Vacuum Science and Technology in Accelerators

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Session 9

Calculations to Support the Design

Aims

- To re-emphasise the importance of conductance limitations on available pumping speed
- To look at some of the techniques available to calculate pressure distributions
- To identify some of the software packages available

Conductance and pumping speed

In accelerator design, we will really only be concerned with the molecular flow regime.

We recall that for a short circular pipe, the conductance is given by

$$C = 12.4 \frac{D^3 / L}{1 + 4D / 3L}$$

l sec^{-1} (for N_2 at 295K)

D,L in cm

So for a short pipe of length 20 cm and diameter 10 cm, this gives a conductance of about 375 l sec^{-1}

Therefore there is little point in fitting a pump much above 400 l sec^{-1} on the end of this pipe.

Conductance and pumping speed

If we have a part of the accelerator with a beam tube of diameter 10 cm and length 30 m, with an outgassing rate of 10^{-11} mbar l sec^{-1},

$$Q \sim 10^{-6} \text{ mbar l sec}^{-1}$$

So to get a pressure of 10^{-9} mbar we need a pumping speed of 1000 l sec^{-1}, or about 3 pumps connected by tubes as on the previous slide.

If, however, the tube is only 2 cm in diameter, the gas load is 2×10^{-7} and in principle, we need only one such pump.

Conductance and pumping speed

However, in this case the pumping tube is much wider than the beam tube. So if we also reduce it to 2 cm in diameter, the conductance becomes less than 5 l sec^{-1} and we need 40 spaced out pumps!

This is rarely viable.

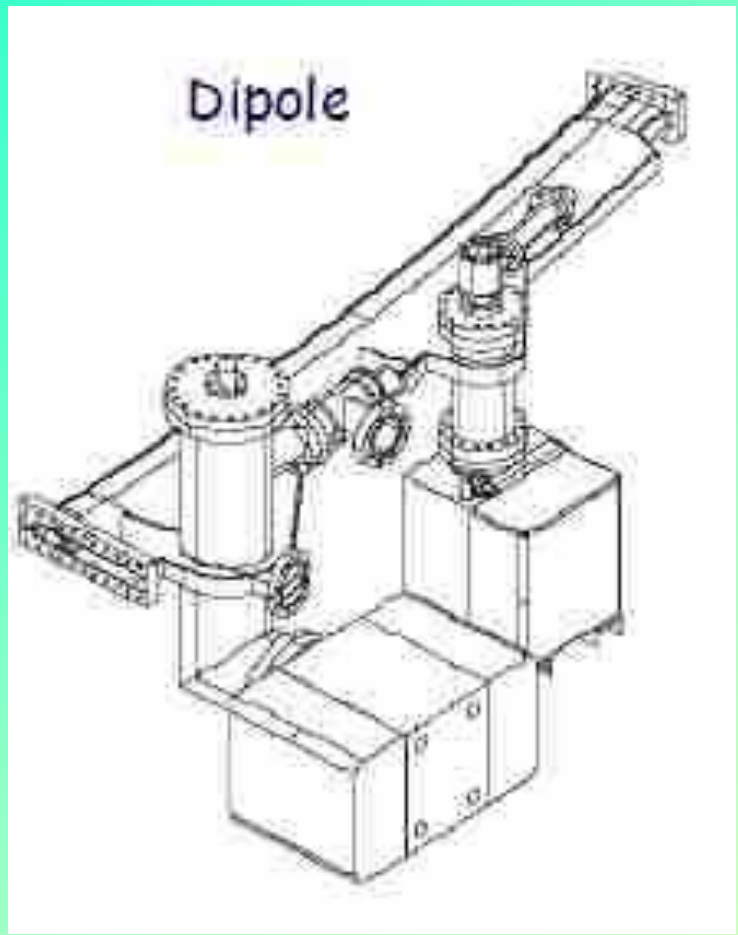
The usual solution (where possible) is to use pumping chambers wider than the beam tube inserted at appropriate intervals.

