Free Electron Lasers

Lecture III

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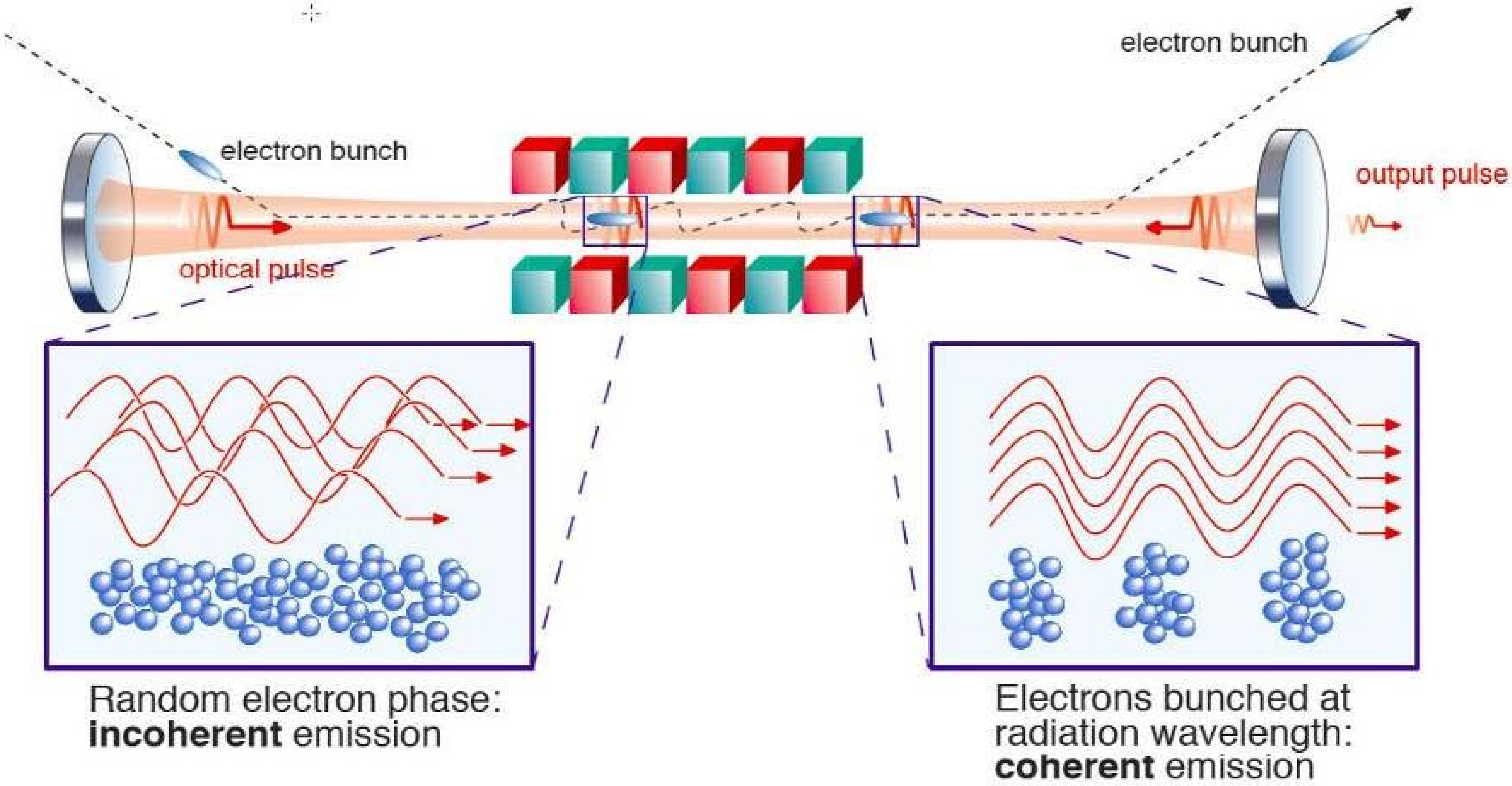
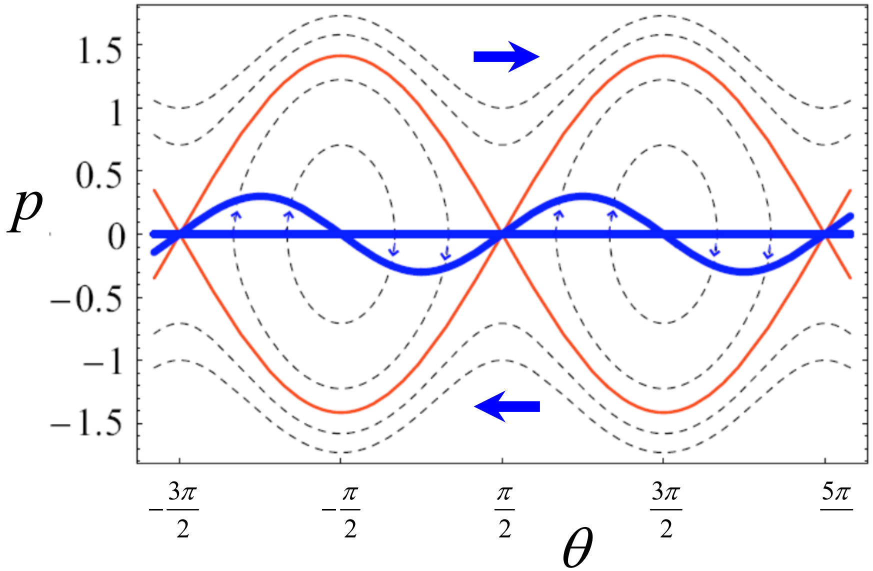
Low Gain mechanism

$$\bar{z} \lesssim 1$$

B.W.J. McNEIL
A SIMPLE MODEL OF THE FREE-ELECTRON-LASER OSCILLATOR FROM LOW INTO HIGH GAIN
Nuclear Instruments and Methods in Physics Research A296 (1990) 388–393

Low Gain – needs cavity feedback





Here we consider the low gain limit with $\bar{z} = 0.24$

The phase-space representation of opposite will be used to look at what happens with the electrons and radiation during the interaction.

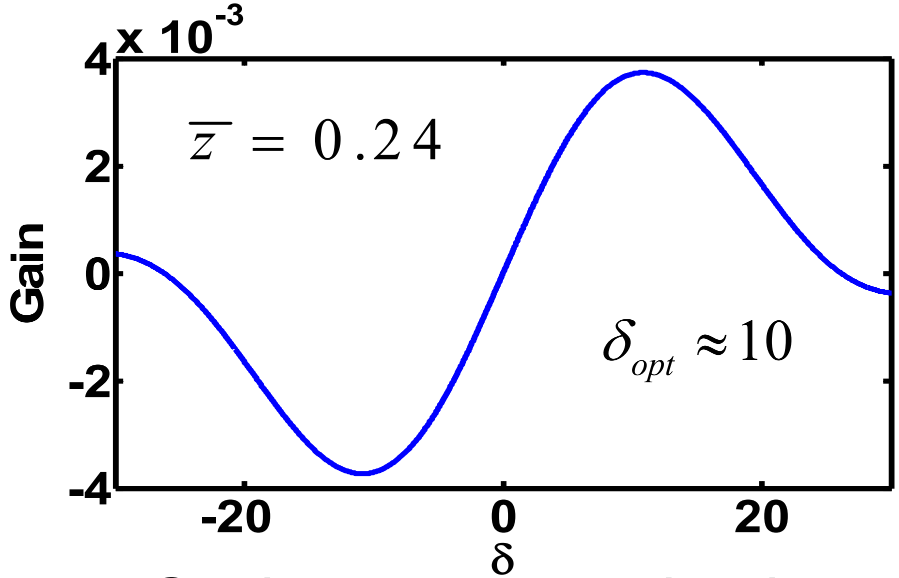
$$\text{Gain} = 4\bar{z}^3 \frac{1}{\Delta^3}\left(1-\cos(\Delta)-\frac{\Delta}{2}\sin(\Delta)\right)$$

where $\Delta = \delta\bar{z}$ and $\delta \equiv p(\bar{z}=0)$,

assumed a constant for all electrons.

The Gain is a maximum for $\Delta \approx 2.6$

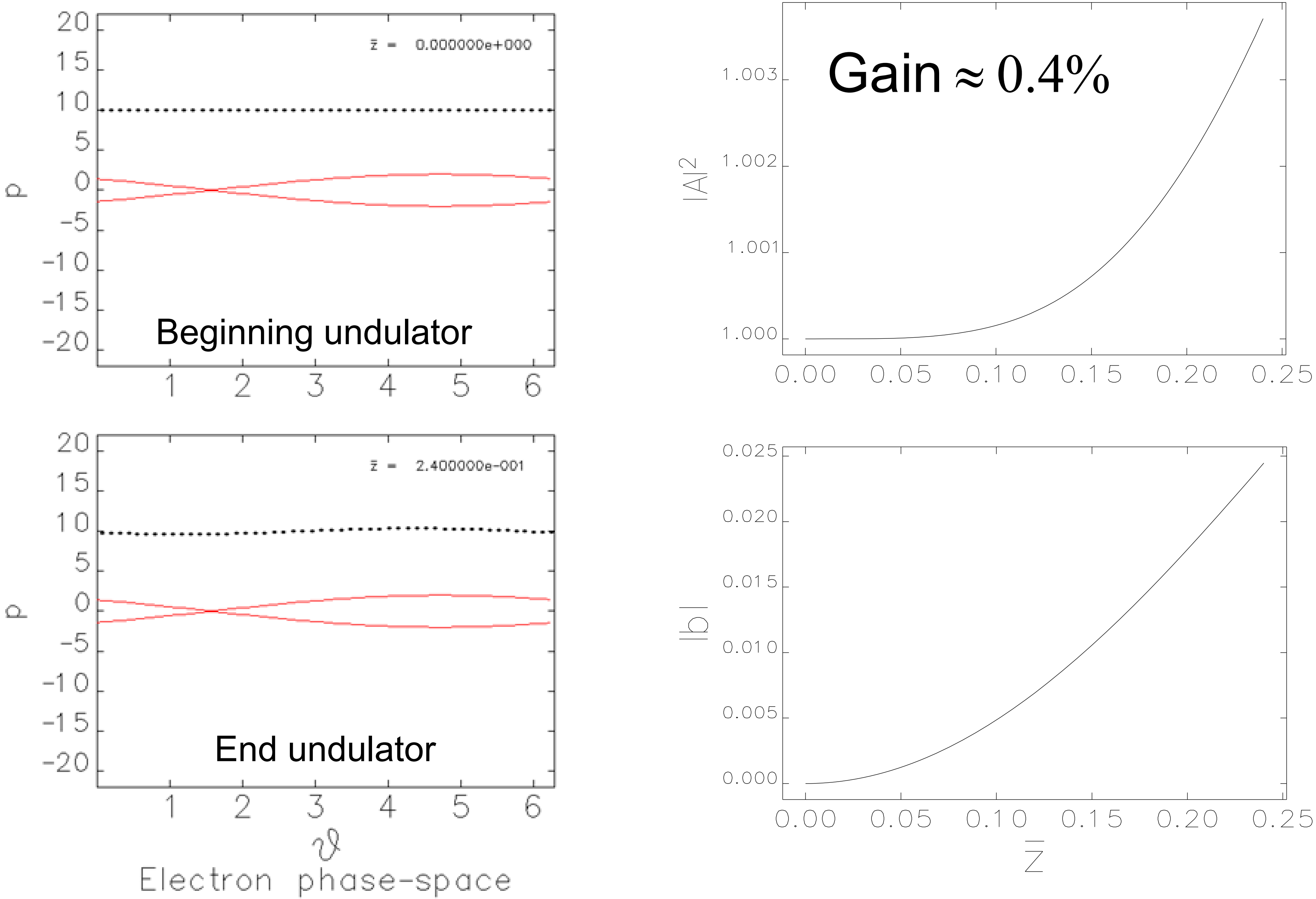
$\Rightarrow \quad \delta_{opt} \approx \dfrac{2.6}{\bar{z}}$ in the low-gain limit.



