Concept of Luminosity

(in particle colliders)

Werner Herr, CERN
Bruno Muratori, Daresbury Laboratory

Slide 1

http://cern.ch/Werner.Herr/COCKCROFT09/lectures/luminosity.pdf
http://cern.ch/Werner.Herr/COCKCROFT09/handout/luminosity.pdf

Why colliding beams ?

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- **Two beams:** $E_1, \vec{p_1}, E_2, \vec{p_2}, m_1 = m_2 = m$
- $E_{cm} = \sqrt{(E_1 + E_2)^2 - (\vec{p_1} + \vec{p_2})^2}$
- **Collider versus fixed target:**

  Fixed target: $\vec{p_2} = \mathbf{0}$ ➡ $E_{cm} = \sqrt{2m^2 + 2E_1 m}$

  Collider: $\vec{p_1} = -\vec{p_2}$ ➡ $E_{cm} = E_1 + E_2$
- **LHC (pp): 14000 GeV versus ≈ 115 GeV**
- **LEP (e^+e^-): 210 GeV versus ?**

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Collider performance issues

- **Available energy**
- **Number of interactions per second (useful collisions)**
- **Total number of interactions**
- **Secondary issues:**
  - **Time structure of interactions (how often and how many at the same time)**
  - **Space structure of interactions (size of interaction region)**
  - **Quality of interactions (background, dead time etc.)**

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Luminosity:

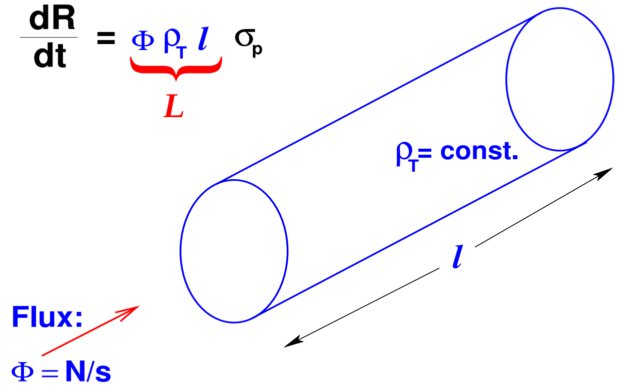
- **We want:**
  - **Proportionality factor between cross section σ_p and number of interactions per second $\frac{dR}{dt}$**

$$\frac{dR}{dt} = \mathcal{L} \times \sigma_p \qquad (\rightarrow \quad \text{units}: \ \text{cm}^{-2}\text{s}^{-1})$$

  - **Relativistic invariant**
  - **Independent of the physical reaction**
  - **Reliable procedures to compute and measure**

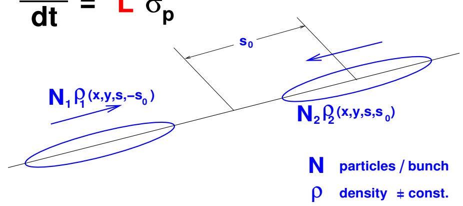
Fixed target luminosity

$$\frac{dR}{dt} = \underbrace{\Phi \rho_T l}_{L} \sigma_p$$



Collider luminosity (per bunch crossing)

$$\frac{dR}{dt} = L\,\sigma_p$$



$$\mathcal{L} \propto K N_1 N_2 \int\int\int\int_{-\infty}^{+\infty} \rho_1(x,y,s,-s_0)\rho_2(x,y,s,s_0)dxdydsds_0$$

s_0 is "time"-variable: $s_0 \;=\; c{\cdot}t$

Kinematic factor: $K = \sqrt{(\vec{v_1}-\vec{v_2})^2 - (\vec{v_1}\times\vec{v_2})^2/c^2}$

Collider luminosity (per beam)

- Assume uncorrelated densities in all planes

→ factorize: $\rho(x,y,s,s_0) = \rho_x(x) \cdot \rho_y(y) \cdot \rho_s(s \pm s_0)$

- For head-on collisions ($\vec{v_1} = -\vec{v_2}$) we get:

$$\mathcal{L} = 2 \cdot N_1 N_2 \cdot f \cdot n_b \cdot \int\int\int\int_{-\infty}^{+\infty} dx dy ds ds_0$$
$$\rho_{1x}(x)\rho_{1y}(y)\rho_{1s}(s-s_0) \cdot \rho_{2x}(x)\rho_{2y}(y)\rho_{2s}(s+s_0)$$

- In principle: should know all distributions

→ Mostly use Gaussian ρ for analytic calculation (in general: it is a good approximation)

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Gaussian distribution functions

- $\rho_{iz}(z) = \frac{1}{\sigma_z\sqrt{2\pi}} \exp\left(-\frac{z^2}{2\sigma_z^2}\right) \quad i = 1,2, \quad z = x,y$

- $\rho_{is}(s \pm s_0) = \frac{1}{\sigma_s\sqrt{2\pi}} \exp\left(-\frac{(s \pm s_0)^2}{2\sigma_s^2}\right)$

- For non-Gaussian profiles not always possible to find analytic form, need a numerical integration

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Luminosity for two beams (1 and 2)

- **Simplest case : equal beams**
  - $\sigma_{1x} = \sigma_{2x}, \quad \sigma_{1y} = \sigma_{2y}, \quad \sigma_{1s} = \sigma_{2s}$
  - **but:** $\sigma_{1x} \neq \sigma_{1y}, \quad \sigma_{2x} \neq \sigma_{2y}$ **is allowed**
- **Further: no dispersion at collision point**

Integration (head-on)

for $\sigma_1 = \sigma_2 \quad \rightarrow \quad \rho_1\rho_2 \;=\; \rho^2$ **:**

$$\mathcal{L} = \frac{2 \cdot N_1 N_2 f n_b}{(\sqrt{2\pi})^6 \sigma_s^2 \sigma_x^2 \sigma_y^2} \int \int e^{-\frac{x^2}{\sigma_x^2}} e^{-\frac{y^2}{\sigma_y^2}} e^{-\frac{s^2}{\sigma_s^2}} e^{-\frac{s_0^2}{\sigma_s^2}} dx dy ds ds_0$$

integrating over s **and** s_0**, using:**

$$\int_{-\infty}^{+\infty} e^{-at^2} dt \quad = \quad \sqrt{\pi/a}$$

$$\mathcal{L} = \frac{2 \cdot N_1 N_2 f n_b}{8(\sqrt{\pi})^4 \sigma_x^2 \sigma_y^2} \int \int e^{-\frac{x^2}{\sigma_x^2}} e^{-\frac{y^2}{\sigma_y^2}} dx dy$$

finally after integration over x and y: $\Longrightarrow \quad \mathcal{L} = \frac{N_1 N_2 f n_b}{4\pi\sigma_x\sigma_y}$

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Luminosity for two (equal) beams

- **Simplest case :**

  $\sigma_{1x} = \sigma_{2x}, \sigma_{1y} = \sigma_{2y}, \sigma_{1s} = \sigma_{2s}$

$$\Longrightarrow \quad \boxed{\mathcal{L} = \frac{N_1 N_2 f n_b}{4\pi\sigma_x\sigma_y}}$$