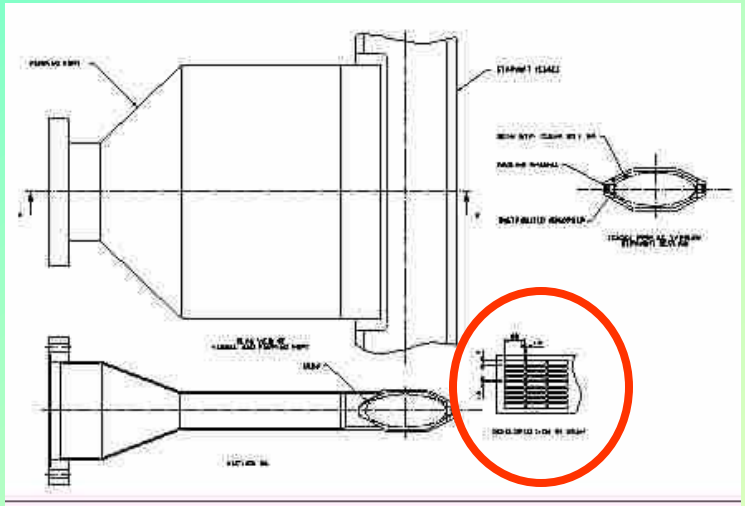
Conductance and pumping speed

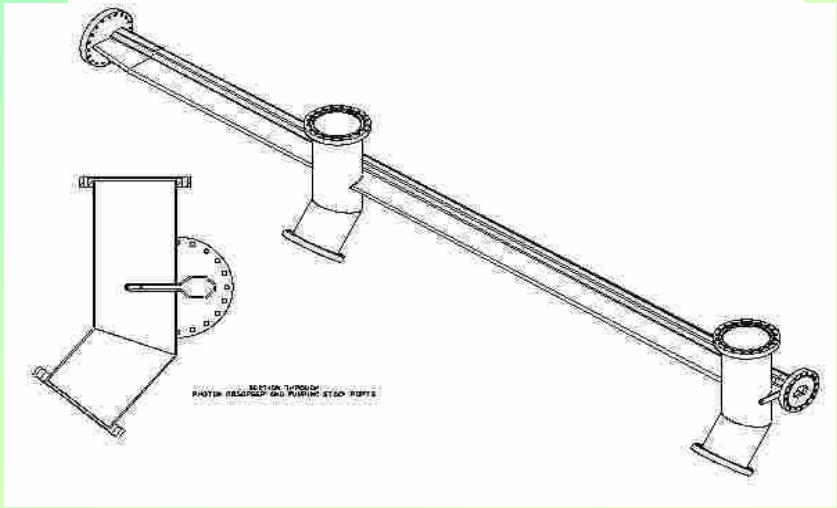
In a real situation, where one puts pumps is constrained and one works quite hard to ensure that the placement of the pumps is reasonably optimum and calculates quite carefully the conductance of the connecting pipework or pumping spouts to get the best capture facture for the pump system.

