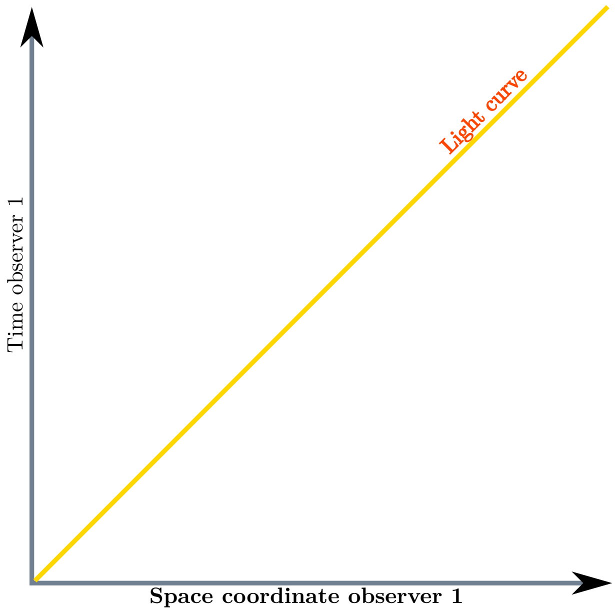
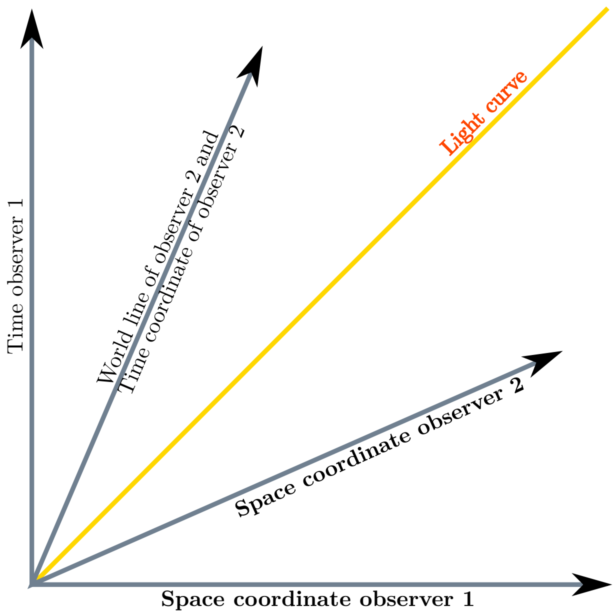
Drawing a spacetime diagram

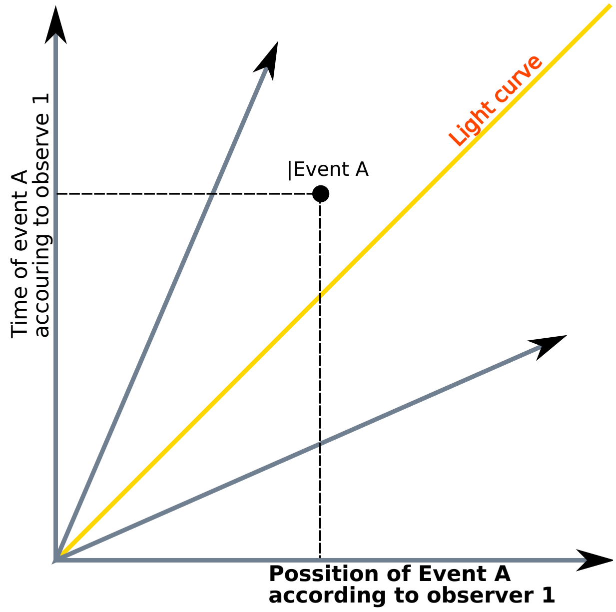
- – Draw horizontal and vertical axes.
  – The vertical axis represents the worldline of a stationary particle, called observer 1.
  – The horizontal line is considered the space axis of observer 1.
  – Draw a diagonal line at 45°. This represents the motion of a beam of light.



