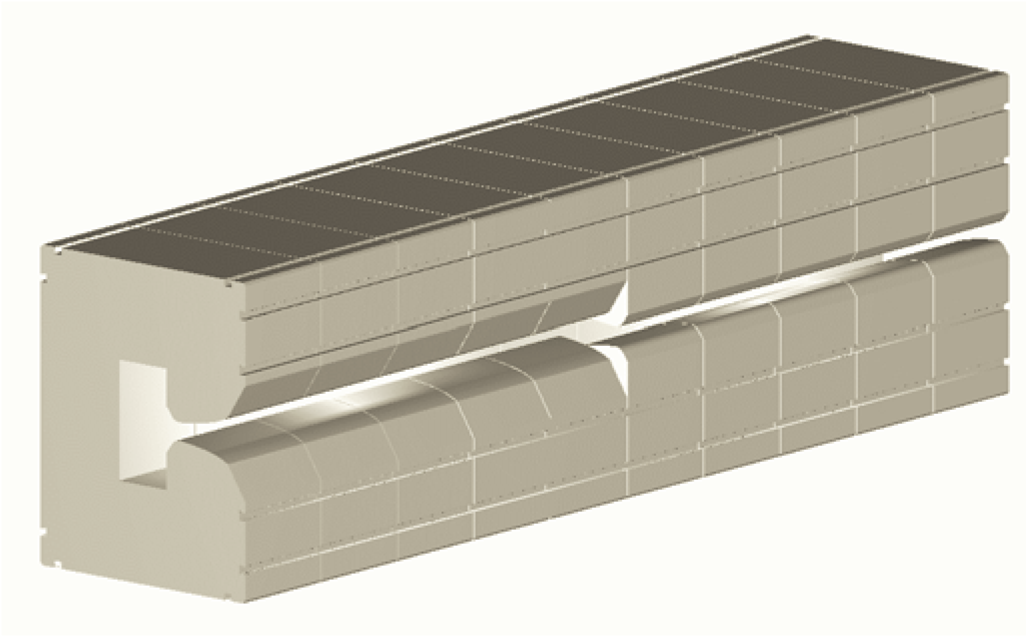
$$n = -\left(\frac{\rho}{B_0}\right)\frac{\partial B}{\partial x}$$