Calculations to Support the Design

Conductance and pumping speed

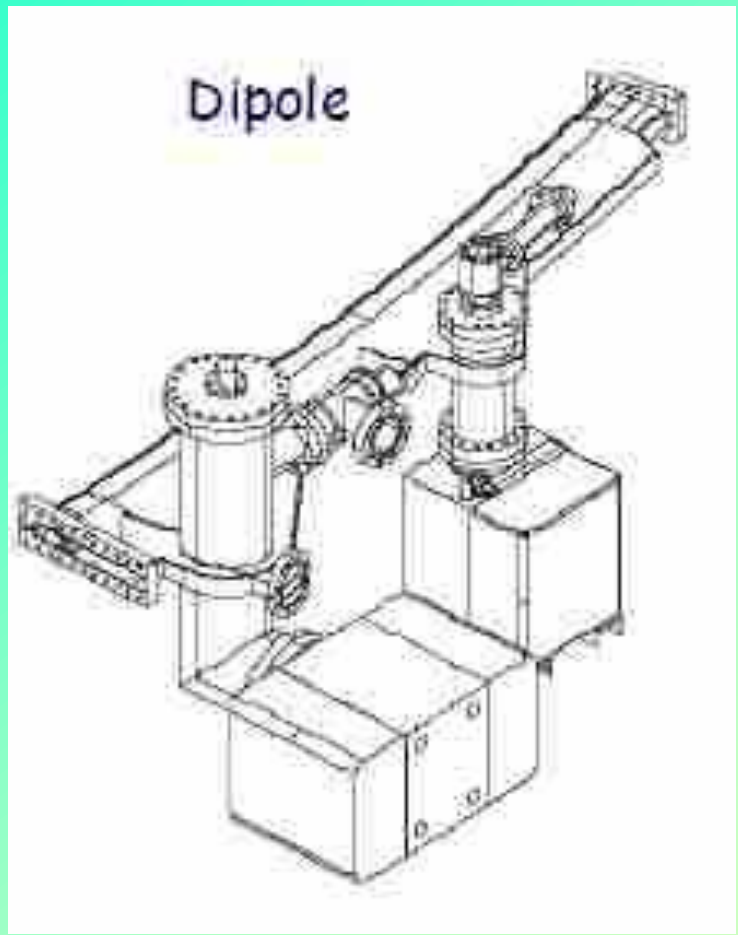


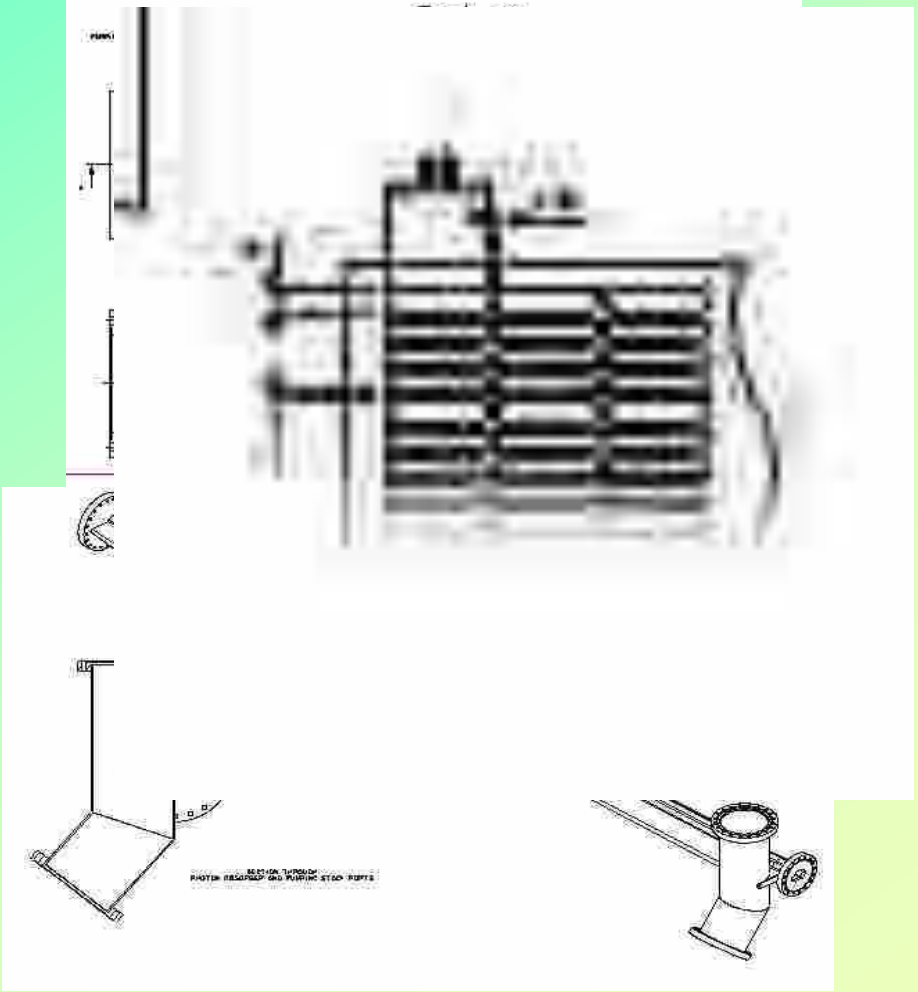




Calculations to Support the Design

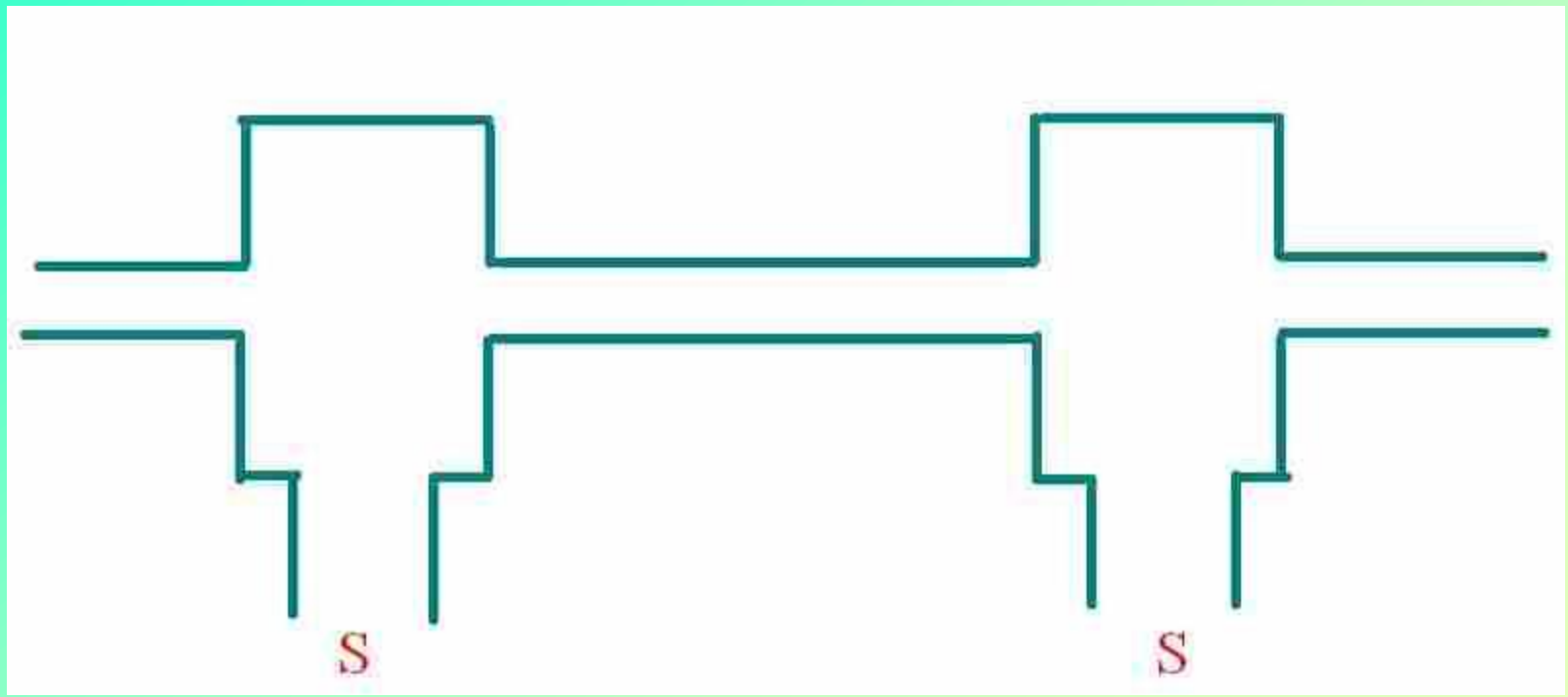
Conductance and pumping speed





Calculations to Support the Design

Conductance and pumping speed



We need to calculate the average pressure along an array of this type