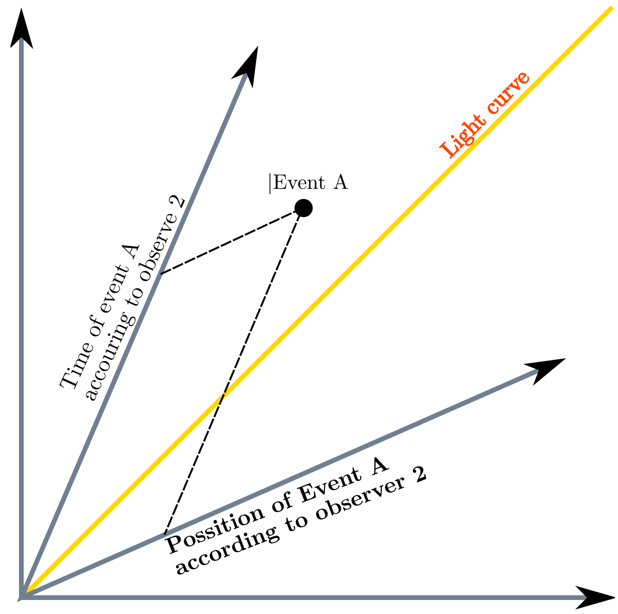
- – Draw a timelike line. This is a line more vertical than the diagonal line at 45°. This represents the worldline of a moving particle, called observer 2. (Moving with respect to the stationary particle). This is called the time axis of observer 2.
  – Draw another line by reflecting the worldline of the moving particle about the 45° lightline. This is the called the space axis of observer 2.



- – A point on the spacetime diagram is called an event. This is a point in space at a specific moment in time.
  – The vertical value of this event is the time as measured by observer 1.
  – The horizontal value event is the position of the event as measured by observer 1.



  – Take a line from the event, parallel to the space axis of observer 2. The value on the time axis where this line crosses it is the time of the event as measured by observer 2.
  – Take a line from the event, parallel to the time axis of observer 2. The value on the space axis where this line crosses it is the position of the event as measured by observer 2.



Questions

Question 1. In the LHC: Energy 7 TeV. Each bunch will contain 1.15×10^{11} protons per bunch at the start of nominal fill. Around 30 cm. long with transverse dimensions of 1mm

(a) Calculate the distance between protons in the lab frame.

(b) Calculate the distance between protons in the rest frame of the electrons.

(c) If the electrons are going in a straight line, draw a spacetime diagram of a signal between electrons.

(d) Explain why, as a first approximation we do not need to consider electrons feeling the others electromagnetic field. (Half of this is easy).

Question 2. Figure 3, shows a spacetime diagram. According to observer 1, events A and B are simultaneous.

(a) Show that according to observer 2, event B occurs before event A.

(b) Draw the axes of another observer who would measure event A before event B.

Question 3. In figure 2 observer 2 is moving at a speed $\frac{\sqrt{5}}{9}c \approx .745c$ where c is the speed of light. The axes measure seconds with respect to both observers.

(a) Show that when observer 2 measures 1 second on her timeline, observer 1 measures 1.5 seconds. Therefore observer 1 thinks observer 2 clock is slow.

(b) When observer 1 measures 1 second on his timeline, what time does observer 2 measure. Does observer 2 think observer 1 clock is fast or slow.

Let's say observer 1 and observer 2 want to directly compare clocks. Both send a light signal to the other after 1 second.

(c) Show that the signal from observer 2 reaches observer 1 at time 2.62 seconds, according to observer 1 clock. (The actual value is $\frac{3}{2}(1 + \frac{\sqrt{5}}{3}.)$

(d) What time, according to observer 2 clock, does the signal from observer 1 arrive?

Question 4. In figure 3:

(a) Identify the events on the diagram A,...,J which represents the following events.

- The moment when the front of the car passes side 1 of the garage.
- The moment when the front of the car passes side 2 of the garage.
- The moment when the rear of the car passes side 1 of the garage.
- The moment when the rear of the car passes side 2 of the garage.