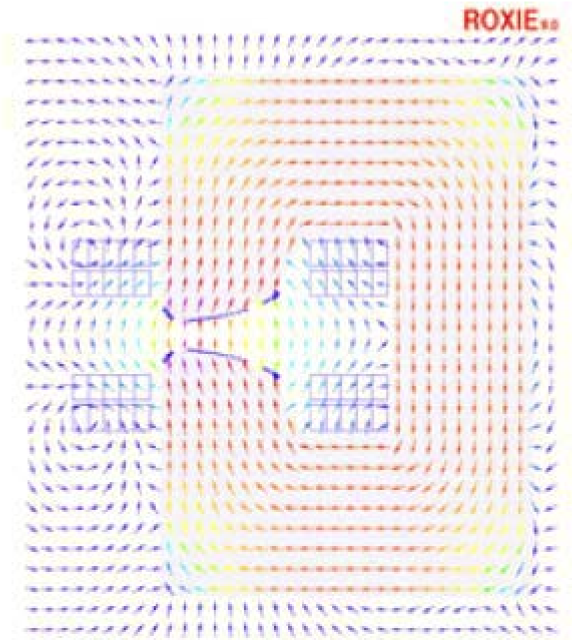
ρ is radius of curvature of the beam

B_o is central dipole field

Combined Function Magnets

CERN Proton Synchrotron

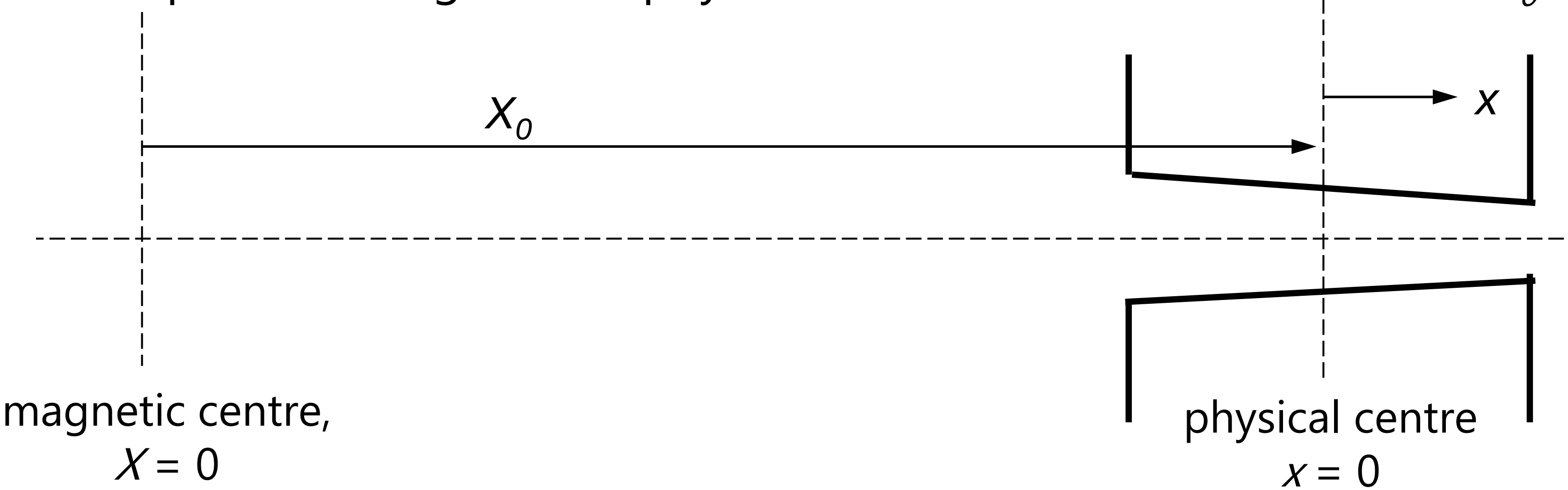




Combined function geometry

Combined function (dipole & quadrupole) magnet:

- beam is at physical centre
- flux density at beam = B_0
- gradient at beam = $\frac{\delta B}{\delta x}$
- magnetic centre is at B and X = 0
- separation magnetic to physical centre = X_0



Pole of a CF dipole & quad magnet

Ref geometry as on previous slide:

Flux density at beam centre: B_0 [T]

Gradient across beam: g [T/m]

Displacement of beam from quadrupole centre: X_0

'Local' displacement from beam centre: x

So: $B_0 = g\, X_0$

$$X_0 = \frac{B_0}{g}$$

And $X = x + X_0$

Quadrupole equation: $Xy = \frac{R^2}{2}$

So pole equation ref beam centre: $y = \frac{R^2}{2} \frac{1}{x+X_0}$

Adjust R to satisfy beam dimensions.