Pressure Distributions – Analytical Method

For a distributed system, one can set up the differential equation

$$c\frac{\partial^2 P}{\partial z^2} - sP + q = 0$$

Where c is the distributed conductance

s is the distributed pumping speed

q is the distributed gas load

For the earlier system this gives a distribution

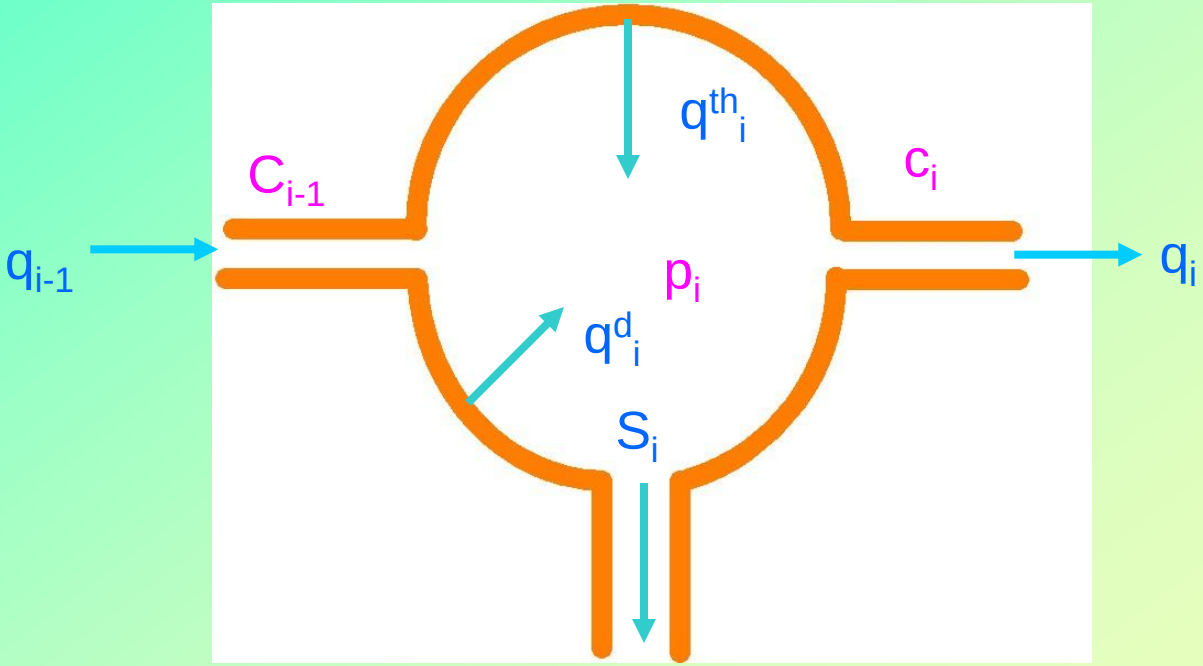
Pressure Distributions – Analytical Method