(b) Does the front of the car pass side 2 of the garage before or after rear of the car passes side 1 of the garage:

- According to the garage observer.
- According to the car observer.

(c) Identify two events which represents the length of the garage according to the garage observer.

Identify two events which represents the length of the car according to the garage observer.

Show that the garage observer measures the car as shorter than the garage.

(d) Identify two events which represents the length of the garage according to the car observer.

Identify two events which represents the length of the car according to the car observer.

Show that the car observer measures the garage as shorter than the car.

Question 5. A mass M at rest decomposes into two masses, m_1 and m_2 moving with Newtonian velocities v_1 and v_2. Use conservation of 4-momentum to:

(a) Calculate v_2 in terms of m_1, m_2 and v_1.

(b) Calculate M in terms of m_1, m_2 and v_1.

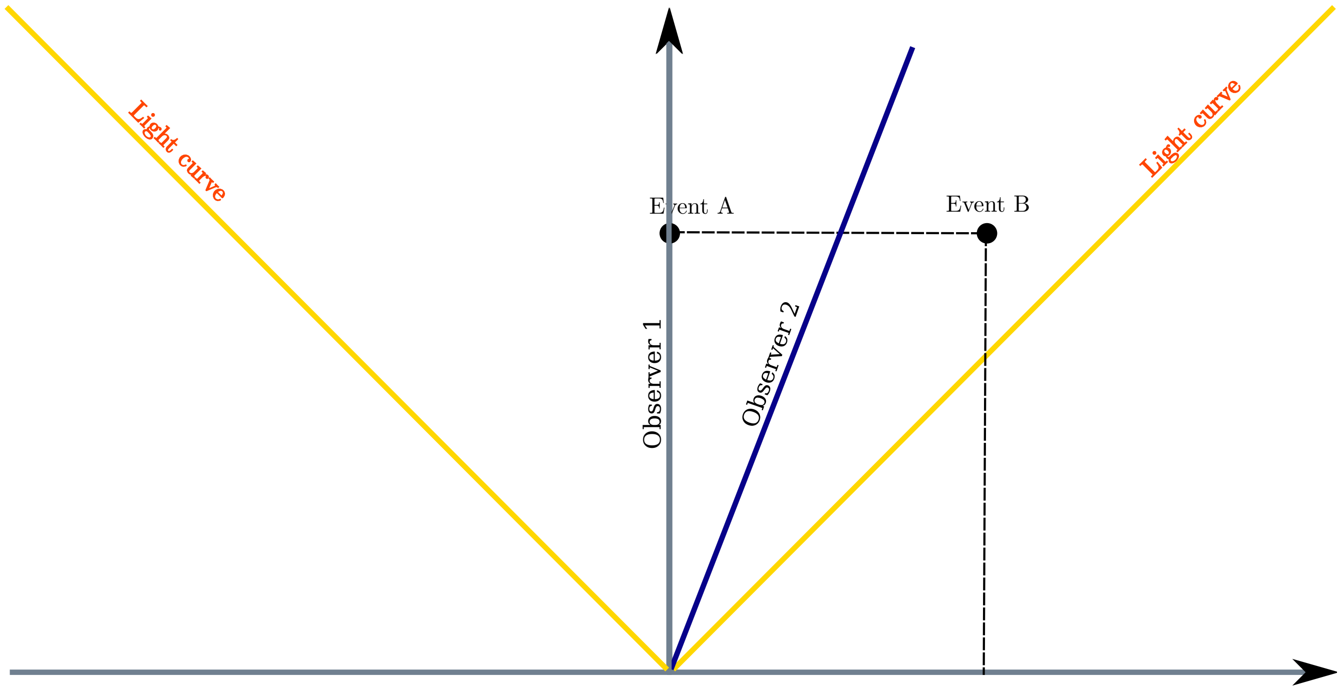


Figure 1: The spacetime diagram representing simultaneous events.