Other combined function magnets

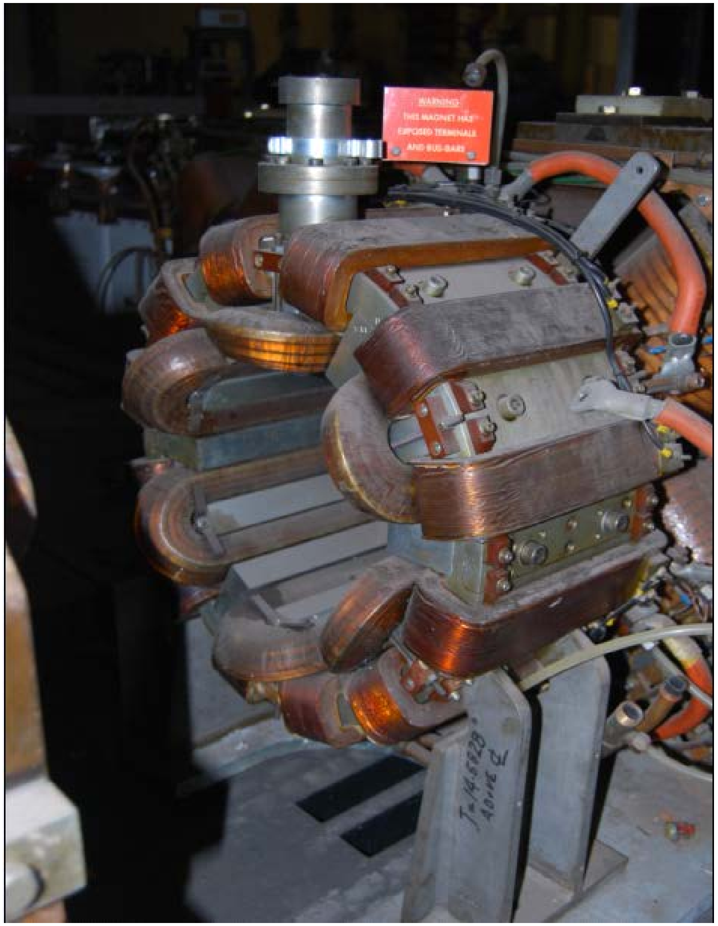
Other combinations:

- dipole, quadrupole and sextupole
- dipole & sextupole (for chromaticity control)
- dipole, skew quad, sextupole, octupole (at DL)

Generated by

- pole shapes given by sum of correct scalar potentials
    - amplitudes built into pole geometry – not variable
- multiple coils mounted on the yoke
    - amplitudes independently varied by coil currents

The SRS multipole magnet



Could develop:

- vertical dipole
- horizontal dipole
- upright quad
- skew quad
- sextupole
- octupole
- others

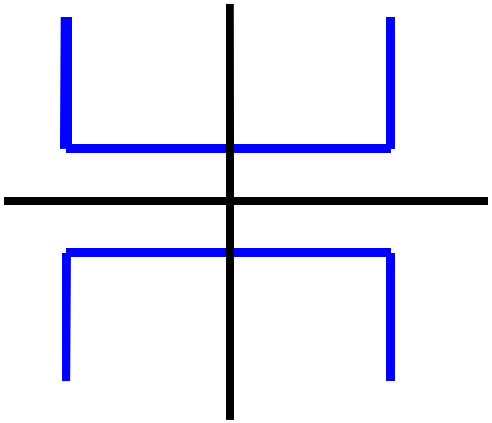
The Practical Pole

Practically, poles are finite, **introducing errors**;

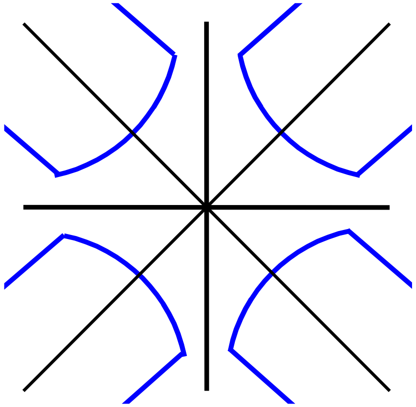
these appear as higher harmonics which degrade the field distribution.

However, the iron geometries have certain symmetries that **restrict** the nature of these errors.

Dipole:



Quadrupole:



Possible symmetries

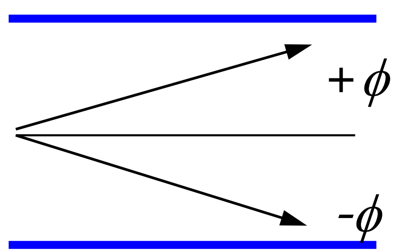
Lines of symmetry:

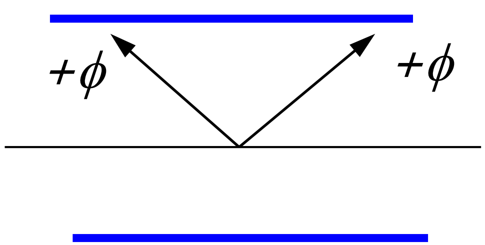
	Dipole	Quad
Pole orientation determines whether pole is normal or skew.	$y = 0$	$x = 0$ $y = 0$
Additional symmetry imposed by pole edges.	$x = 0$	$y = \pm x$