Clearly, applying this approach to a real distributed system is only reasonable if the system itself is reasonably uniform i.e. there are no point sources of gas for example.

The formalism can be extended, but it becomes messy.

Pressure Distributions – Finite Element Method

For instance, if the cross section varies, this can be difficult to handle analytically. For reasonably simple systems, one can use a finite element like formalism.



Pressure Distributions – Finite Element Method

Then, from the detailed balance of gas flow at each element, one can set up a system of equations which can be expressed as

$$\vec{p} = \vec{C}^{-1} \vec{q}$$

Where $\vec{p}$ is a vector of the pressures

$\vec{q}$ is a vector of the desorptions

$\vec{C}$ is a system matrix incorporating the inter-element conductances and element pumping speeds

Pressure Distributions – Finite Element Method

The system matrix looks something like

$$\begin{pmatrix}
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\cdots & \cdots & -c_{i-2} & 0 & 0 & \cdots & \cdots \\
\cdots & -c_{i-2} & s_{i-1}+c_{i-2}+c_{i-1} & -c_{i-1} & 0 & 0 & \cdots \\
\cdots & 0 & -c_{i-1} & s_i+c_{i-1}+c_i & -c_i & 0 & \cdots \\
\cdots & 0 & 0 & -c_i & s_{i+1}+c_i+c_{i+1} & -c_{i+1} & \cdots \\
\cdots & \cdots & 0 & 0 & -c_{i+1} & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots
\end{pmatrix}$$

Pressure Distributions – Finite Element Method

The equations are comparatively easy to set up, and modest systems can be calculated using MathCad, Matlab or similar packages.

There are dedicated computer packages available for calculations based on these methods.

Examples are Vaccalc and Vactrak, both originating in SLAC.

Pressure Distributions

The main problems with the finite element method are

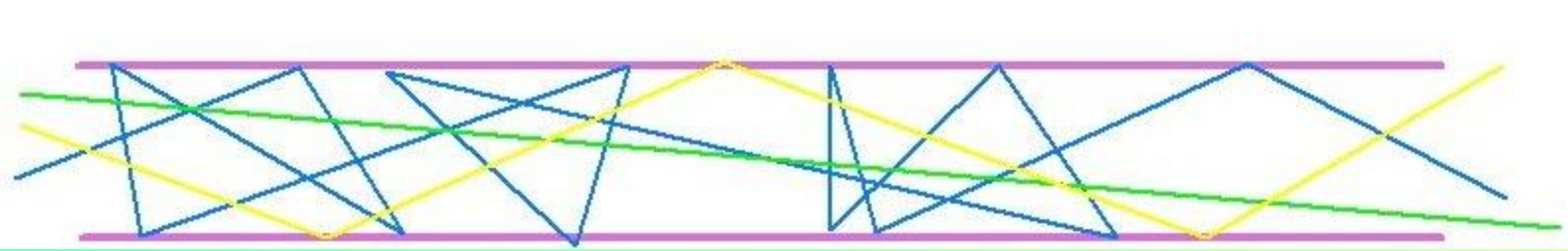
- calculating the interelemental conductances
- beaming effects are ignored

For anything apart from simple quasi one dimensional systems, this can cause significant inaccuracies.

One solution is to use a Monte Carlo simulation.

Pressure Distributions – Monte Carlo Simulation

Calculations use the test particle Monte Carlo method



Particles are generated at random at the entrance and are followed through the system until they are either trapped (pumped) or exit at either end.