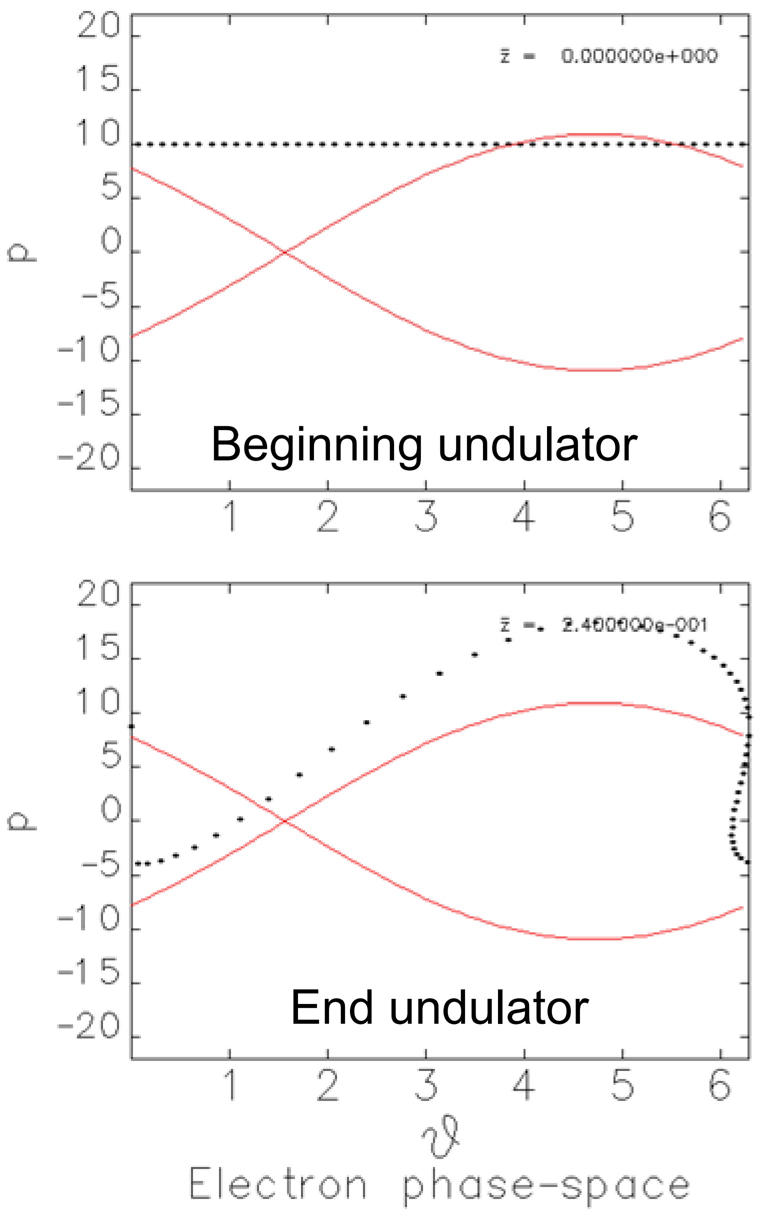
Oscillatory terms dominate

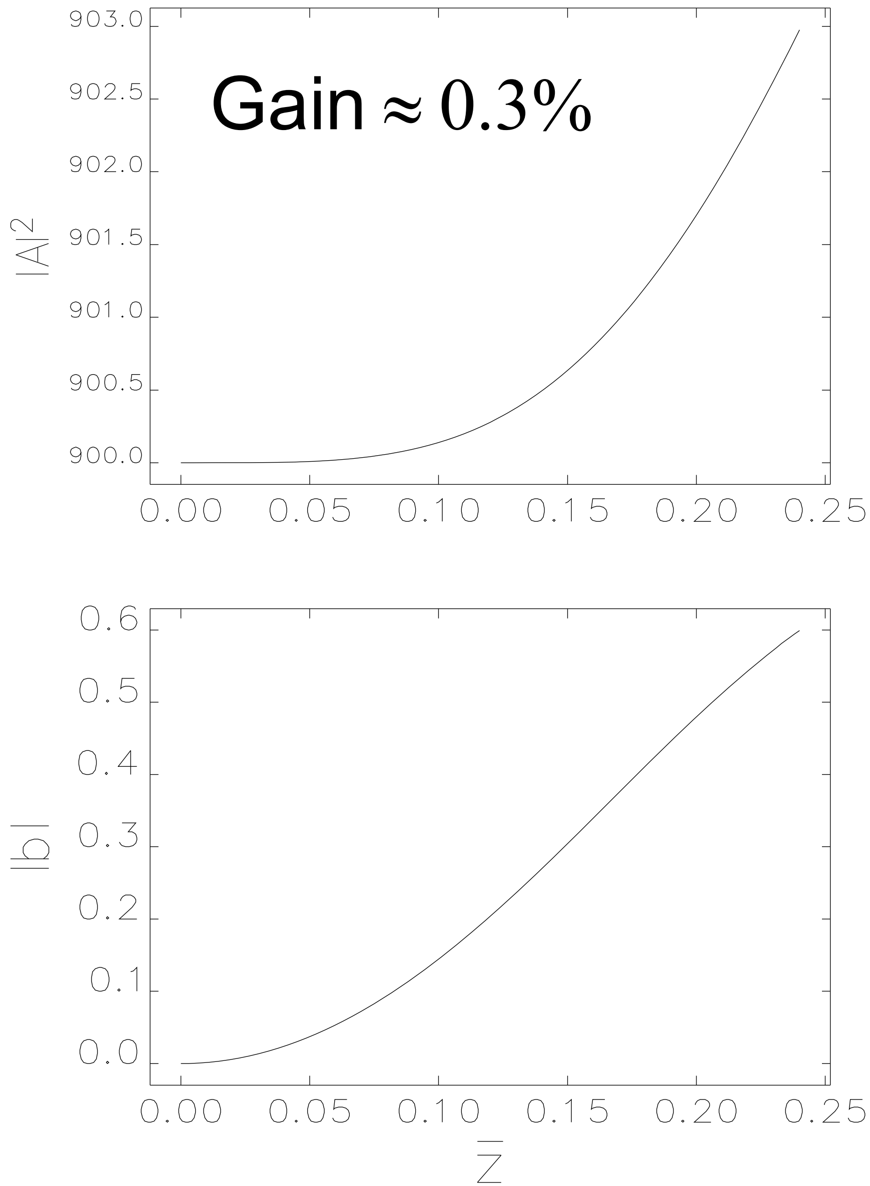
FEL saturates when Gain=Cavity Losses

Low Gain – early stages

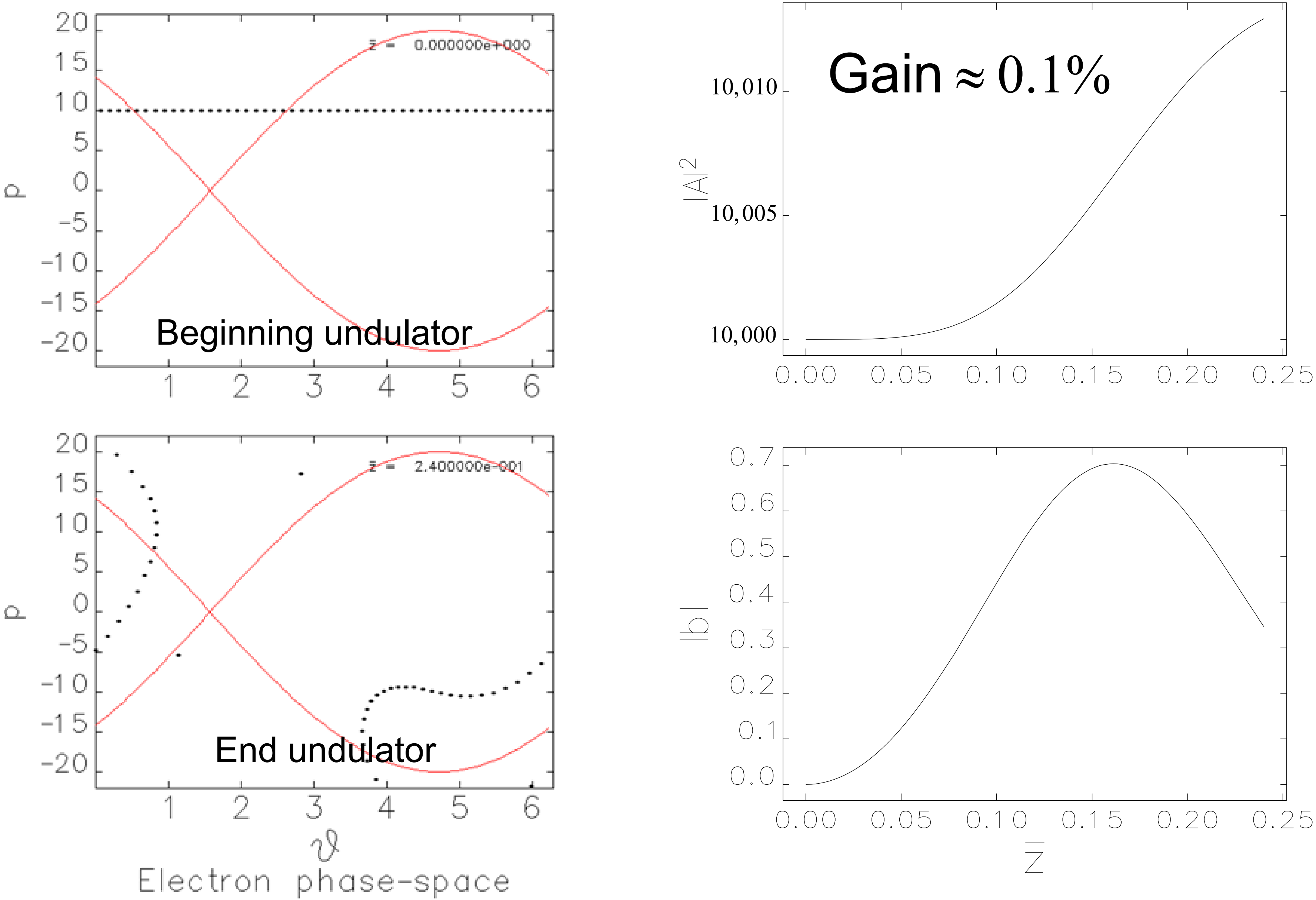


Low Gain – intermediate





Low Gain – saturated



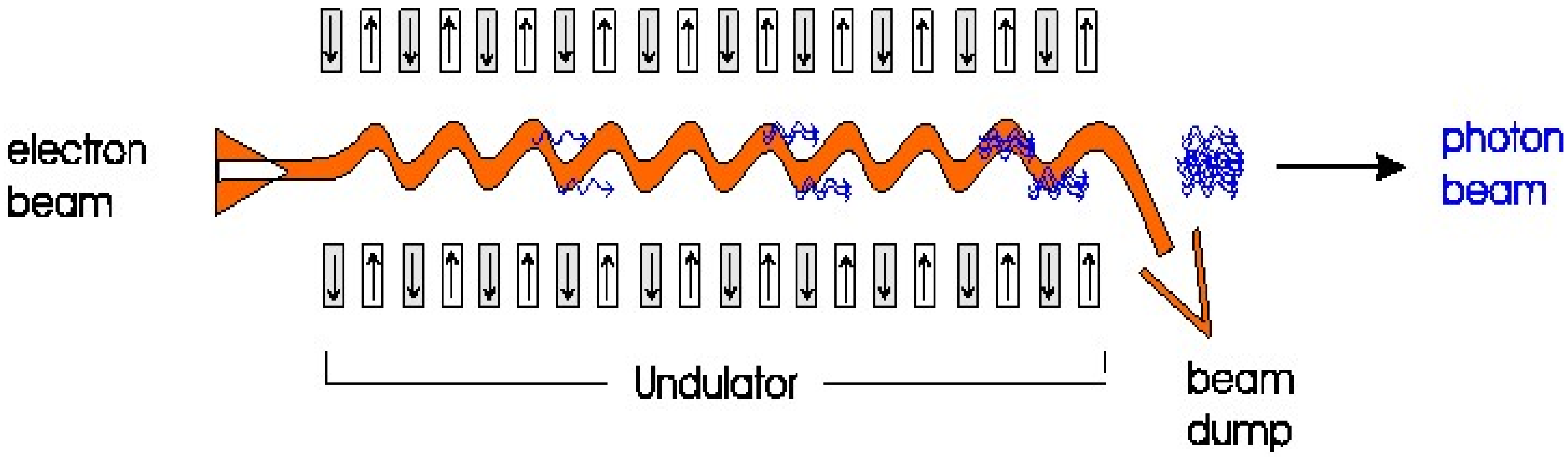
High Gain mechanism

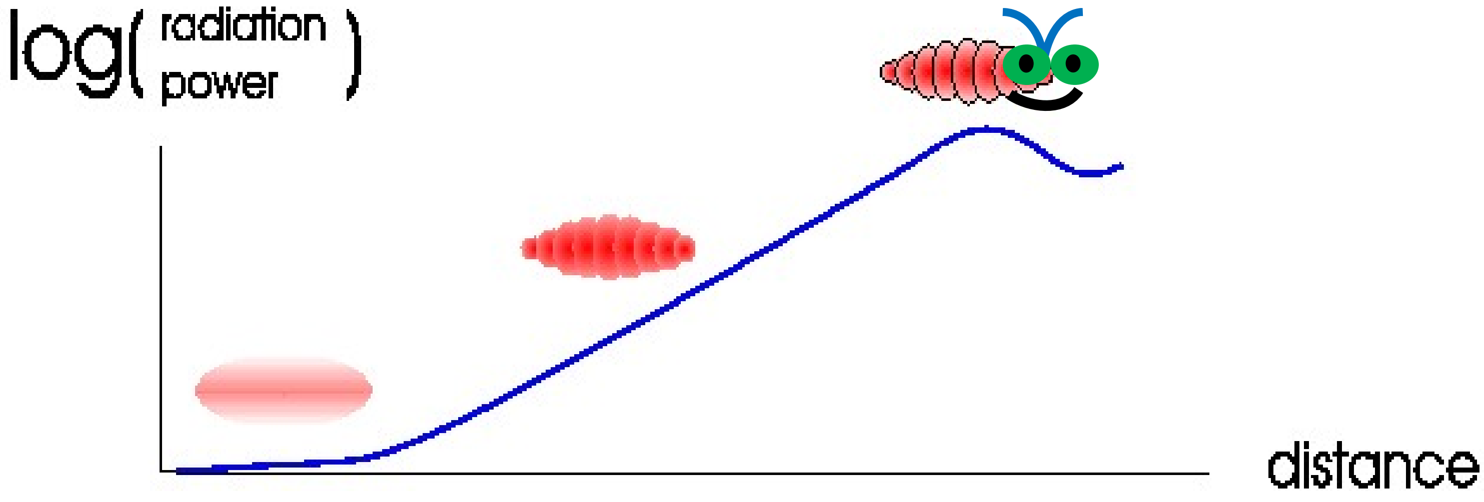
$$\bar{z} > 1$$

$$A(\bar{z}) \approx \frac{A_0}{3} e^{\frac{\sqrt{3}}{2}\bar{z}} = \frac{A_0}{3} e^{\frac{\sqrt{3}}{2}\frac{z}{l_g}}$$

$$l_g = \lambda_w / 4\pi\rho$$

Single pass high-gain amplifier