The additional constraints imposed by the symmetrical pole edges limits the values of n that have non-zero coefficients

Dipole symmetries

Type	Symmetry	Constraint
Pole orientation	$\phi(\theta) = -\phi(-\theta)$	all $J_n = 0$
Pole edges	$\phi(\theta) = \phi(\pi - \theta)$	K_n non-zero only for $n = 1, 3, 5$, etc.





So, for a fully symmetric dipole, only 6, 10, 14 etc. pole errors can be present.

Quadrupole symmetries

Type	**Symmetry**	**Constraint**
Pole orientation	$\phi(\theta) = -\phi(-\theta)$	All $J_n = 0$
	$\phi(\theta) = -\phi(\pi - \theta)$	$K_n = 0$ for all odd n
Pole edges	$\phi(\theta) = \phi(\frac{\pi}{2} - \theta)$	K_n non-zero for $n = 2, 6, 10$, etc.

So, for a fully symmetric quadrupole, only 12, 20, 28 etc. pole errors can be present.

Sextupole symmetries

Type	Symmetry	Constraint
Pole orientation	$\phi(\theta) = -\phi(-\theta)$ $\phi(\theta) = -\phi(\frac{2\pi}{3} - \theta)$ $\phi(\theta) = -\phi(\frac{4\pi}{3} - \theta)$	All $J_n = 0$ $K_n = 0$ where *n is* **not** a multiple of 3
Pole edges	$\phi(\theta) = \phi(\frac{\pi}{3} - \theta)$	K_n non-zero only for $n = 3, 9, 15$, etc.

So, for a fully symmetric sextupole, only 18, 30, 42 etc. pole errors can be present.

Summary - ‘Allowed’ Harmonics

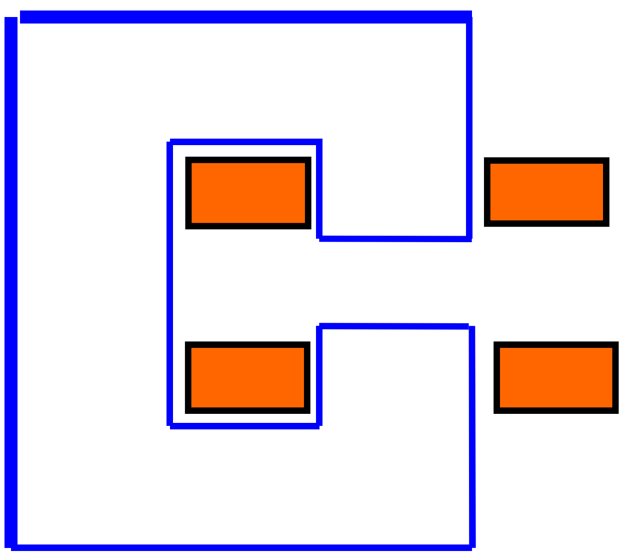
Summary of ‘allowed harmonics’ in **fully symmetric** magnets:

Fundamental geometry	**‘Allowed’ harmonics**
Dipole, $n = 1$	$n = 3, 5, 7, \ldots\ldots$ (6 pole, 10 pole, etc.)
Quadrupole, $n = 2$	$n = 6, 10, 14, \ldots.$ (12 pole, 20 pole, etc.)
Sextupole, $n = 3$	$n = 9, 15, 21, \ldots$ (18 pole, 30 pole, etc.)
Octupole, $n = 4$	$n = 12, 20, 28, \ldots.$ (24 pole, 40 pole, etc.)