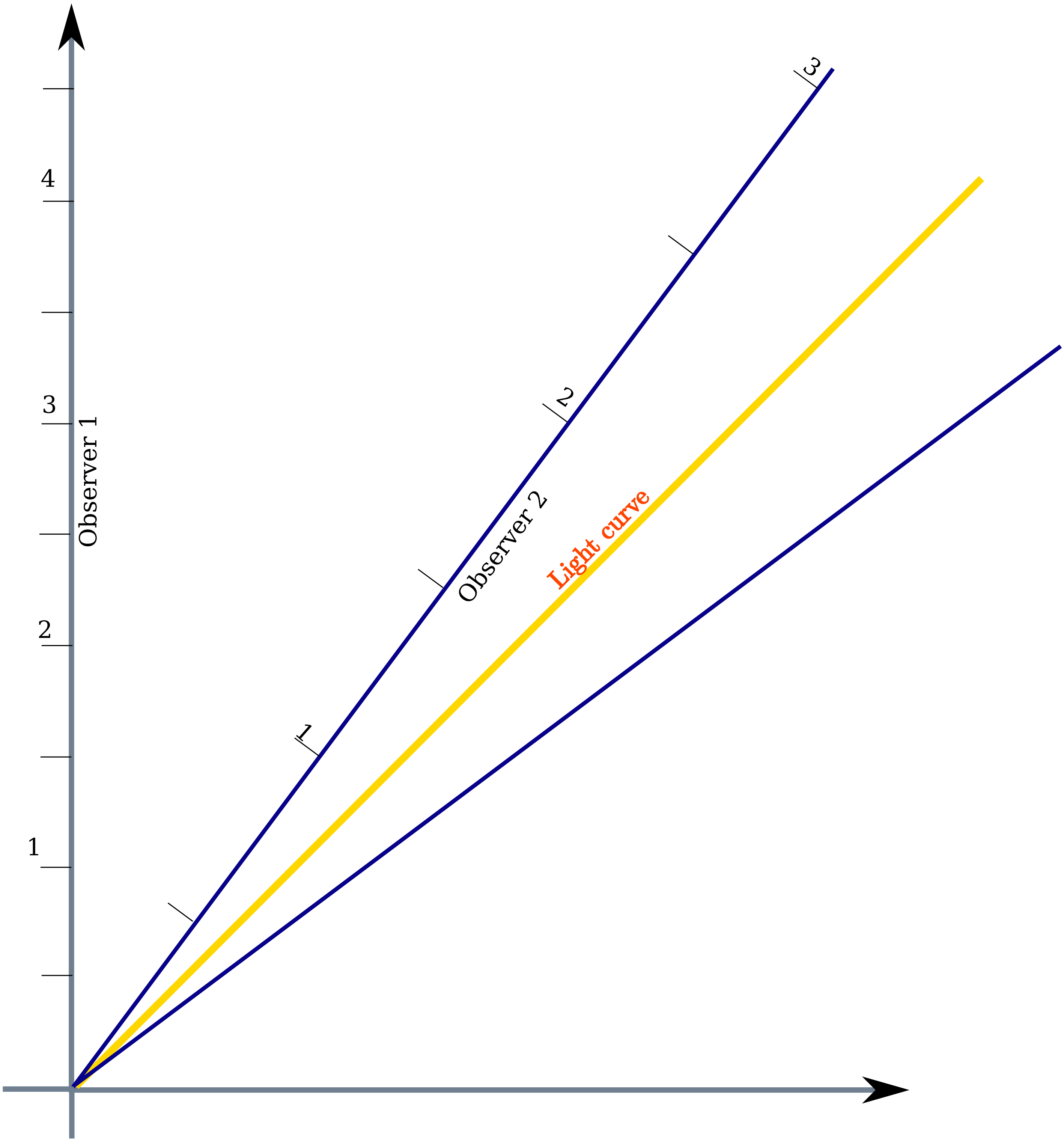


Figure 2: The spacetime diagram represents the “paradox” of