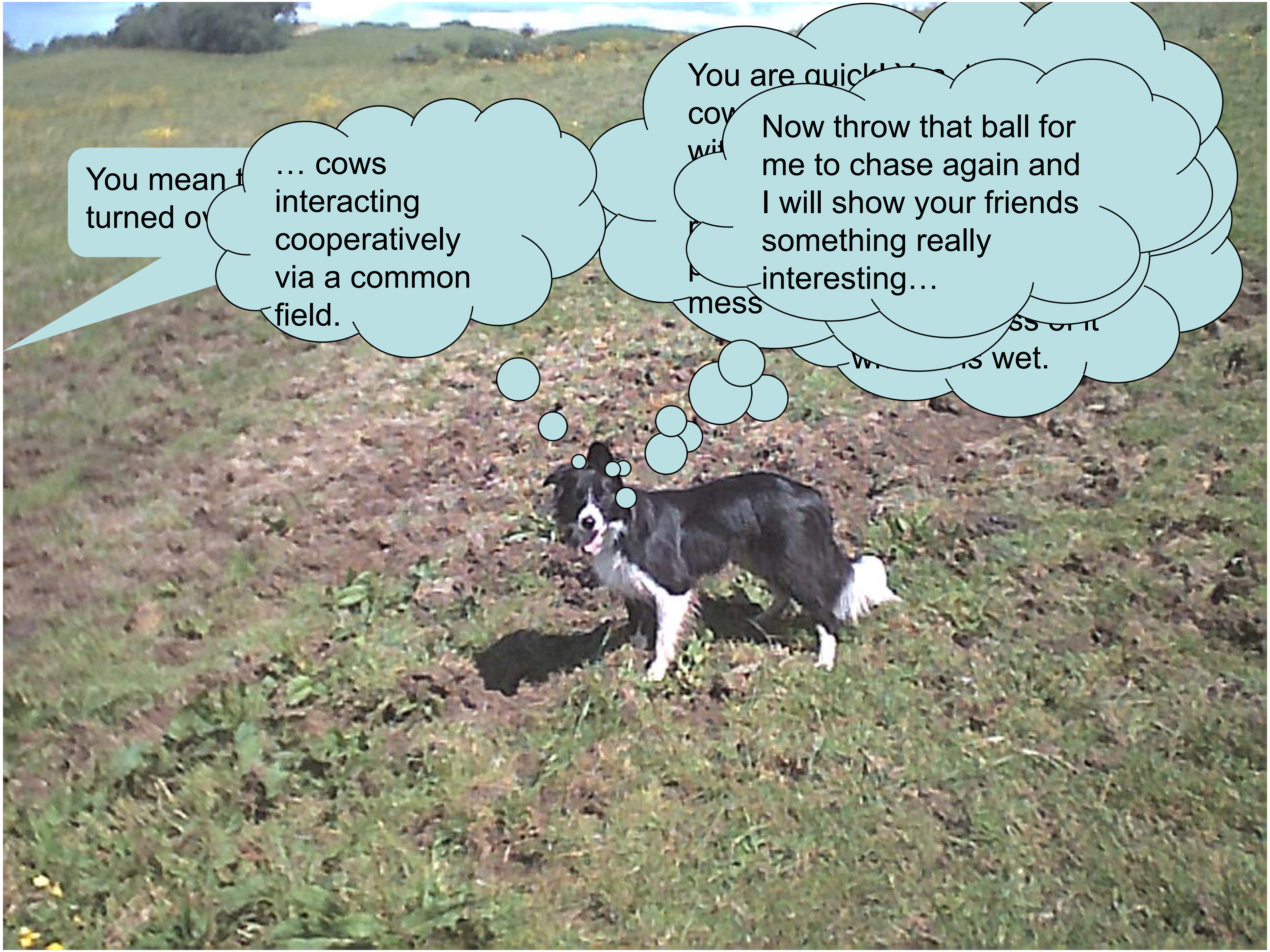
Dougal the Wonder-dog…

…with Hamish and his herd of hairy friends explains …

‘Cooperative (or collective) phenomena in systems of coupled particles’


If you throw my ball for me in the field, I'll show you what I mean!
OK, lets go!
My friends the cows know a thing or two about that

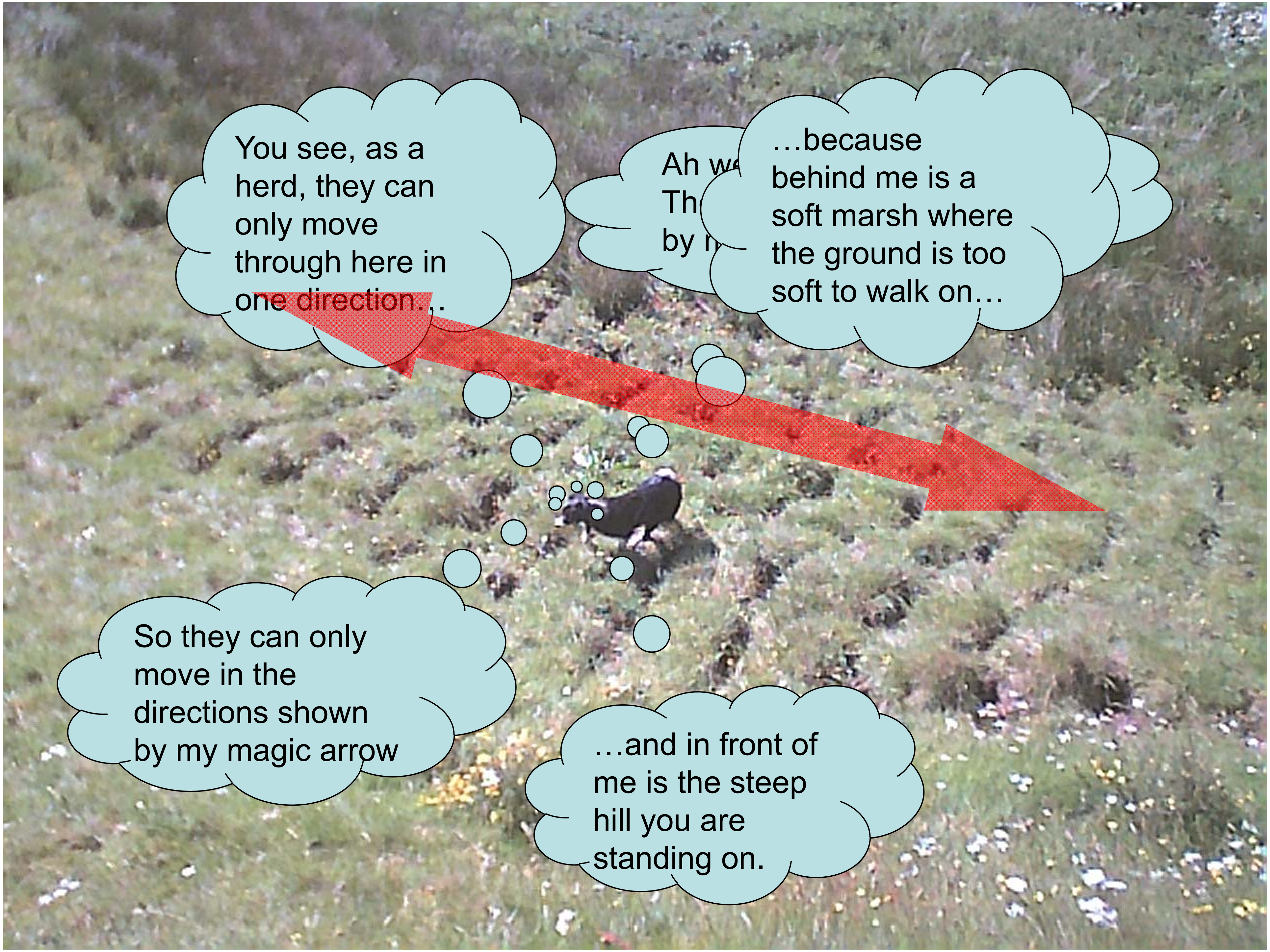

Dougal, you come here! Don’t you dare run away!
An electromagnetic field that is!
Look at my friends the cows in the distance. They behave just like particles in a field.