Asymmetries generating harmonics (i)

Two sources of asymmetry generate 'forbidden' harmonics:

i) magnetic asymmetries - significant at low permeability:

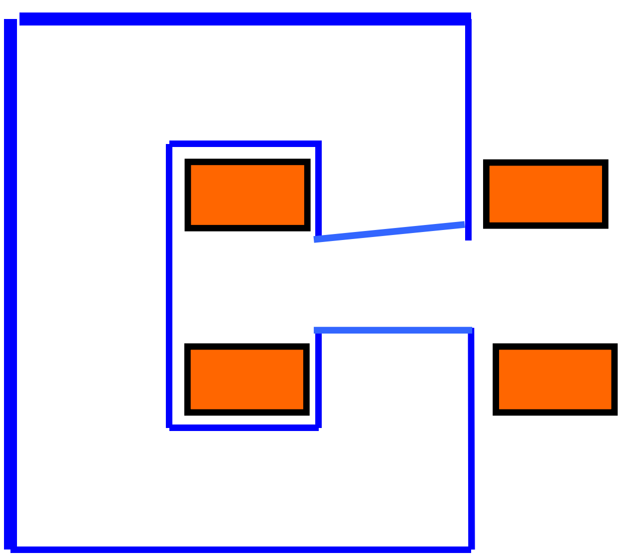


e.g. C core dipole not completely symmetrical about pole centre, but negligible effect with high permeability.

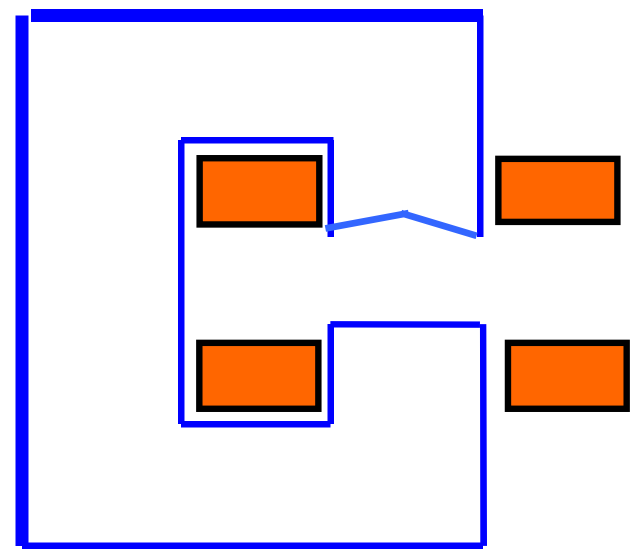
Generates n = 2,4,6, etc.

Asymmetries generating harmonics (ii)

ii) asymmetries due to small manufacturing errors in dipoles:



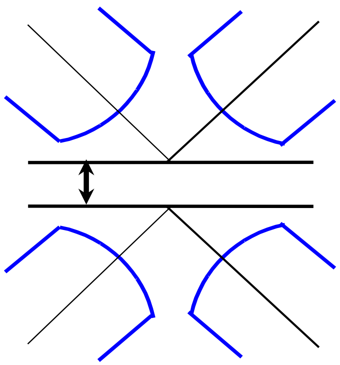
n = 2, 4, 6 etc.



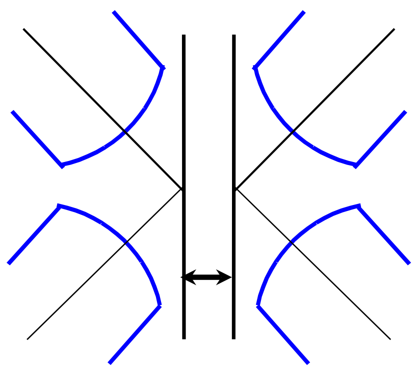
n = 3, 6, 9, etc.