time dilation

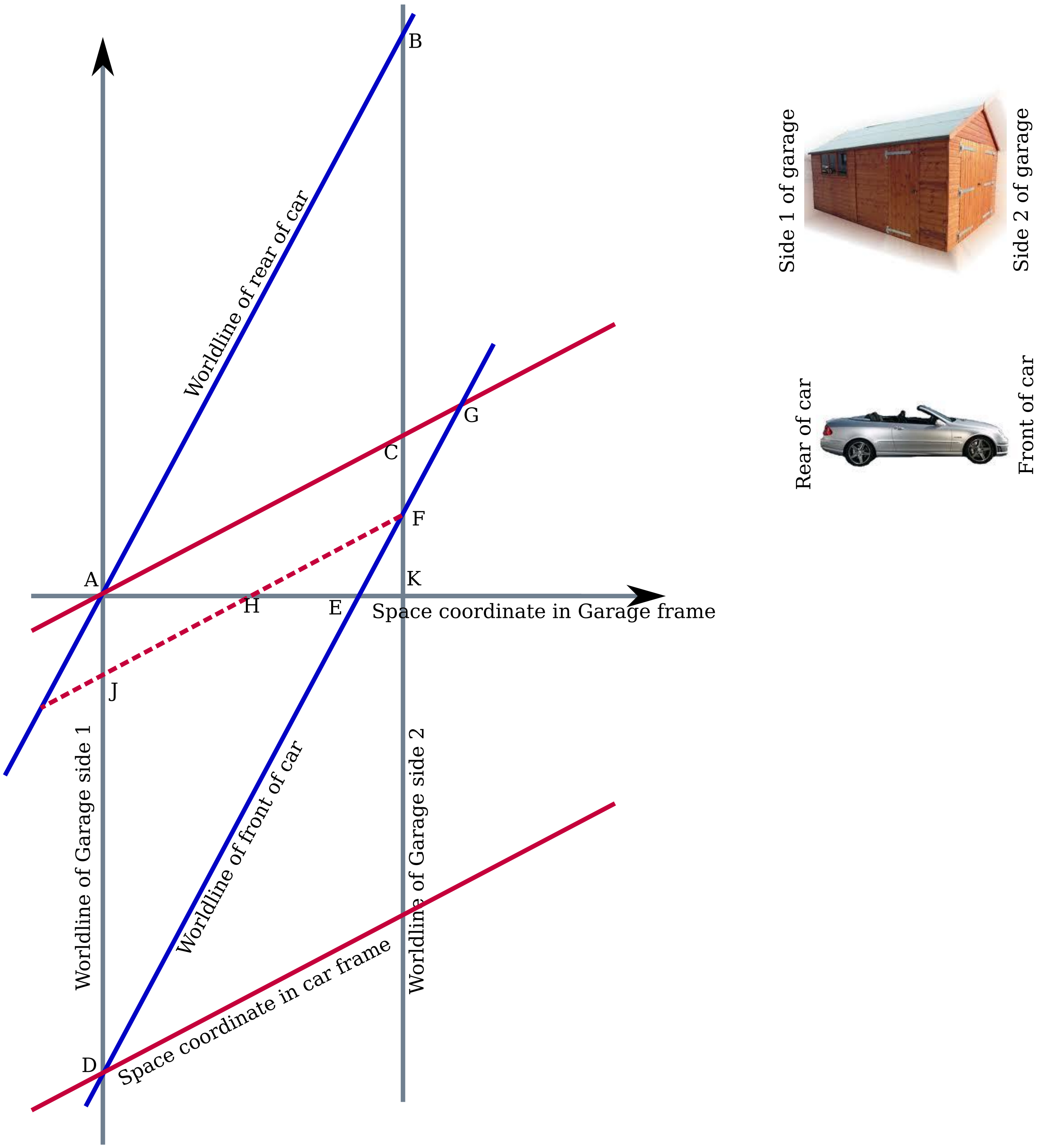
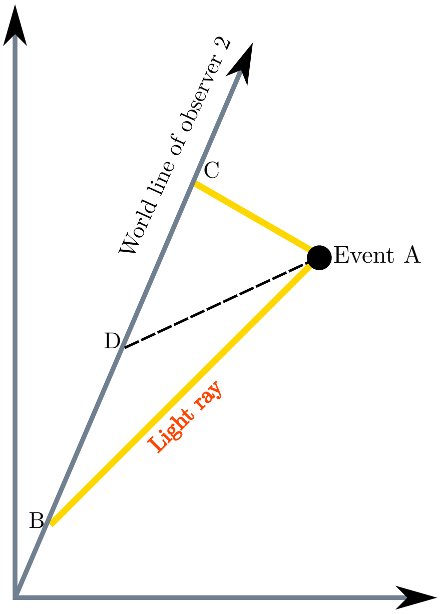