You mean
turned
… cows interacting cooperatively via a common field.
You are
mess
wet.
Now throw that ball for me to chase again and I will show your friends something really interesting…


Come on Dougal! Hurry up!
I’m coming,
I’m coming!


Right!! That's enough of that! What are you going to show us?
Phew!...Tha
for an old m
Take this ball from me and we will go over to that small hill about 50m behind me.

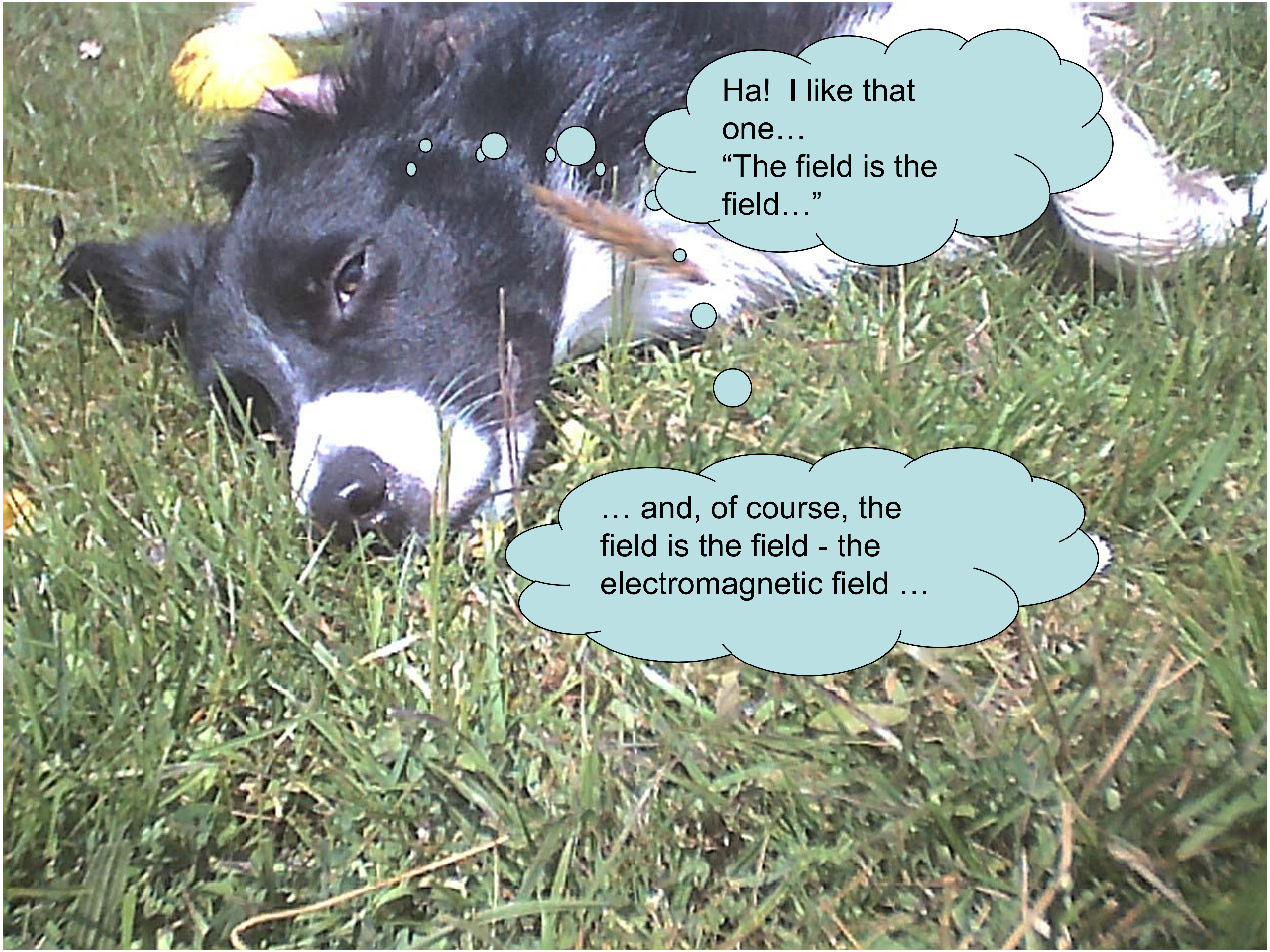

What on earth
What are all these strange ridges in the field?

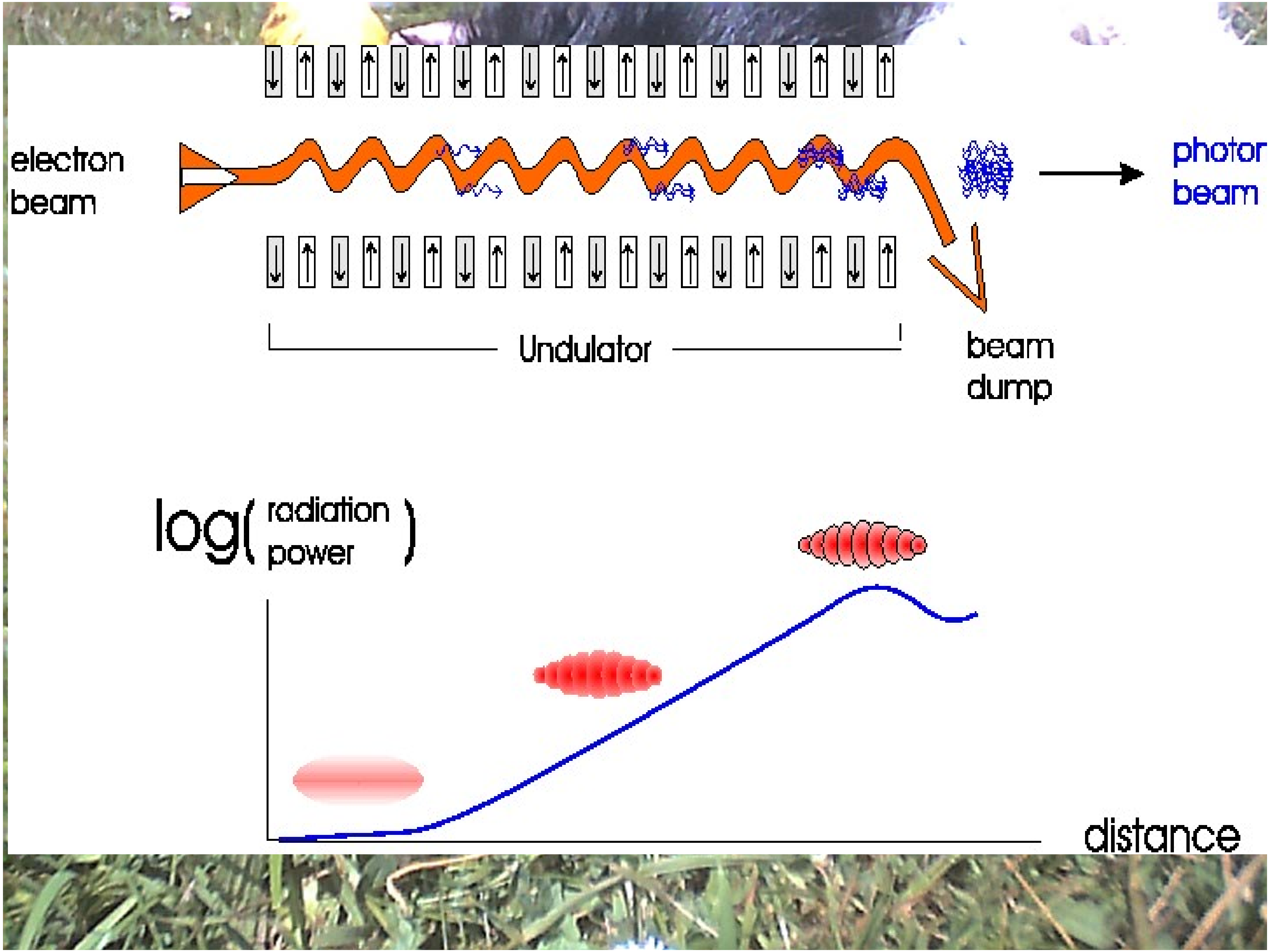

You see, as a herd, they can only move through here in one direction…
…because behind me is a soft marsh where the ground is too soft to walk on…
…and in front of me is the steep hill you are standing on.
So they can only move in the directions shown by my magic arrow


So the first cow, in placing its hoof in
the same, and
third cow even more likely, and the fourth cow…
So what has that got to do with the ridges in the field?
…and made a small
And look…
When walking, the cows tend to place a hoof on lower ground, where another cow has placed it's before…
Yes, yes, you got it! It is a positive feedback process. The cows behave *collectively* in where they place their hooves – they communicate via the common field!


Even those with different step sizes are forced to take the same step size when walking through here…
…look how evenly spaced the ridges are – this is the average step size of my friends, the cows.
…they all fall into step!


So, if you imagine my friends hooves are electrons…


Ha! I like that one… "The field is the field…"
… and, of course, the field is the field - the electromagnetic field …



electron beam
Undulator
beam dump
photon beam
log(radiation power)
distance

Dougal is a bit older now…

…but he does have a new friend: Donald!