- **Typical example: round beams, e.g. hadrons**

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Luminosity for two (unequal) beams

- **Generalized for unequal beam sizes:**

  $\sigma_{1x} \neq \sigma_{2x} \neq \sigma_{1y} \neq \sigma_{2y}, \quad but : \sigma_{1s} \approx \sigma_{2s}$

$$\Longrightarrow \quad \boxed{\mathcal{L} = \frac{N_1 N_2 f n_b}{2\pi\sqrt{\sigma_{1x}^2 + \sigma_{2x}^2}\sqrt{\sigma_{1y}^2 + \sigma_{2y}^2}}}$$

- **Typical example: flat beams, e.g. leptons**

Examples

	Energy (GeV)	$\mathcal{L}_{max}$ $cm^{-2}s^{-1}$	rate s^{-1}	σ_x/σ_y $\mu m/\mu m$	Particles per bunch
SPS ($p\bar{p}$)	315x315	$6\ 10^{30}$	$4\ 10^{5}$	60/30	$\approx 10\ 10^{10}$
Tevatron ($p\bar{p}$)	1000x1000	$100\ 10^{30}$	$7\ 10^{6}$	30/30	$\approx 30/8\ 10^{10}$
HERA (e^+p)	30x920	$40\ 10^{30}$	40	250/50	$\approx 3/7\ 10^{10}$
LHC (pp)	7000x7000	$10000\ 10^{30}$	10^{9}	17/17	$\approx 11\ 10^{10}$
LEP (e^+e^-)	105x105	$100\ 10^{30}$	≤ 1	200/2	$\approx 50\ 10^{10}$
PEP (e^+e^-)	9x3	$8000\ 10^{30}$	NA	150/5	$\approx 2/6\ 10^{10}$

What else ?

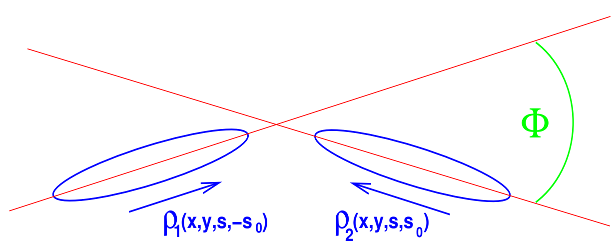
- What about linear colliders ?

→ See later ...

Complications

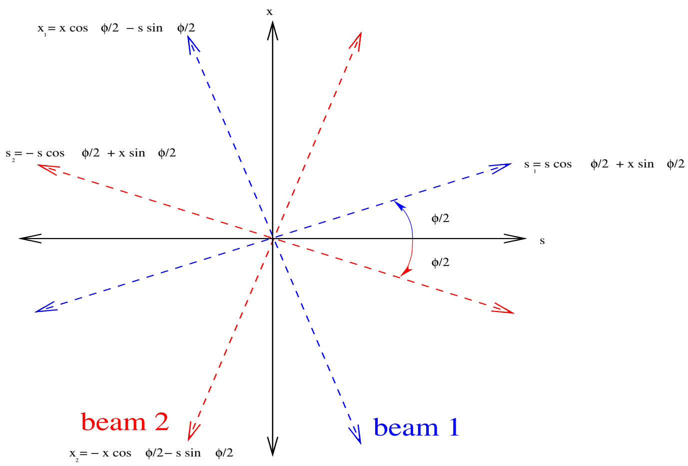
- Crossing angle
- Hour glass effect
- Displaced waist: $\delta\beta^*/\delta s = \alpha^* \neq 0$
- Collision offset (wanted or unwanted)
- Non-Gaussian profiles
- Dispersion at collision point
- Strong coupling
- etc.

Collisions at crossing angle



- Needed to avoid unwanted collisions
  - For colliders with many bunches: LHC, CESR, KEKB
  - For colliders with coasting beams

Collisions angle geometry (horizontal plane)



Crossing angle

Assume crossing in horizontal (x, s)- plane.
Transform to new coordinates:

$$\begin{cases} x_1 = x\cos\frac{\phi}{2} - s\sin\frac{\phi}{2}, & s_1 = s\cos\frac{\phi}{2} + x\sin\frac{\phi}{2}, \\ x_2 = x\cos\frac{\phi}{2} + s\sin\frac{\phi}{2}, & s_2 = s\cos\frac{\phi}{2} - x\sin\frac{\phi}{2} \end{cases}$$

$$\mathcal{L} = 2\cos^2\frac{\phi}{2} N_1 N_2 f n_b \int\int\int\int_{-\infty}^{+\infty} dx\,dy\,ds\,ds_0$$
$$\rho_{1x}(x_1)\rho_{1y}(y_1)\rho_{1s}(s_1 - s_0)\rho_{2x}(x_2)\rho_{2y}(y_2)\rho_{2s}(s_2 + s_0)$$

Integration (crossing angle)

- tutorial ..

Hint:

use as before:

$$\int_{-\infty}^{+\infty} e^{-at^2} dt \quad = \quad \sqrt{\pi/a}$$

and:

$$\int_{-\infty}^{+\infty} e^{-(at^2+bt+c)} dt \quad = \quad \sqrt{\pi/a} \cdot e^{\frac{b^2-ac}{a}}$$

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Crossing angle

After integration over t and transverse coordinates:

$$\mathcal{L} = \frac{N_1 N_2 f n_b}{8\pi^{\frac{3}{2}} \sigma_s} \quad 2\cos\frac{\phi}{2} \quad \int_{-\infty}^{+\infty} \frac{e^{-As^2}}{\sigma_x \sigma_y} ds$$

with:

$$A = \frac{\sin^2\frac{\phi}{2}}{\sigma_x^2} + \frac{\cos^2\frac{\phi}{2}}{\sigma_s^2}$$

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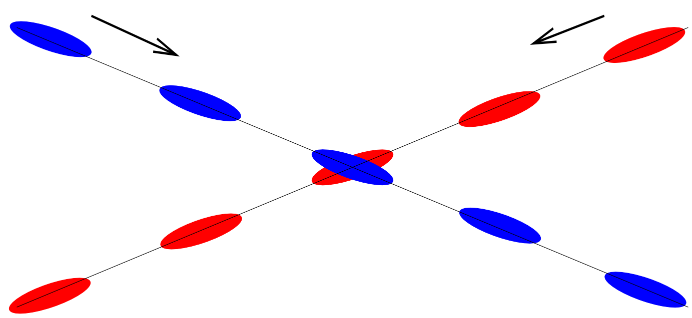
Crossing angle

- Crossing Angle $\Rightarrow$ $$\mathcal{L} = \frac{N_1 N_2 f n_b}{4\pi\sigma_x\sigma_y} \cdot S$$
- S is the reduction factor
- For small crossing angles and $\sigma_s \gg \sigma_{x,y}$
- $$\Rightarrow S = \frac{1}{\sqrt{1+(\frac{\sigma_s}{\sigma_x}\tan\frac{\phi}{2})^2}} \approx \frac{1}{\sqrt{1+(\frac{\sigma_s}{\sigma_x}\frac{\phi}{2})^2}}$$

Example LHC:

Φ = 285 μrad, σ_s = 7.5 cm, S = 0.84

Large crossing angle



→ **Large crossing angle: large loss of luminosity**

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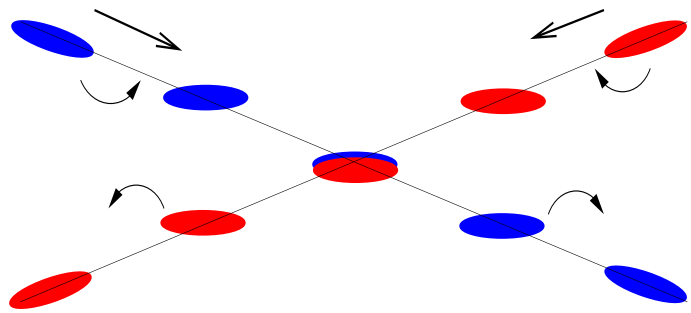
"crab" crossing scheme



- "crab" crossing recovers geometric loss factor
- feasibility needs to be demonstrated

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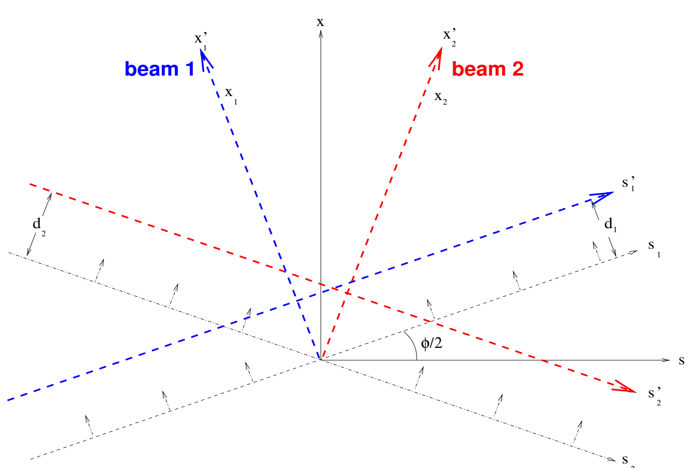
Offset and crossing angle



Offset and crossing angle

- **Transformations with offsets in crossing plane:**

$$\begin{cases} x_1 = d_1 + x\cos\frac{\phi}{2} - s\sin\frac{\phi}{2}, & s_1 = s\cos\frac{\phi}{2} + x\sin\frac{\phi}{2}, \\ x_2 = d_2 + x\cos\frac{\phi}{2} + s\sin\frac{\phi}{2}, & s_2 = s\cos\frac{\phi}{2} - x\sin\frac{\phi}{2} \end{cases}$$

- **Gives after integration over y and s_0:**

$$\mathcal{L} = \frac{\mathcal{L}_0}{2\pi\sigma_s\sigma_x} 2\cos^2\frac{\phi}{2} \int\int e^{-\frac{x^2\cos^2(\phi/2)+s^2\sin^2(\phi/2)}{\sigma_x^2}} e^{-\frac{x^2\sin^2(\phi/2)+s^2\cos^2(\phi/2)}{\sigma_s^2}}$$

$$\times \quad e^{-\frac{d_1^2+d_2^2+2(d_1+d_2)x\cos(\phi/2)-2(d_2-d_1)s\sin(\phi/2)}{2\sigma_x^2}} dxds.$$

Offset and crossing angle

After integration over x:

$$\mathcal{L} = \frac{N_1 N_2 f n_b}{8\pi^{\frac{3}{2}}\sigma_s} \quad 2\cos\frac{\phi}{2} \quad \int_{-\infty}^{+\infty} W \cdot \frac{e^{-(As^2+2Bs)}}{\sigma_x\sigma_y} ds$$

with:

$$A = \frac{\sin^2\frac{\phi}{2}}{\sigma_x^2} + \frac{\cos^2\frac{\phi}{2}}{\sigma_s^2} \qquad B = \frac{(d_2-d_1)\sin(\phi/2)}{2\sigma_x^2}$$

and $W = e^{-\frac{1}{4\sigma_x^2}(d_2-d_1)^2}$

$\Longrightarrow$ **Luminosity with correction factors**

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Luminosity with correction factors

$$\mathcal{L} = \frac{N_1 N_2 f n_b}{4\pi\sigma_x\sigma_y} \cdot W \cdot e^{\frac{B^2}{A}} \cdot S$$

- W: **correction for beam offset**
- S: **correction for crossing angle**
- $e^{\frac{B^2}{A}}$: **correction for crossing angle and offset**

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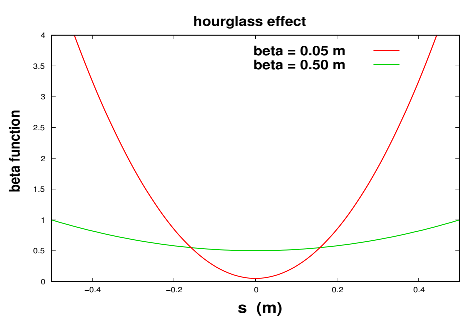
Hour glass effect



- β**-functions depends on position s**
- $\beta(s) \approx \beta^*(1 + \left(\frac{s}{\beta^*}\right)^2)$

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Hour glass effect

- β-functions depends on position s

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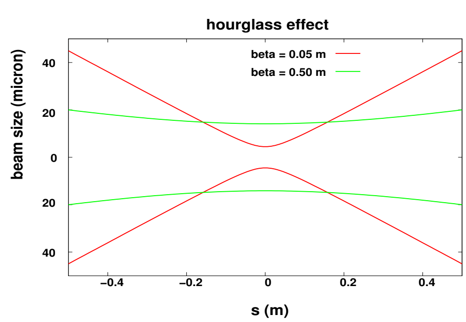
Hour glass effect



- Beam size σ $\quad (\propto \sqrt{\beta^*(s)})$ depends on position s
- For smaller β^*: very fast increase with distance to interaction point

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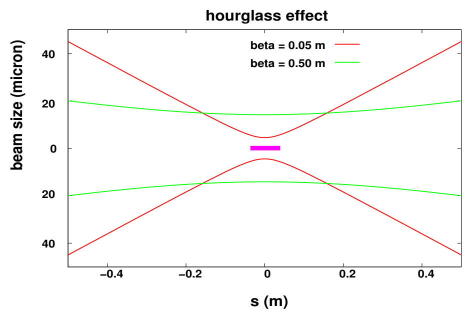
Hour glass effect - short bunches