Figure 3: The spacetime diagram represents the "paradox" of the car and the garage.

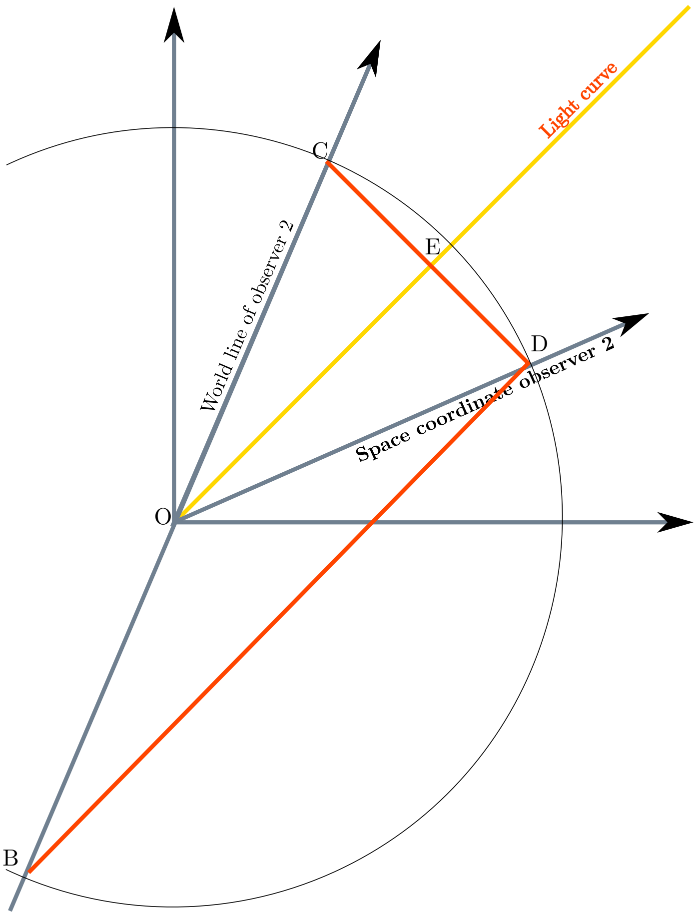
Appendix: Why is the space coordinate axis where it is?



We start with **radar time**. Observer 2 sends a light signal at event B to event A. The signal is immediately returned and intersects the worldline of observer 2 at C. Observer 2 defines the the time of event A as the same time as event D which is **half way** between B and C.

We define the space coordinate axis of observer 2 to be all the points with time 0 according to observer 2.

It is now simple geometry to show that the space coordinate axis of observer 2 is the reflection of the worldline of observer 2 through the light curve.



In the diagram on the right, BD and CD are at 45°, therefore BCD is a right angled triangle.

Thus the circle which circumscribes BCD must have BC as a diameter. Since O bisects BC then O is the origin of the circle.

Thus the length OC equals OD so the triangle OCD is an isosceles triangle.

However the light curve OE is at right angles to C and therefore must bisect the triangle. Thus OD is the reflection of OC through OE.