Can assume periodic BC over one potential well:

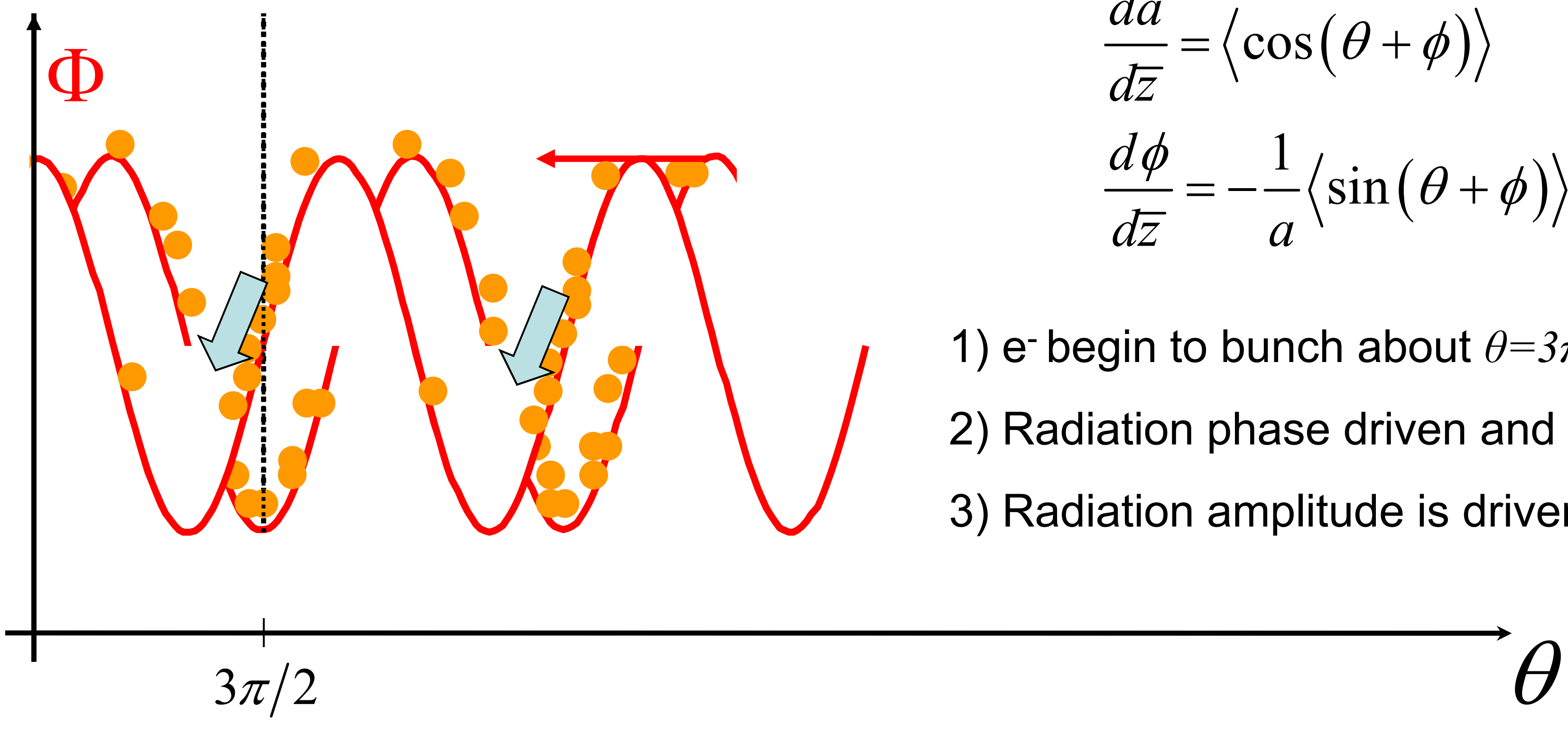
Letting: $A(\bar{z}) = a(\bar{z})e^{i\phi(\bar{z})} \Rightarrow$

$$\frac{d\theta_j}{d\bar{z}} = p_j$$

$$\frac{dp_j}{d\bar{z}} = -2a\cos\left(\theta_j + \phi\right)$$

$$\frac{da}{d\bar{z}} = \left\langle \cos\left(\theta + \phi\right) \right\rangle$$

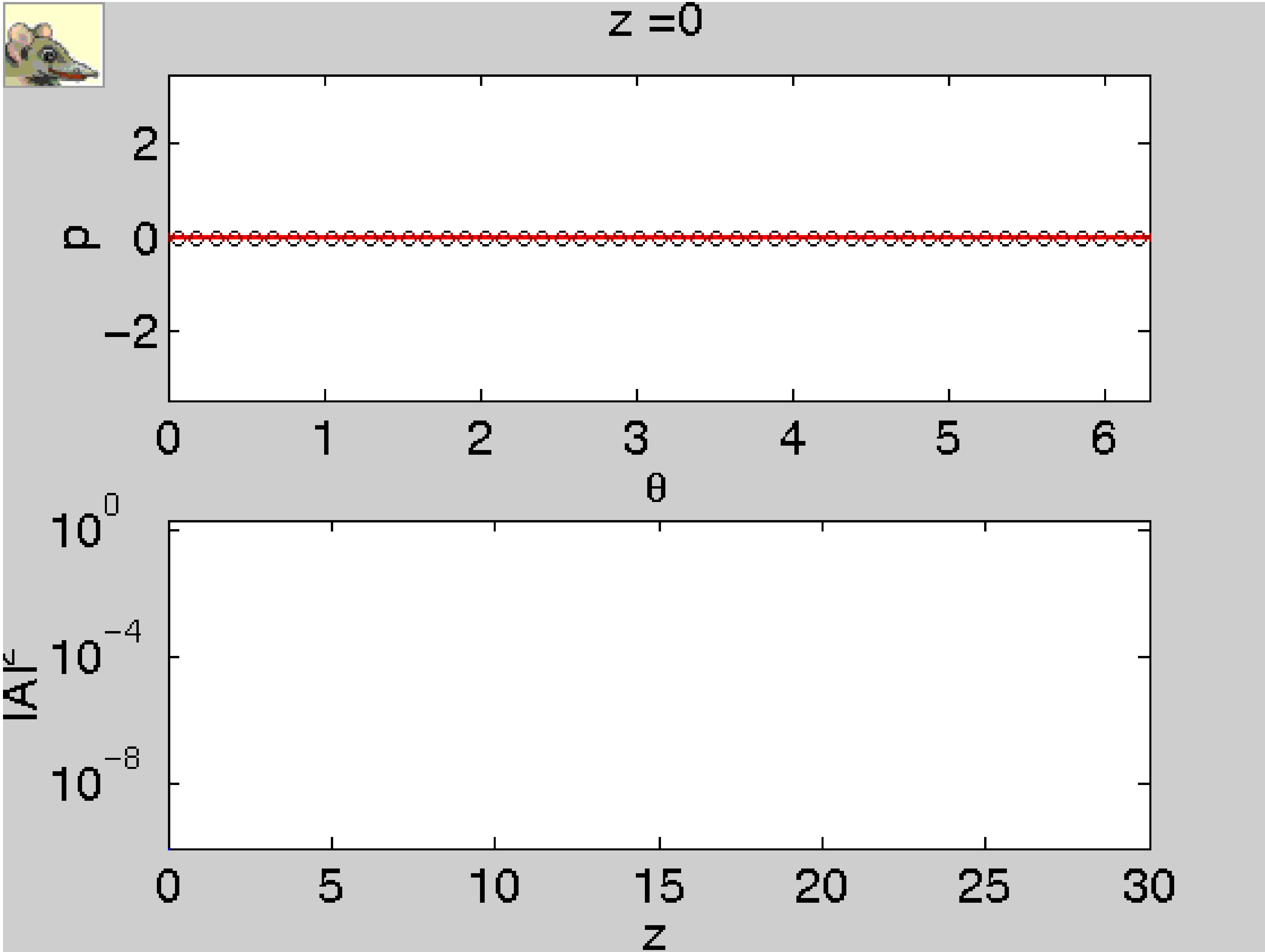
$$\frac{d\phi}{d\bar{z}} = -\frac{1}{a}\left\langle \sin\left(\theta + \phi\right) \right\rangle$$



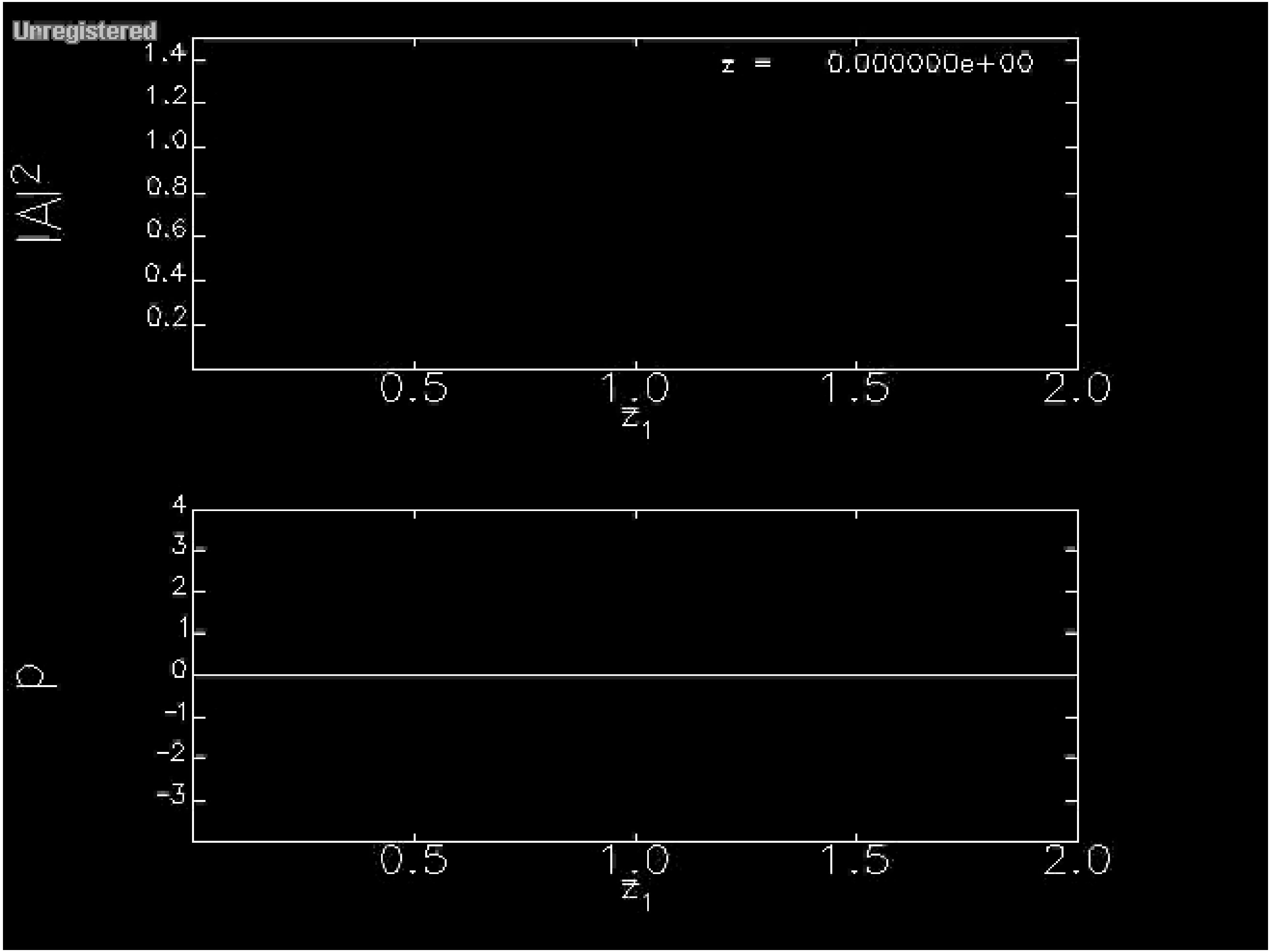
1) e⁻ begin to bunch about $\theta = 3\pi/2$