- Small variation of beam size along bunch

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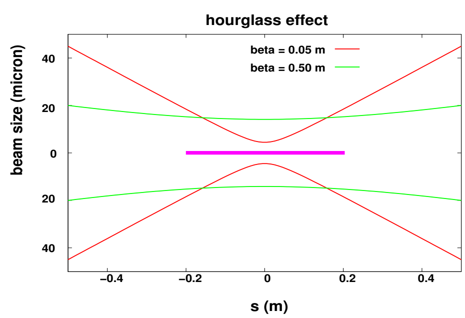
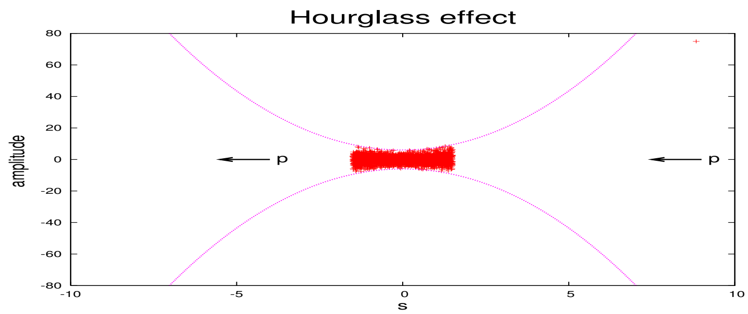
Hour glass effect - long bunches



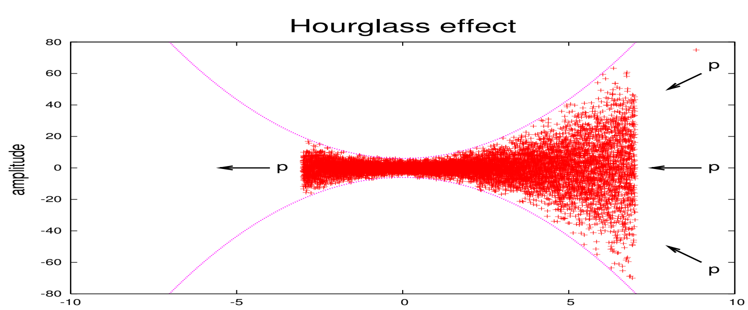
- Significant effect for long bunches and small β^*

Hour glass effect - short bunches



- Small variation of beam size along bunch

Hour glass effect - long bunches



- Significant effect for long bunches and small β^*

Hour glass effect

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- β**-functions depends on position s**
- **Usually:** $\beta(s) = \beta^*(1 + \left(\frac{s}{\beta^*}\right)^2)$

→ **i.e.** $\sigma \implies \sigma(s) \neq$ **const.**

→ $\sigma(s) = \sigma^* \sqrt{(1 + \left(\frac{s}{\beta^*}\right)^2)}$

- **Important when** β^* **comparable to the r.m.s. bunch length** σ_s **(or smaller !)**

Hour glass effect

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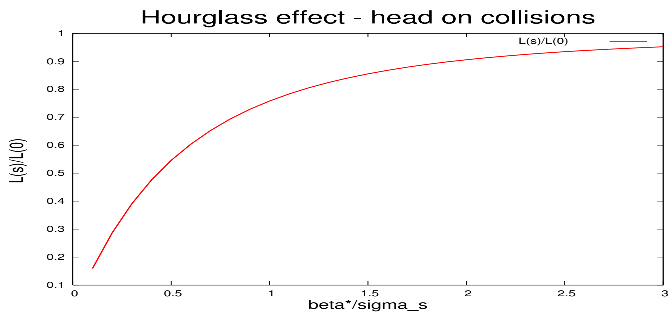
- **Without crossing angle and for symmetric, round Gaussian beams we get the relative luminosity reduction as:**

$$\frac{\mathcal{L}(\sigma_s)}{\mathcal{L}(0)} = \int_{-\infty}^{+\infty} \frac{1}{\sqrt{\pi}} \frac{e^{-u^2}}{[1 + (\frac{u}{u_x})^2]} du = \sqrt{\pi} \cdot u_x \cdot e^{u_x^2} \cdot \mathrm{erfc}(u_x)$$

Using the expression: $u_x = \beta^* / \sigma_s$

Hour glass effect



➡ **Hourglass reduction factor as function of ratio β^*/σ_s.**

Luminosity loss with longitudinal displacement

- **Loss of luminosity if beams collide with longitudinal displacement: i.e. β-waist not at collision point ($\delta\beta^*/\delta s = \alpha^* \neq 0$):**

➤ $$\frac{\mathcal{L}(\sigma_s)}{\mathcal{L}(0)} = \int_{-\infty}^{+\infty} \frac{1}{\sqrt{\pi}} \frac{e^{-(u-u_w)^2}}{[1+(\frac{u}{u_x})^2]} du$$

Using the expression: $u_w = s_w/\sigma_s$

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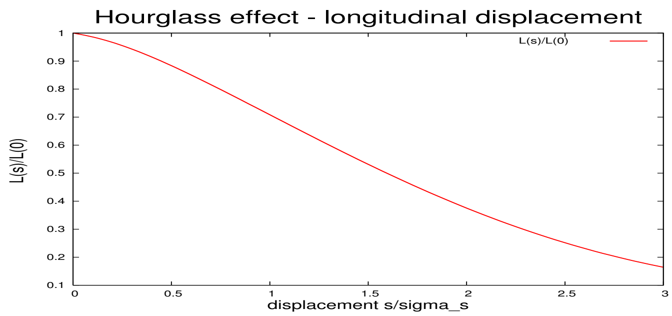
Hour glass effect



➡ **Hourglass reduction factor with longitudinal displacement s_w as function of ratio s_w/σ_s.**

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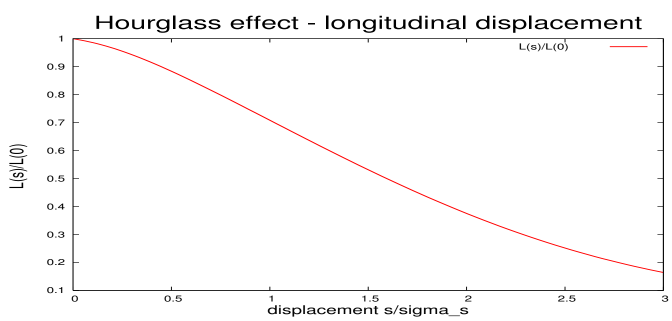
Hour glass effect



➡ **Where could that become important ???**

Calculations for the LHC

- $N_1 = N_2 = 1.15 \times 10^{11}$ **particles/bunch**
- $n_b = 2808$ **bunches/beam**
- $f = 11.2455$ kHz, $\phi = 285$ μ**rad**
- $\beta_x^* = \beta_y^* = 0.55$ m
- $\sigma_x^* = \sigma_y^* = 16.6$ μm, $\sigma_s = 7.7$ cm

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- **Simplest case (Head on collision):**

$$\mathcal{L} = 1.200 \times 10^{34} \text{ cm}^{-2}\text{s}^{-1}$$

- **Effect of crossing angle:**

$$\mathcal{L} = 0.973 \times 10^{34} \text{ cm}^{-2}\text{s}^{-1}$$

- **Effect of crossing angle & Hourglass:**

$$\mathcal{L} = 0.969 \times 10^{34} \text{ cm}^{-2}\text{s}^{-1}$$

What about large crossing angle and long bunches ???