Asymmetries generating harmonics (iii)

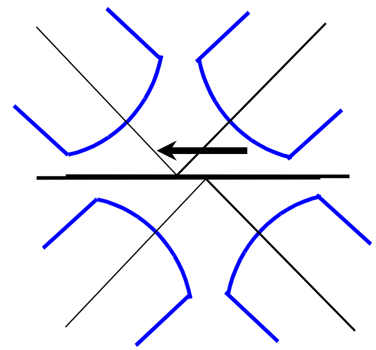
ii) asymmetries due to small manufacturing errors in quadrupoles:



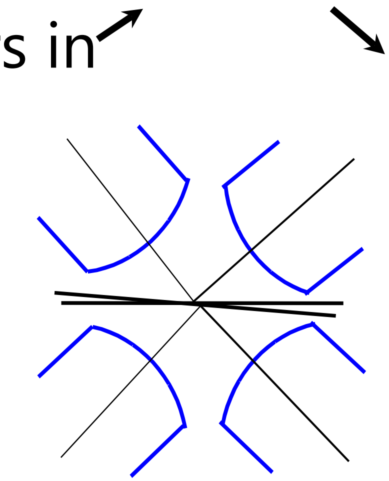
n = 4 -ve



n = 4 +ve



n = 3



n = 2 (skew)

n = 3

These errors are bigger than the finite μ type; can seriously affect machine behaviour and must be controlled.

Current Affairs

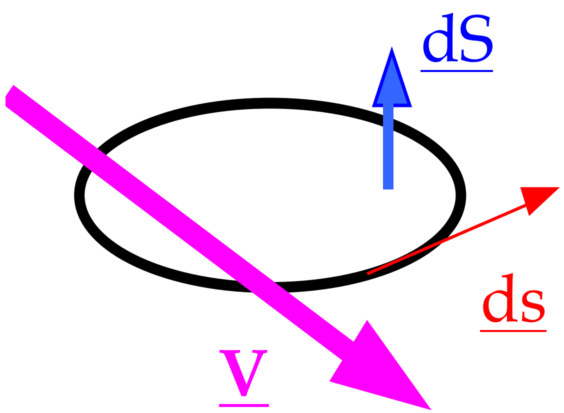
FIELDS DUE TO COILS

Introduction of currents

Now for $\boldsymbol{j} \neq 0$ $\nabla \times \boldsymbol{H} = \boldsymbol{j}$

To expand, use Stoke's Theorem for any vector $\boldsymbol{V}$ and a closed curve $\boldsymbol{s}$:

$$\int \boldsymbol{V}.d\boldsymbol{s} = \iint \boldsymbol{\nabla} \times \boldsymbol{V}.d\boldsymbol{S}$$



Apply this to: $\nabla \times \boldsymbol{H} = \boldsymbol{j}$

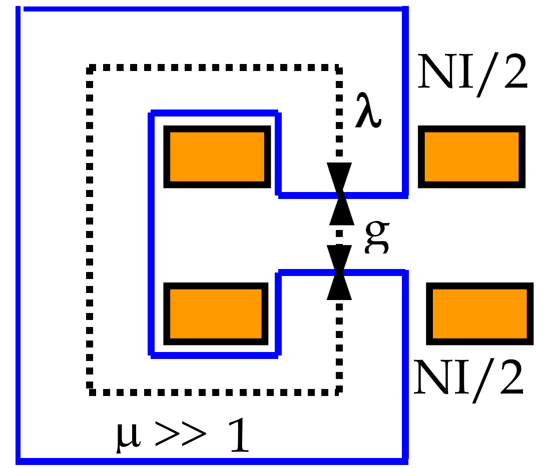
then in a magnetic circuit: (Ampere's equation)

$$\boxed{\int \boldsymbol{H}.d\boldsymbol{s} = NI}$$

NI (Ampere-turns) is total current cutting S

Excitation current in a dipole

B is approximately constant round the loop made up of λ and g, (but see below);



But in iron, $\mu \gg 1$

and $H_{iron} = \frac{H_{air}}{\mu}$

So $$\boxed{B_{air} = \frac{\mu_0 NI}{g + \frac{\lambda}{\mu}}}$$

g, and λ/μ are the 'reluctance' of the gap and iron.