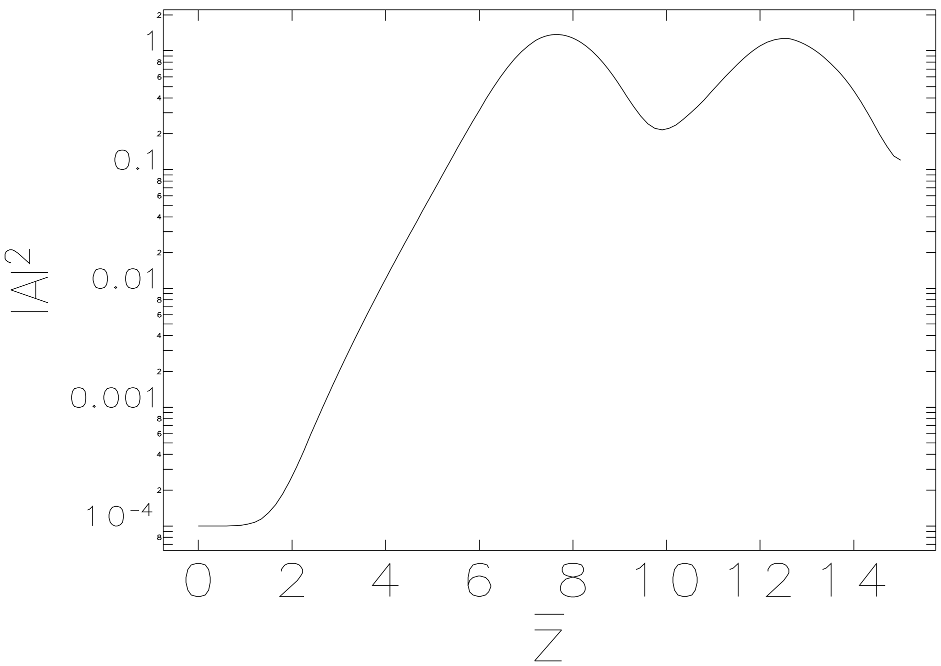
2) Radiation phase driven and shifts

3) Radiation amplitude is driven



z =0
p
2
0
−2
0
1
2
3
4
5
6
θ
$|A|^2$
10^0
10^{-4}
10^{-8}
0
5
10
15
20
25
30
z


Unregistered
z = 0.000000e+00
1.4
1.2
1.0
0.8
0.6
0.4
0.2
$|A|^2$
0.5
1.0
1.5
2.0
z_1
4
3
2
1
0
-1
-2
-3
ρ
0.5
1.0
1.5
2.0
z_1



Scaled intensity

Because the equations are universally scaled (depend only on initial conditions) and because $|A_{sat}|^2 \approx 1$, can see from scaling of A that the rho parameter is a measure of the efficiency of the high-gain FEL amplifier:

$$\rho |A|^2 \equiv \frac{P_{rad}}{P_{beam}} \quad \Rightarrow \quad \rho \approx \frac{P_{rad}^{(sat)}}{P_{beam}}$$

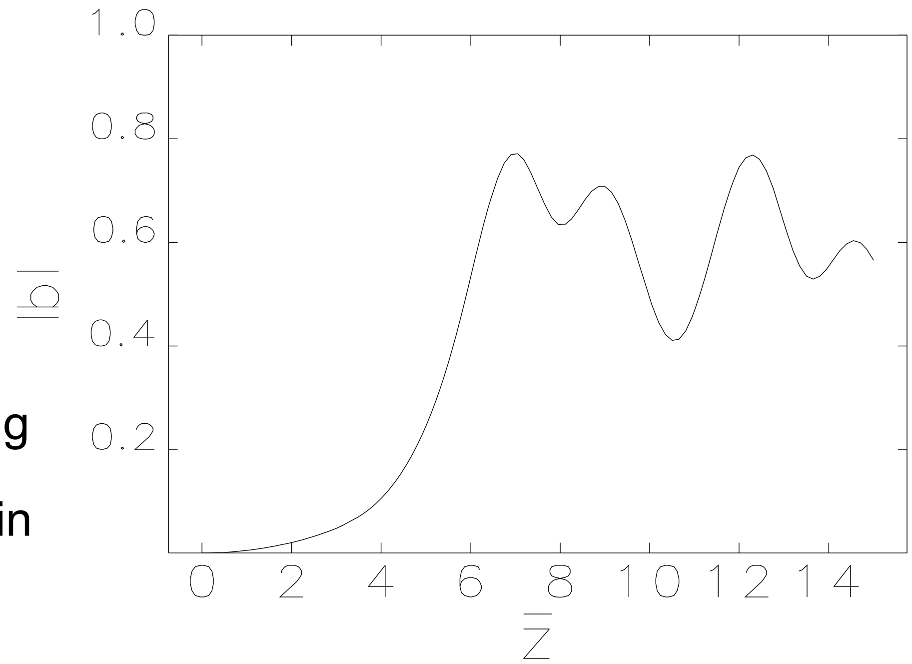
Also see that the saturated radiated power:

$$P_{rad}^{(sat)} \approx \rho P_{beam}$$

As: $P_{beam} \propto I$, and: $\rho \propto I^{1/3}$,

see that: $P_{rad}^{(sat)} \propto I^{4/3}$, demonstrating

that, in the steady-state, the high-gain FEL interaction is a ***cooperative*** or ***collective*** interaction.



Bunching parameter

Some resources

If you want to download some Fortran and Matlab codes that solve the FEL equations and plot their solutions you can obtain them from:

http://phys.strath.ac.uk/eurofel/rebs/rebs.htm

Pulse effects in the high gain

Conventional laser Vs FEL pulses

Conventional laser pulse interacts with all of the active medium $\overline{z}$

PHYSICAL REVIEW A VOLUME 40, NUMBER 8 OCTOBER 15, 1989

Superradiance in the high-gain free-electron laser

R. Bonifacio, B. W. J. McNeil, and P. Pierini

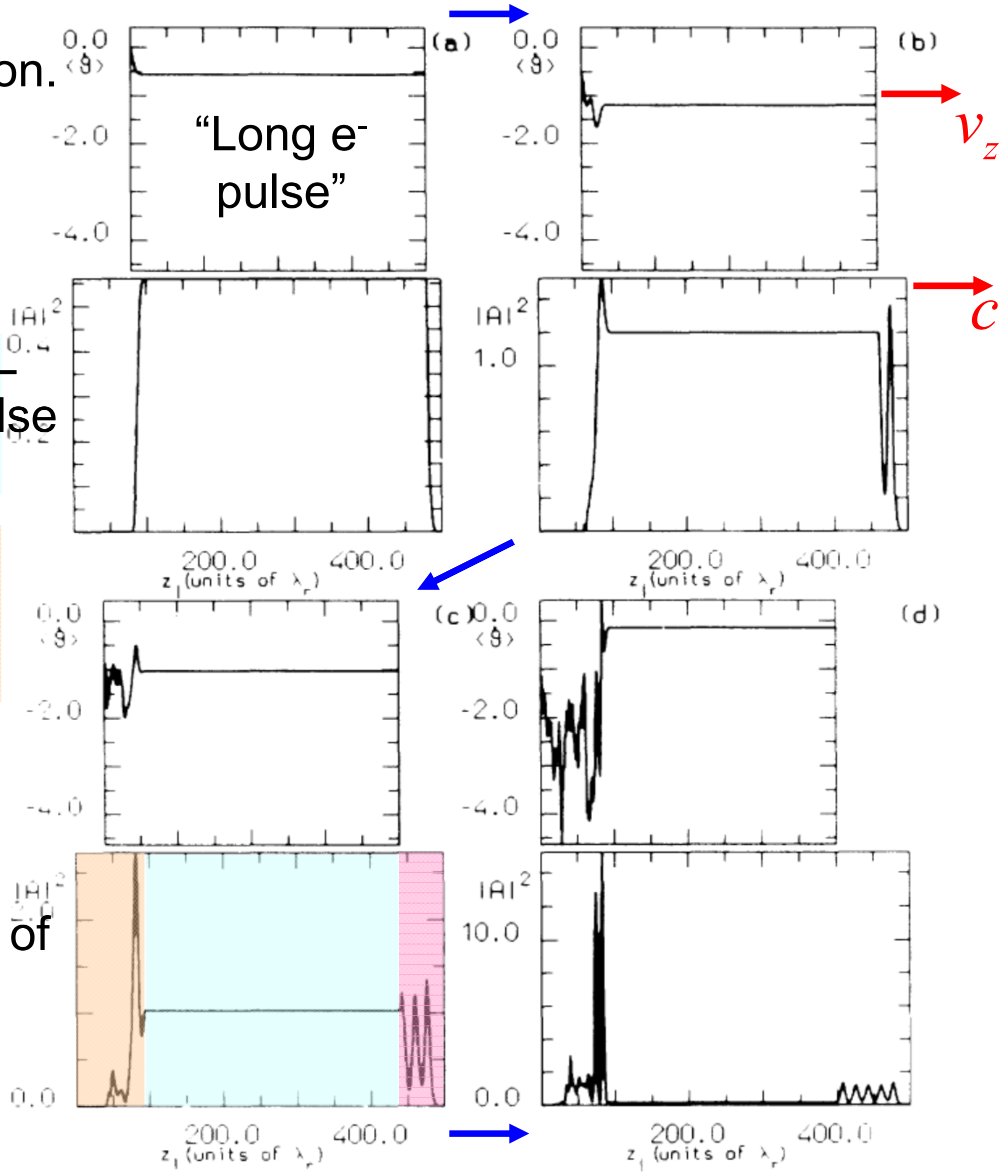
There are 3 main regions of interaction.
From shaded RHS of simulation :

- Vacuum region where - radiation propagates in vacuum.

- Steady state region - where the FEL interaction in the uniform electron pulse has been identical throughout.

- Slippage region – the tail of the electron pulse where no radiation enters from 'behind'.

The slippage region gives rise to ***superradiance***, or more correctly ***superfluorescence*** in which the radiation power scales as I^2 instead of the $I^{4/3}$ scaling of the steady state.



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Superradiance in the high-gain free-electron laser

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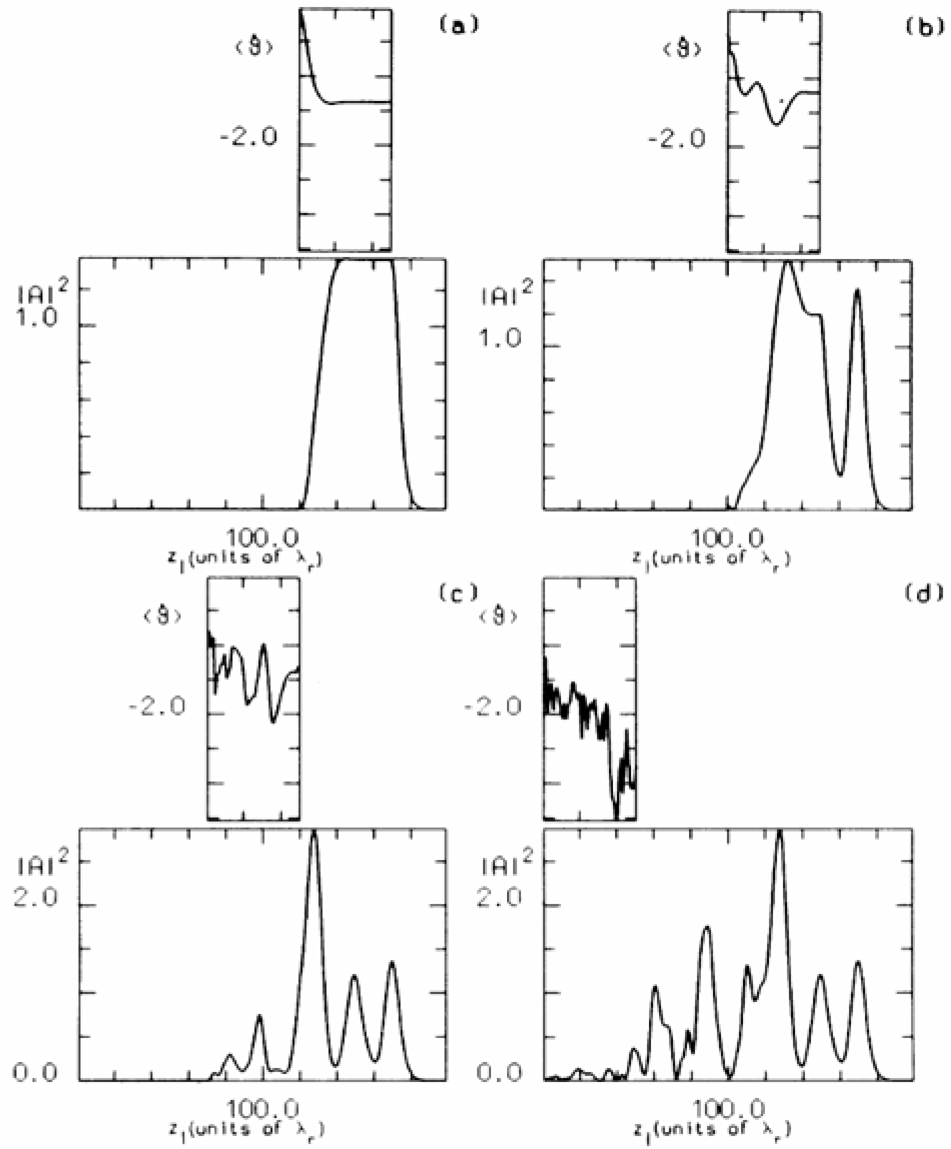
The electron pulse length is clearly important. It is best measured with respect to the relative slippage between the electron pulse and the radiation pulse in one gain length. This is defined as the ***cooperation length***:

$$l_c = \frac{1-\bar{\beta}_z}{\bar{\beta}_z} l_g = \frac{1-\bar{\beta}_z}{\bar{\beta}_z} \frac{\lambda_w}{4\pi\rho} = \frac{\lambda_r}{4\pi\rho}$$