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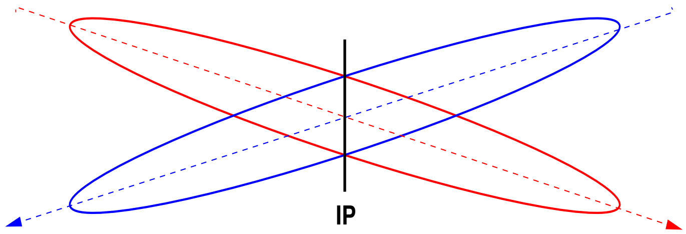
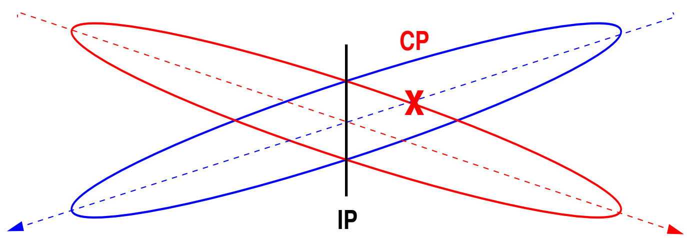
Large crossing angle - long bunches



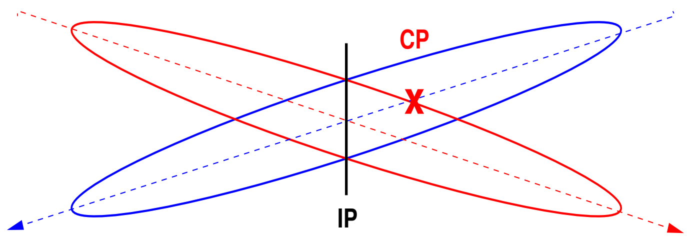
- Assume crossing angle in horizontal plane
- Large crossing angle: large loss of luminosity

Large crossing angle



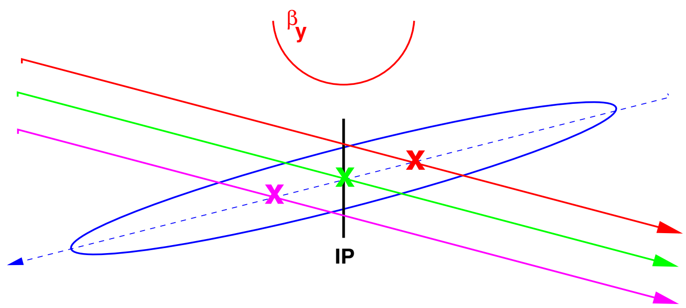
- For large amplitude particles: collision point longitudinally displaced

Large crossing angle



- For large amplitude particles: collision point longitudinally displaced
- Can introduce coupling (transverse and synchro betatron, bad for flat beams)

Large crossing angle



- A particle's collision point amplitude dependent
- Different (vertical) β functions at collision points

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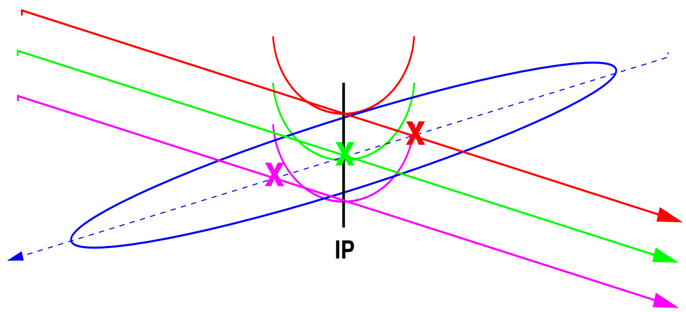
Large crossing angle



- A particle's collision point amplitude dependent
- Different β functions at collision points (hour glass !)

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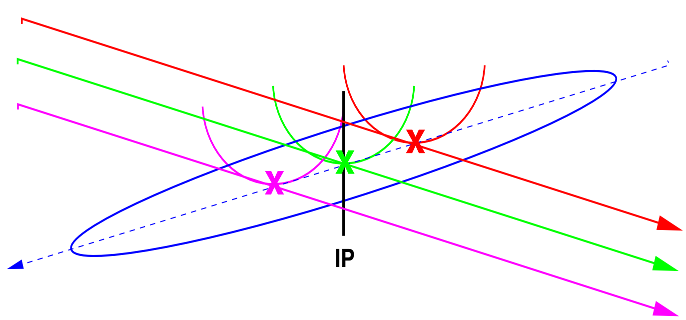
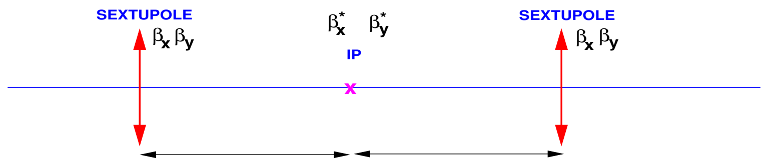
"crab waist" scheme



- Make vertical waist (β_y^{min}) amplitude (x) dependent
- All particles in both beams collide in minimum β_y region

"crab waist" scheme optical setup



$\Delta\mu_y = \frac{\pi}{2}$ $\Delta\mu_y = \frac{\pi}{2}$

$\Delta\mu_x = \pi$ $\Delta\mu_x = \pi$

for the sextupole strength:

$k = \frac{1}{2\theta}\frac{1}{\beta_y^*\beta_y}\sqrt{\frac{\beta_x^*}{\beta_x}}$ **and:** $\beta_y = \beta^* + \frac{(s-x/\theta)^2}{\beta_y^*}$

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"crab waist" scheme

- Make vertical waist (minimum of β) amplitude (x) dependent
- Can be done with two sextupoles
- First tried at DAPHNE (Frascati) in 2008
- Geometrical gain small
- Smaller vertical tune shift as function of horizontal coordinate
  - Less betatron and synchrotron coupling
  - Good remedy for flat (i.e. lepton) beams with large crossing angle

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If the beams are not Gaussian ??

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Exercise:

- **Assume flat distributions (normalized to 1)**
  $\rho_1 = \rho_2 = \frac{1}{2a}$, **for** $[-a \leq z \leq a]$, $z = x, y$

  Calculate r.m.s. in x and y:

$$< (x,y)^2 > = \int_{-\infty}^{+\infty} (x,y)^2 \cdot \rho(x,y) dxdy$$

and

$$\mathcal{L} = \int_{-\infty}^{+\infty} \rho_1(x,y)\rho_2(x,y)\ dxdy$$

- **Compute:** $\mathcal{L} \cdot \sqrt{< x^2 > \cdot < y^2 >}$
- **Repeat for various distributions and compare**

What about unequal bunch lengths ?

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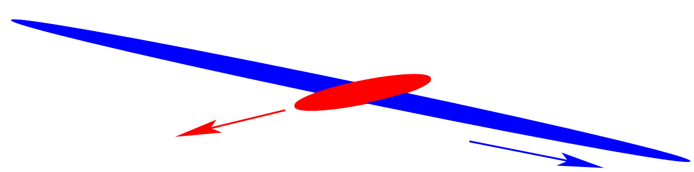

- **Overlap not optimum**
- **Typical case: lepton hadron colliders (HERA, LHeC)**

What about unequal bunch lengths ?