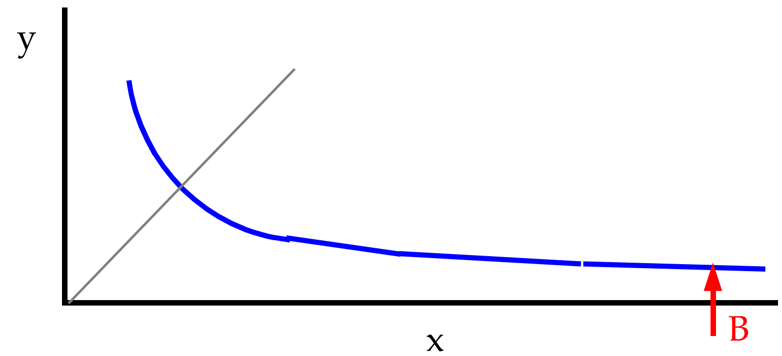
Approximation ignoring iron reluctance ($\frac{\lambda}{\mu} \ll g$):

$$\boxed{NI = -\frac{Bg}{\mu_0}}$$

Excitation current in quad & sextupole

For quadrupoles and sextupoles, the required excitation can be calculated by considering fields and gap at large *x*.

For example: **Quadrupole:**



Pole equation: $xy = \frac{R^2}{2}$

On *x* axes $B_y = gx$

where *g* is gradient in T/m

At large *x* (to give vertical lines of *B*):

$$NI = gx\frac{R^2}{2x\mu_0}$$

i.e. $NI = \frac{gR^2}{2\,\mu_0}$ (per pole)

The same method for a **Sextupole** (coefficient g_S) gives:

$NI = \frac{g_S R^3}{3\mu_0}$ (per pole)

General solution for magnets order *n*

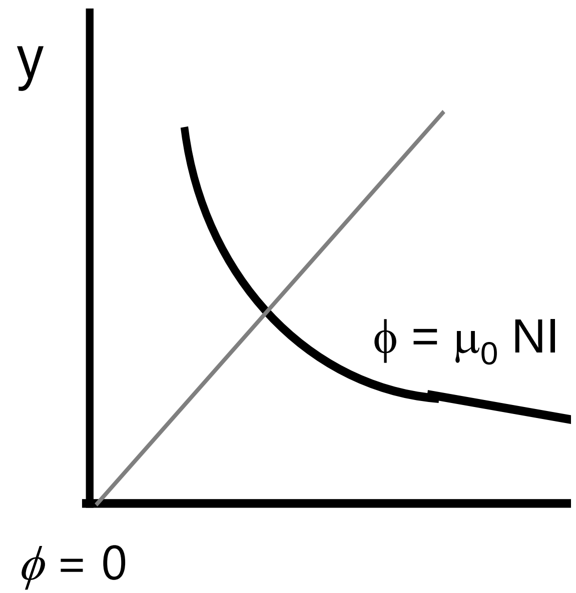
In air (remote currents!) $\boldsymbol{B} = \mu_0 \boldsymbol{H}$

$\boldsymbol{B} = -\nabla\phi$

Integrating over a limited path

(not circular) in air: $NI = \frac{\phi_1 - \phi_2}{\mu_0}$

ϕ_1, ϕ_2 are the scalar potentials at two points in air.
Define $\phi = 0$ at magnet centre;
then potential at the pole is: $\mu_0 NI$



Apply the general equations for magnetic field harmonic order *n* for non-skew magnets (all $J_n = 0$) giving: $NI = \frac{1}{n} \frac{1}{\mu_0} \frac{B_r}{R^{n-1}} R^n$

Where:

NI is excitation per pole

R is the inscribed radius (or half gap in a dipole)

$\frac{B_r}{R^{n-1}}$ is magnet strength in T/m$^{(n-1)}$

Further Reading

- CERN Accelerator School on Magnets
  Bruges, Belgium; June 2009
  https://arxiv.org/html/1105.5069v1
- United States Particle Accelerator School
  Magnet and RF Cavity Design, January 2016
  http://uspas.fnal.gov/materials/16Austin/austin-magnets.shtml
- J.D. Jackson, *Classical Electrodynamics*
- J.T. Tanabe, *Iron Dominated Electromagnets*