The length of the electron pulse with respect to this length determines how the electron/radiation FEL interaction evolves:

$$\frac{l_e}{l_c} \lesssim 1 \quad \text{- Short pulse}$$

$$\frac{l_e}{l_c} \gg 1 \quad \text{- Long pulse}$$



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Superradiance in the high-gain free-electron laser

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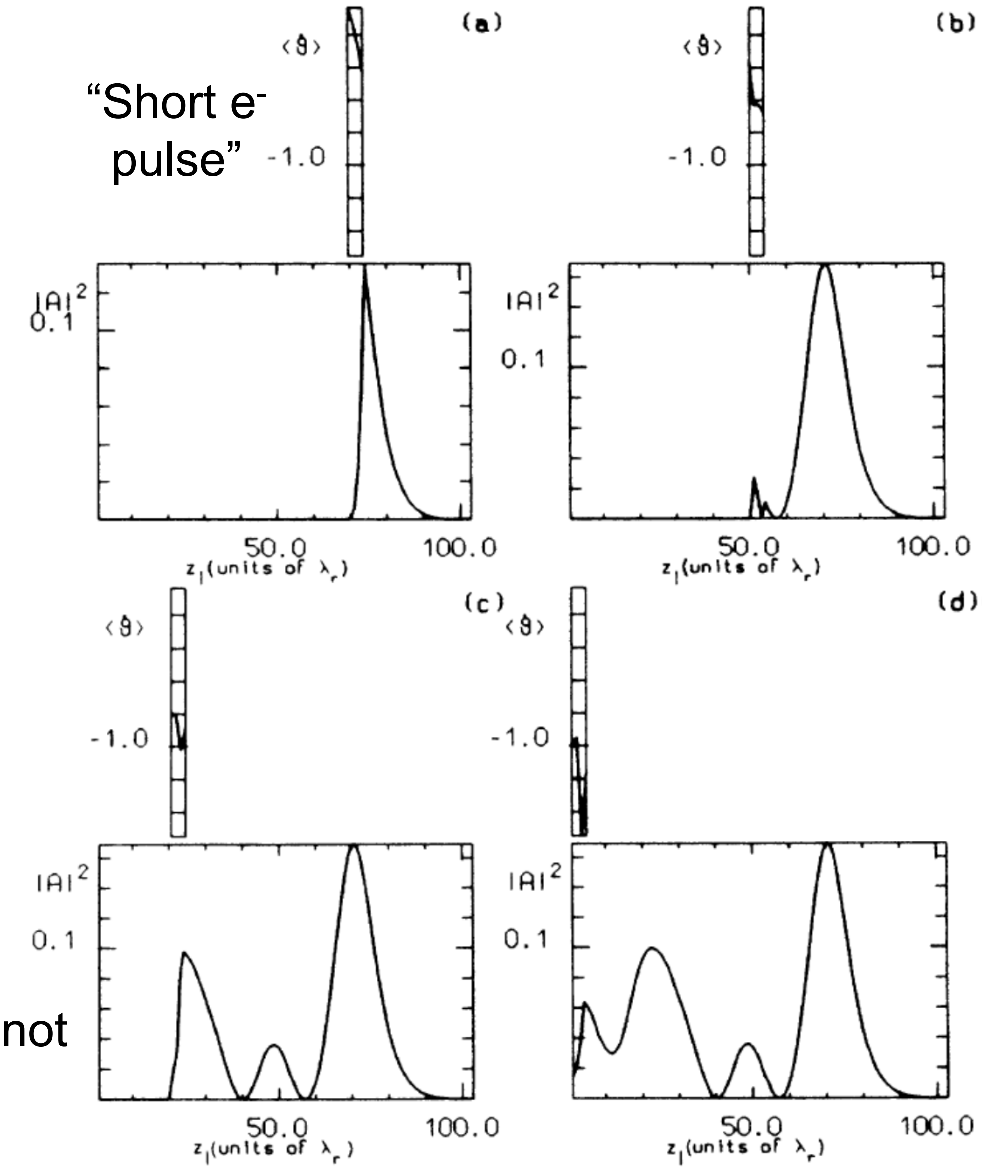
Short electron pulses generate a single ‘spike’ followed by a series of sub-spikes.

This may offer a method of generating a single attosecond timescale pulse in the XUV and x-ray regions of the spectrum.

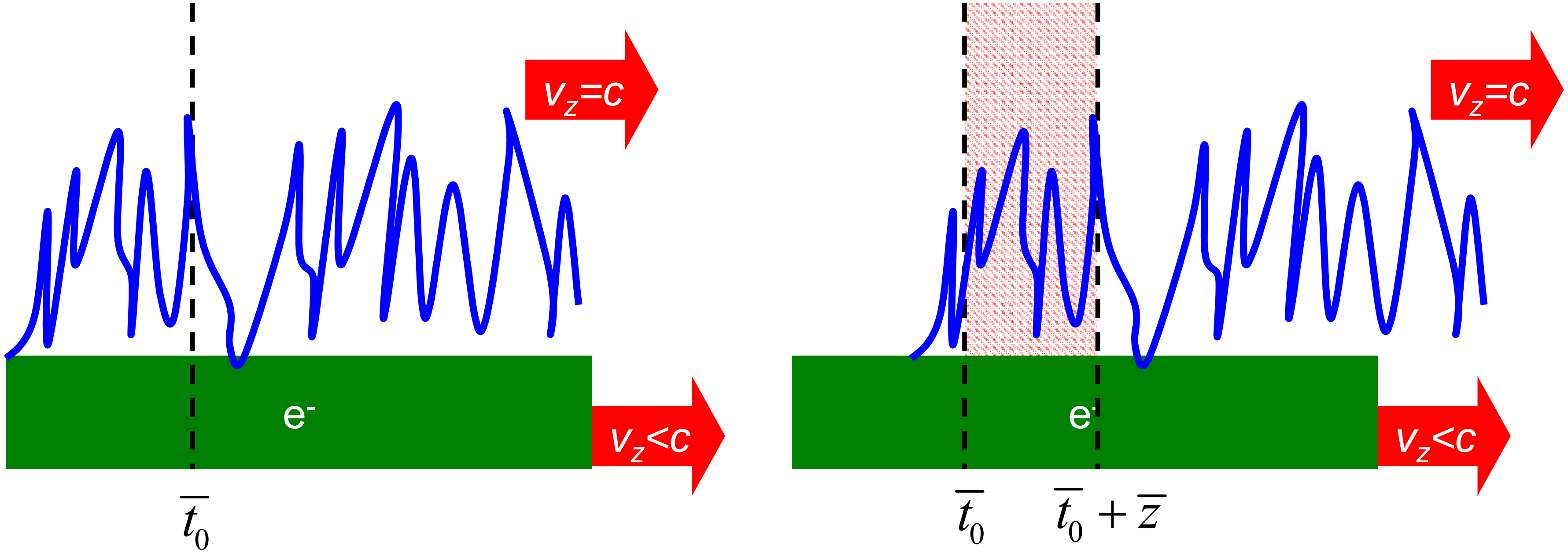
There is a further important effect that must be taken into account. Here, we assumed the electron current was ‘uniform’ i.e. the electrons are distributed so that the initial electron bunching:

$$b\left(\bar{z}=0,\bar{t}\right)=\frac{I\left(\bar{t}\right)}{I_{pk}}\left(\frac{1}{N}\sum_{j=1}^{N}e^{-i\theta_j(\bar{z})}\right)\Bigg|_{\bar{t}} = \text{constant} \quad \forall\ \bar{t}$$

Electron shot-noise ensures that this is not so, and has important consequences.



FEL pulses starting from noise in a High-Gain amplifier (SASE)



(Note: in the scaled variables a radiation wavefront propagates a distance $\bar{z}$ with respect to the electron rest frame.)

Regions of radiation pulse evolve independently of other regions. Hence there can be many regions that evolve independently from different initial source terms due to noise* $b(\bar{z}=0,\bar{t}) \neq \text{constant}$. This leads to a noisy temporal and spectral radiation pulse.

*B.W.J. McNeil, M.W. Poole and G.R.M. Robb, PRST-AB, **6**, 070701 (2003)

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Spectrum, Temporal Structure, and Fluctuations in a High-Gain Free-Electron Laser Starting from Noise

R. Bonifacio,[1,2] L. De Salvo,[1] P. Pierini,[2] N. Piovella,[1] and C. Pellegrini[3]

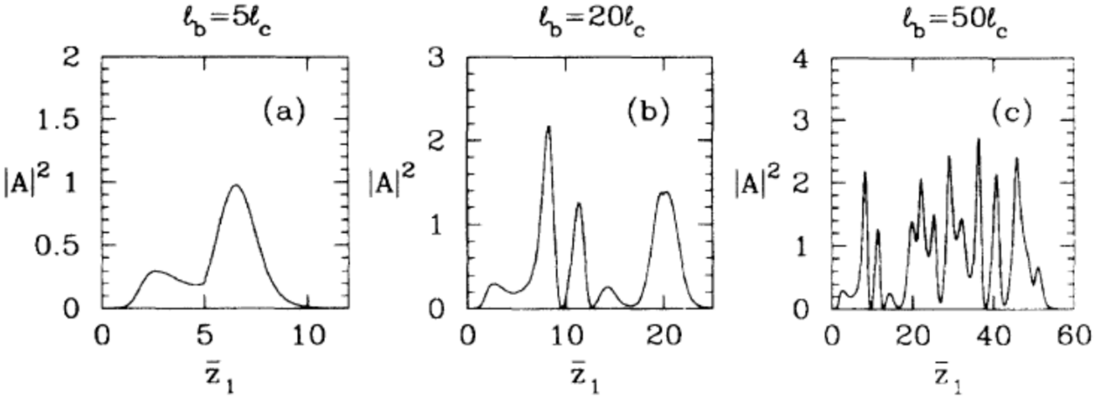


FIG. 1. Results of the numerical model: temporal structure of the radiated pulse, $|A|^2$ vs $\bar{z}_1$, at the first saturation, for three values of the electron bunch length, at $z = 14\ell_g$ and for $\langle|b_0|^2\rangle = 10^{-6}$: (a) $\ell_b = 5\ell_c$, (b) $\ell_b = 20\ell_c$, and (c) $\ell_b = 50\ell_c$. The temporal scale is in units of $z_1 = (z - v_{\|}t)/\ell_c$.