- Relevant with a crossing angle
- For lepton-hadron colliders usually:
  - $\sigma_s^p \gg \sigma_s^e$
  - For example HERA and LHeC: $\sigma_s^p \approx 8$ cm, $\sigma_s^e \leq 1$ cm
- Same procedures applied, but now with $\sigma_s^p \neq \sigma_s^e$

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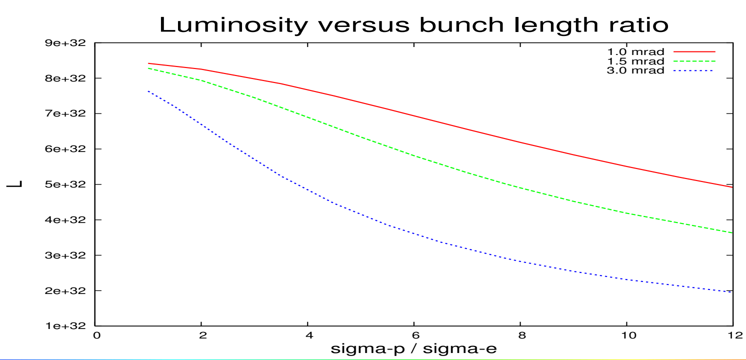
Luminosity correction

- Result from numerical integration for $\sigma_s^p \neq \sigma_s^e$
- Vary proton/electron bunch length ratio



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Integrated luminosity

- $\mathcal{L}_{\text{int}} = \int_0^T \mathcal{L}(t)dt$
- **The figure of merit:**
  $\mathcal{L}_{\text{int}} \cdot \sigma_p =$ **number of events**
- **Experiments: continuous recording of** $\mathcal{L}$
- **For studies: assume some life time behaviour.**
  E.g. $\mathcal{L}(t) \longrightarrow \mathcal{L}_0 \exp\left(-\frac{t}{\tau}\right)$
- **Contributions to life time from: intensity decay, emittance growth etc.**

Integrated luminosity

- **Knowledge of preparation time allows optimization of** $\mathcal{L}_{\text{int}}$

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Integrated luminosity

- **Typical run times LEP:**
  t_r $\approx$ 8 - 10 hours
- **For LHC long preparation time t_p expected**

→ **Optimum combination of t_r and t_p gives maximum luminosity**

→ **t_r is usually a "free" parameter, i.e. can be chosen**

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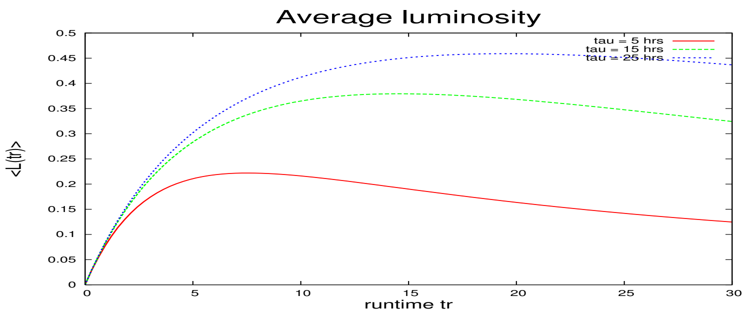
Maximising Integrated Luminosity

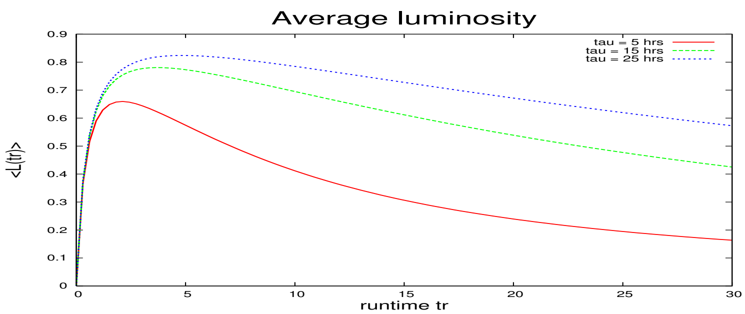
- **Assume exponential decay of luminosity**
  $\mathcal{L}(t) = \mathcal{L}_0 \cdot e^{-t/\tau}$
- **Average luminosity $< \mathcal{L} >$**
  $< \mathcal{L} >= \frac{\int_0^{t_r} dt \mathcal{L}(t)}{t_r + t_p} = \mathcal{L}_0 \cdot \tau \cdot \frac{1 - e^{-t_r/\tau}}{t_r + t_p}$
- **(Theoretical) maximum for:**
  $t_r \approx \tau \cdot \ln(1 + \sqrt{2 t_p/\tau} + t_p/\tau)$
- **Example LHC: $t_p \approx$ 10h, $\tau \approx$ 15h, $\Rightarrow t_r \approx$ 15h**
- **Exercise: Would you improve τ (long t_r) or t_p ?**

Average luminosity for different run times



- **Average luminosity, τ_p = 10 hrs, for different beam lifetimes**

Average luminosity for different run times



- **Average luminosity, τ_p = 0.5 hr, shorter run times preferred**

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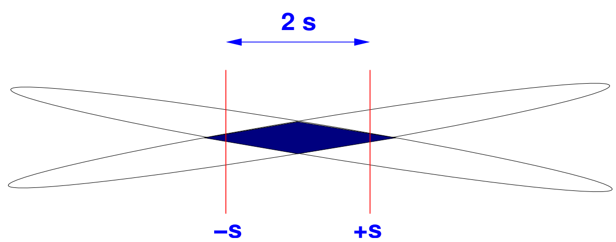
Luminous region



- Density distribution of interaction vertices
- Fraction of collisions occur ± s from the IP ?
- Important for experiments !

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Luminous region

- Depends on σ_x, σ_y, σ_s and crossing angle ϕ
- Integrate only along a finite longitudinal length:

$$\mathcal{L}_0 = \int_{-\infty}^{+\infty} \mathcal{L}(s')ds' \longrightarrow \mathcal{L}(S) = \int_{-S}^{+S} \mathcal{L}(s')ds'$$

$$\mathcal{L}(S) = \left(\frac{N_1 N_2 f n_b}{8\pi\sigma_x^*\sigma_y^*}\right) \frac{2\cos\frac{\phi}{2}}{\sqrt{\pi}\sigma_s} \sqrt{\frac{\pi}{A}}\ \mathrm{erf}\left(\sqrt{A}\ S\right)$$

- Real figure of merit: $\mathcal{L}_{\mathrm{int}}(S) = \int_0^T \int_{-S}^{+S} \mathcal{L}(s',t)ds'dt$

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Some results for LHC

- $\sigma_s = 7.7$ cm, $\beta^* = 0.55$ m, $\phi = 285$ μrad:
- 100% **lumi** $\rightarrow$ $S = \pm$ 12 cm
- 90% **lumi** $\rightarrow$ $S = \pm$ 7 cm
- 80% **lumi** $\rightarrow$ $S = \pm$ 5.5 cm

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Interactions per crossing

- **Luminosity/fn_b $\propto$ N_1N_2**
- **In LHC: crossing every 25 ns**
- **Per crossing approximately 20 interactions**
- **May be undesirable (pile up in detector)**
- $\Longrightarrow$ **more bunches n_b, or smaller N ??**

Beware: maximum (peak) luminosity $\mathcal{L}_{max}$
is not the whole story ... !