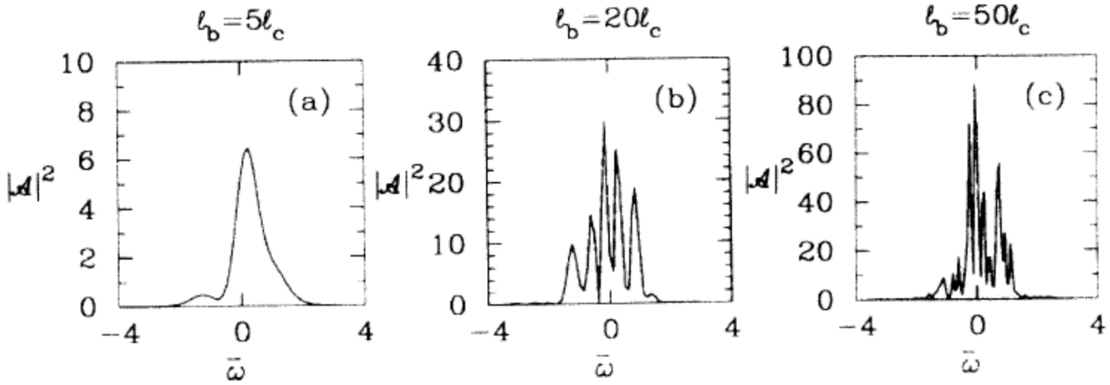
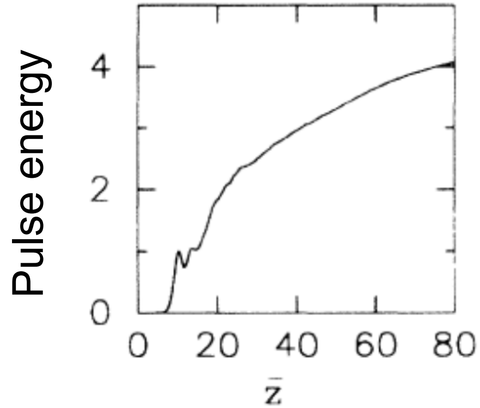
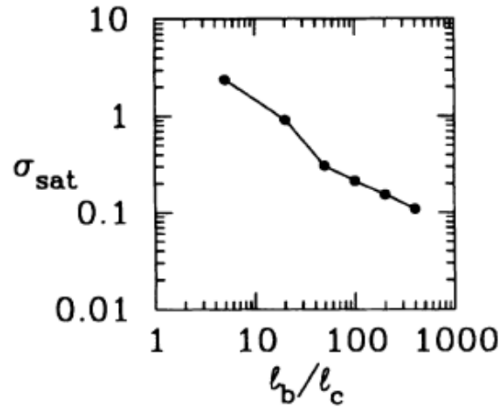


FIG. 3. Spectrum of the radiated pulses, for the same cases

The spike widths are $\lesssim 2\pi l_c$





Saturation length – shot-to-shot fluctuations.

Self Amplified Spontaneous Emission (SASE) in the x-ray

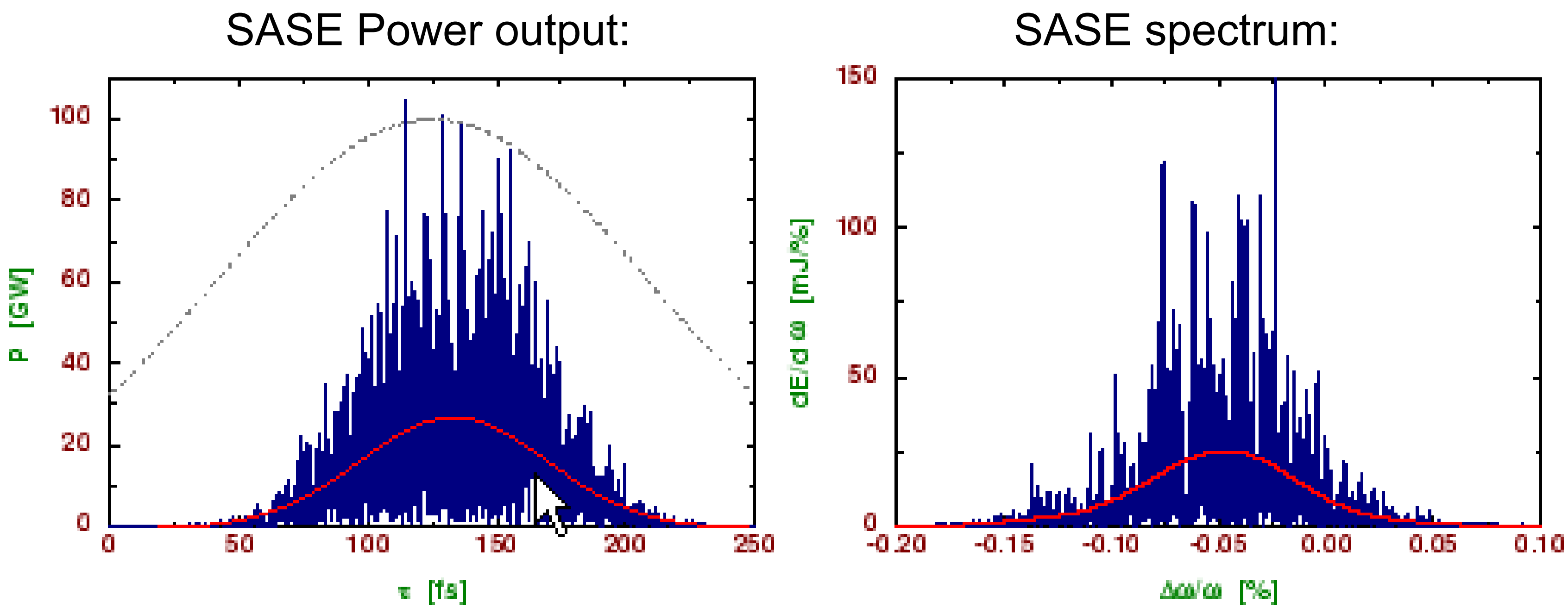
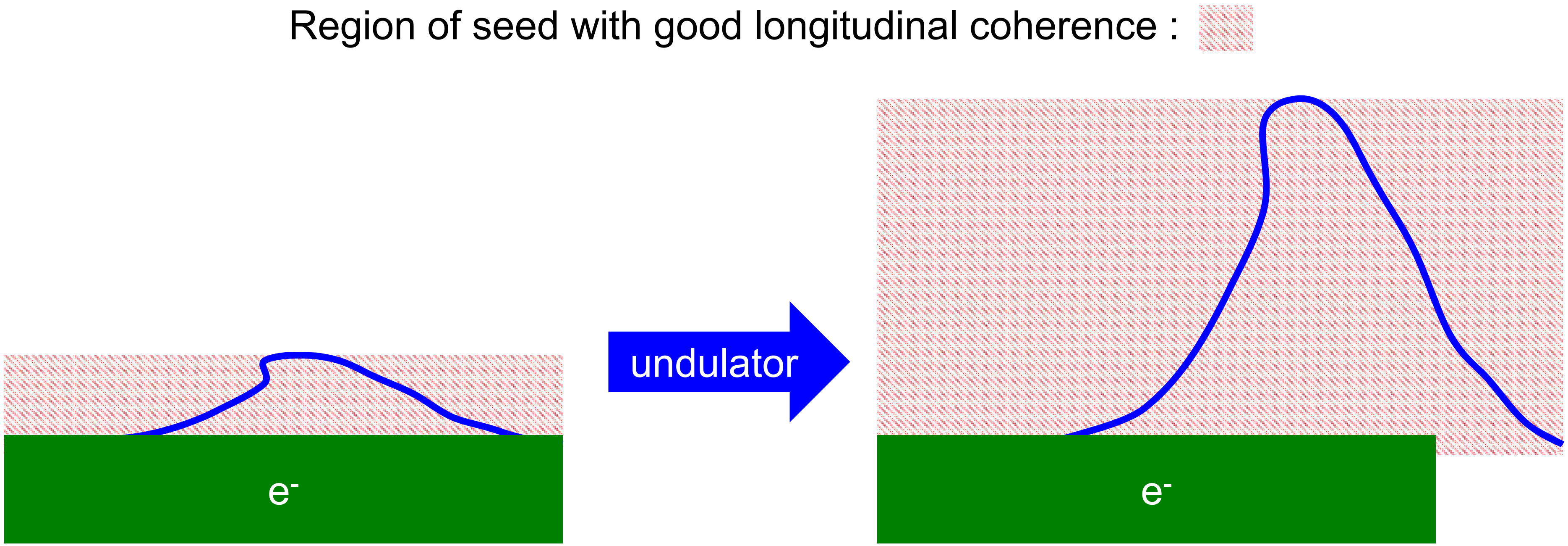


Figure 5: Typical temporal (left) and spectral (right) structure of the radiation pulse from a SASE XFEL at a wavelength of 1Å. The red lines correspond to averaged values. The dashed line represents the axial density profile of the electron bunch. Note that the growth rate in the electron bunch tail is reduced due to the reduced current. Therefore, the radiation pulse length of 100fs (FWHM) is about a factor of two shorter than the electron bunch.

Seeded FEL – improves SASE



Longitudinal coherence of radiation pulse is inhereted from that of seed if $P_{seed} >> P_{noise}$

Estimate of the initial SASE power

The radiation power evolves like: $$P_{rad}(\bar{z}) \approx \frac{P_{rad}(0)}{9} \exp(\sqrt{3}\bar{z}) \qquad (1)$$

And the saturation power: $$P_{sat} \approx \rho P_{beam} \qquad (2)$$

From the analysis of *, the power in the linear regime is: $$P_{rad}(\bar{z}) = \frac{2\sqrt{\pi}}{3^{5/4}} \frac{\exp\left(\sqrt{3}\bar{z}\right)}{\sqrt{\bar{z}}N_\lambda} \rho^2 P_{beam} \qquad (3)$$

N_λ is # e$^-$ in radn. wavelength.

From last 2 eqns.: $$\ln\left(\bar{z}_{sat}\right) - 2\sqrt{3}\bar{z}_{sat} - \ln\left(\frac{4\pi\rho^2}{3^{5/2}N_\lambda^2}\right) = 0$$

$$\Rightarrow \quad \bar{z}_{sat} \approx \ln\left(N_\lambda/\rho\right)/\sqrt{3} \quad \text{for: } 2\sqrt{3}\bar{z}_{sat} \gg \ln\left(\bar{z}_{sat}\right) \qquad (4)$$

Equating (1) & (3): $$P_{rad}(0) \approx 2\frac{3^{3/4}\sqrt{\pi}}{\sqrt{\bar{z}_{sat}}N_\lambda}\rho^2 P_{beam}$$ and using (4):

$$P_{rad}(0) \approx \frac{6\sqrt{\pi}}{N_\lambda\sqrt{\ln\left(N_\lambda/\rho\right)}}\rho^2 P_{beam} \quad \Rightarrow \quad |A_0|^2 = \frac{6\sqrt{\pi}\rho}{N_\lambda\sqrt{\ln\left(N_\lambda/\rho\right)}}$$

* Kwang-Je Kim, Phys. Rev. Lett., **57**, 1871, (1986)