Luminosity measurement

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- One needs to get a signal proportional to interaction rate → Beam diagnostics
- Large dynamic range: 10^{27} cm^{-2}s^{-1} to 10^{34} cm^{-2}s^{-1}
- Very fast, if possible for individual bunches
- Used for optimization
- For absolute luminosity need calibration

Luminosity calibration

(e^+e^-)

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- Use well known and calculable process
- $e^+e^- \rightarrow e^+e^-$ elastic scattering (Bhabha scattering)
- Have to go to small angles ($\sigma_{el} \propto \Theta^{-3}$)
- Small rates at high energy ($\sigma_{el} \propto \frac{1}{E^2}$)

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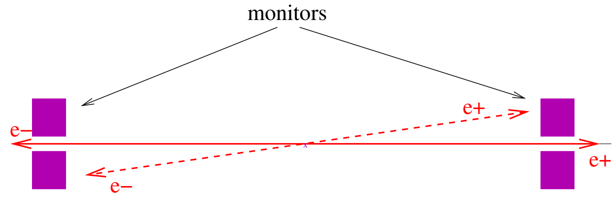
Luminosity calibration



- Measure coincidence at small angles
- Low counting rates, in particular for high energy !
- Background may be problematic

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Luminosity calibration

(hadrons, e.g. pp or $p\bar{p}$)

- Must measure beam current and beam sizes
- Beam size measurement:
  - Wire scanner or synchrotron light monitors
  - Measurement with beam ... → remember luminosity with offset
  - Move the two beams against each other in transverse planes (van der Meer scan)

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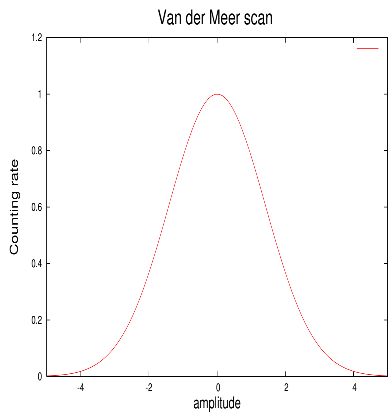
Luminosity optimization



- Record counting rates *R(d)* as function of movement *d*
- Since *R(d)* is proportional to luminosity *L(d)*
- Get ratio of luminosity *L(d)/L(0)*

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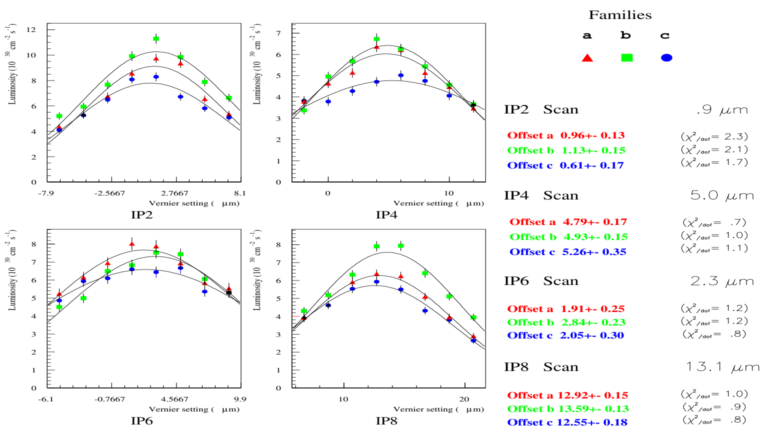
Luminosity optimization

- From ratio of luminosity $\mathcal{L}(\mathrm{d})/\mathcal{L}_0$
- Remember: $W = e^{-\frac{1}{4\sigma^2}(d_2-d_1)^2}$
- Determines σ
- ... and centres the beams !
- Others:
  - Beam-beam deflection scans LEP
  - Beam-beam excitation

Scan in LEP with bunch trains



Absolute value of $\mathcal{L}$ (pp or $p\bar{p}$)

- **By total rate and optical theorem**

  (also: luminosity independent determination of σ_{tot}):

  - $\sigma_{tot} \cdot \mathcal{L} = N_{inel} + N_{el}$ **(Total counting rate)**
  - $\lim_{t\to 0} \frac{d\sigma_{el}}{dt} = (1+\rho^2)\frac{\sigma_{tot}^2}{16\pi} = \frac{1}{\mathcal{L}}\frac{dN_{el}}{dt}|_{t=0}$

  → $\mathcal{L} = \frac{(1+\rho^2)}{16\pi}\frac{(N_{inel}+N_{el})^2}{(dN_{el}/dt)_{t=0}}$

- **Luminosity determined from experimental rates**

Absolute value of $\mathcal{L}$ (pp or $p\bar{p}$)

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- **By Coulomb normalization:**
  - **Coulomb amplitude exactly calculable:**
  - $\lim_{t\to 0} \frac{d\sigma_{el}}{dt} = \frac{1}{\mathcal{L}}\frac{dN_{el}}{dt}|_{t=0} = \pi \mid f_C + f_N \mid^2$
    $\simeq \pi \mid \frac{2\alpha_{em}}{-t} + \frac{\sigma_{tot}}{4\pi}(\rho + i)e^{b\frac{t}{2}} \mid^2 \simeq \frac{4\pi\alpha_{em}^2}{t^2} \mid_{|t|\to 0}$
  - **Fit gives:** σ_{tot}, ρ, b **and** $\mathcal{L}$
- **Can be done measuring only elastic scattering (No N$_{inel}$ needed !)**

Optics for luminosity measurement

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- **Want to measure small momentum transfer reactions:**
  - $\mid t \mid \to 0$
  - **Small beam divergence** σ' **needed !**
- **Since** $\sigma' = \sqrt{\epsilon/\beta^*}$
  - **Large** β^* **at collision point needed ...**

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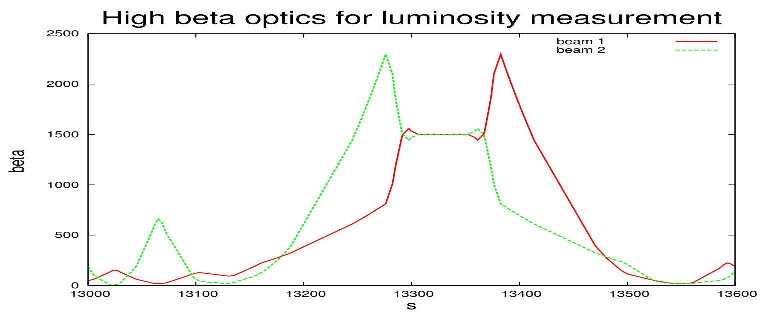
High β^* optics for luminosity measurement



- For LHC: β^* 1500 - 2600 m foreseen

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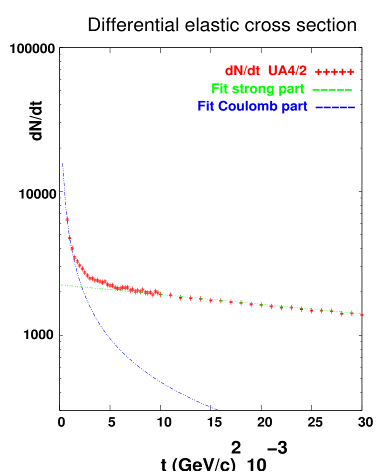

- Measure dN/dt at small t (0.01 < (GeV/c)**2) and extrapolate to t = 0.0
- Needs special optics to allow measurement at very small t
- Measure total counting rate N_{el} + N_{inel} Needs good detector coverage
- Often use slightly modified method, precision 1 – 2 %

Luminosity in linear colliders

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- Mainly (only) $e+e-$ colliders
- Past collider: SLC (SLAC)
- Under consideration: CLIC, ILC
- Special issues:
  - Particles collide only once (dynamics) !
  - Particles collide only once (beam power) !

→ Must be taken into account

Luminosity in linear colliders

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- Basic formula:

$$\text{From :} \quad \mathcal{L} = \frac{N^2 \; f \; n_b}{4\pi\sigma_x\sigma_y} \quad \text{to :} \quad \mathcal{L} = \frac{N^2 \; f_{rep} \; n_b}{4\pi\sigma_x\sigma_y}$$

- Replace frequency f by repetition rate f_{rep}.
- And introduce effective beam sizes $\overline{\sigma_x}, \overline{\sigma_y}$:

$$\mathcal{L} = \frac{N^2 \; f_{rep} \; n_b}{4\pi\overline{\sigma_x} \; \overline{\sigma_y}}$$

Luminosity in linear colliders

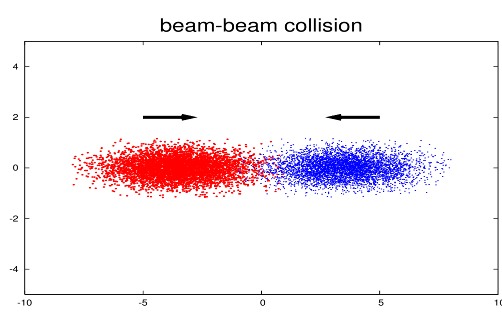
- Using the enhancement factor H_D:

$$\mathcal{L} = \frac{N^2\ f_{rep}\ n_b}{4\pi\overline{\sigma_x}\ \overline{\sigma_y}} \rightarrow \mathcal{L} = \frac{H_D \cdot N^2\ f_{rep}\ n_b}{4\pi\sigma_x\ \sigma_y}$$

- Enhancement factor H_D takes into account reduction of nominal beam size by the disruptive field (pinch effect)
- Related to disruption parameter $\mathcal{D}$:

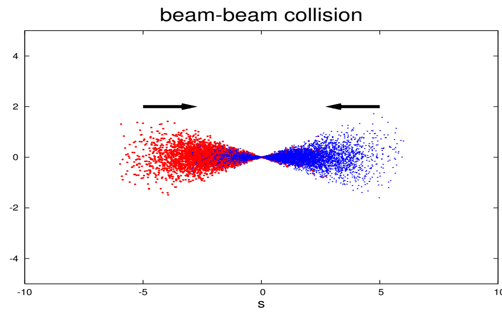
$$\mathcal{D}_{x,y} = \frac{2r_e N\sigma_z}{\gamma\sigma_{x,y}(\sigma_x + \sigma_y)}$$

Pinch effect - disruption



- Additional focusing by opposing beams

Pinch effect - disruption



➤ **Additional focusing by opposing beams**

Luminosity in linear colliders

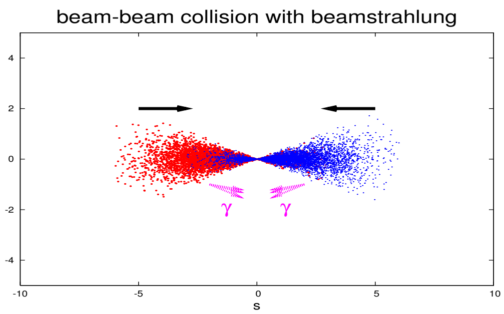
- **For weak disruption $\mathcal{D} \ll 1$ and round beams:**

$$H_D = 1 \; + \; \frac{2}{3\sqrt{\pi}}\mathcal{D} \; + \; \mathcal{O}(\mathcal{D}^2)$$

- **For strong disruption and flat beams: computer simulation necessary, maybe can get some scaling**

Beamstrahlung

- Disruption at interaction point is basically a strong "bending" (can be very strong !)
- Results in strong synchrotron radiation: beamstrahlung



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Beamstrahlung

- Disruption at interaction point is basically a strong "bending" (can be very strong !)
- Results in strong synchrotron radiation: beamstrahlung
- This causes (unwanted):
  - Spread of centre-of-mass energy
  - Pair creation and detector background
- Again: luminosity is not the only important parameter

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Beamstrahlung Parameter Y

- **Measure of the mean field strength in the rest frame normalized to critical field B_c:**

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$$Y = \frac{< E + B >}{B_c} \approx \frac{5}{6} \frac{r_e^2 \gamma N}{\alpha \sigma_z (\sigma_x + \sigma_y)}$$

with:

$$B_c = \frac{m^2 c^3}{e\hbar} \approx 4.4 \times 10^{13} G$$

Energy loss and power consumption

- **Average fractional energy loss δ_E:**

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$$\delta_E = 1.24 \frac{\alpha \sigma_z m_e}{\lambda_C E} \frac{Y}{(1 + (1.5Y)^{2/3})^{1/2}}$$

where E is beam energy at interaction point and λ_C the Compton wavelength.

Luminosity in linear colliders

- Using the beam power P_b and beam energy E in the luminosity:

$$\mathcal{L} = \frac{H_D \cdot N^2 \ f_{rep} \ n_b}{4\pi\sigma_x \ \sigma_y} \quad \rightarrow \quad \mathcal{L} = \frac{H_D \cdot N \cdot P_b}{eE \cdot 4\pi\sigma_x \ \sigma_y}$$

- Beam power P_b related to AC power consumption P_{AC} via efficiency η_b^{AC}

$$P_b \quad = \quad \eta_b^{AC} \quad \cdot \quad P_{AC}$$

Figure of merit in linear colliders

- Luminosity at given energy normalized to power consumption and momentum spread due to beamstrahlung:

$$M = \frac{\mathcal{L}E}{\sqrt{\delta_b} P_{AC}}$$

- With previous definition (and reasonably small beamstrahlung) this becomes:

$$M = \frac{\mathcal{L}E}{\sqrt{\delta_b} P_{AC}} \quad \propto \quad \frac{\eta_b^{AC}}{\sqrt{\epsilon_y^*}}$$

- These are optimized in the linear collider design

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- **These are optimized in the linear collider design**

Not treated :

- Coasting beams (e.g. ISR)
- Asymmetric colliders:
  - Asymmetric energies (e.g. PEP)
  - Asymmetric charges (e.g. p-Pb)
  - Ring - LINAC colliders

How to cook high Luminosity ?

- **Get high intensity**
- **Get small beam sizes (small ϵ and β^*)**
- **Get many bunches**
- **Get small crossing angle (if any)**
- **Get exact head-on collisions**
- **Get short bunches**