Pressure Distributions – Monte Carlo Simulation

Monte Carlo calculations make no assumptions about the vacuum system except that the particles do not interact and all scattering is diffuse and elastic.

The pressure exerted on any surface is then proportional to the number of molecules striking that surface. The pressure distribution along a complex string like an accelerator can be most easily determined by counting the axial distribution of particles passing through a transparent imaginary facet along the axis.

Pressure Distributions – Monte Carlo Simulation

The entire vacuum system is split up into a number of facets, the number depending on the complexity of individual vessels.

Pumping is simulated by making facets “sticky” i.e. assigning them a capture coefficient.

Desorption (thermal or stimulated) can be simulated by generating particles at the appropriate facet inside the system.

Pressure Distributions – Monte Carlo Simulation

Such calculations require the generation of many test particles and are run until a termination condition is satisfied.

For a pressure distribution it will be until a stable output is achieved.

If one is using such a calculation to determine a transmission coefficient for a differential pumping stage for example, then the termination condition will be when a sufficient number of particles has been transmitted.

Pressure Distributions – Monte Carlo Simulation

Several different dedicated computer programs have been written at major accelerator laboratories and most are readily available.

One in reasonably widespread use is MolFlow written by Roberto Kersevan at ESRF.

Pressure Distributions – Monte Carlo Simulation

Monte Carlo simulations work well on modern desk top computers and reasonable statistics can be obtained in a reasonable time for moderately complex systems.

However, it can take a long time to set up the geometrical parameters of a moderately complex system and this process can be error prone.

In carrying out the vacuum design of an accelerator, we have seen that it is an iterative procedure to provide a satisfactory solution.

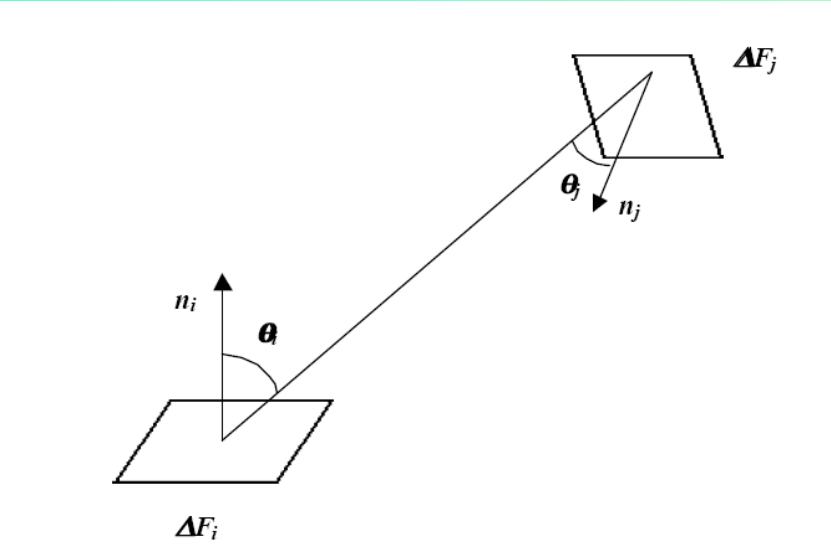
Pressure Distributions

For quasi one dimensional systems, an alternative method which can be much quicker to set up and run is the view factor method or method of angular coefficients.

This actually is a method very similar to that used in calculating the properties of optical systems.

Pressure Distributions – Method of Angular Coefficients

In this method, the molecular flux propagating from one elemental surface to another is considered



The differential angular coefficient is then given by

$$\partial\phi_{ij} = K(r_i, r_j)\,dF_j$$

where

$$K(r_i, r_j) = \begin{cases} \dfrac{\cos\theta_i \cos\theta_j}{\pi \left| r_i - r_j \right|^2} & 0<\theta_i<\pi/2 \text{ and } 0<\theta_j<\pi/2 \\ 0 & \pi/2<\theta_i<\pi \text{ and/or } \pi/2<\theta_j<\pi \end{cases}$$

Pressure Distributions – Method of Angular Coefficients

The probability of direct molecular interchange between the surfaces F_i and F_j is then

$$\phi_{i,j} = \int_{F_j} dF_j \int_{F_i} K(r_i, r_j) dF_i$$

If we define β_i as 1- α_i where α_i is the sticking coefficient of the i^{th} surface, then it is possible to set up a set of equations

$$q = q_i^0 + \beta_i \sum_{j=1}^{N} q_j K_{ij} \Delta F_j$$

which can be solved numerically or can be recast and solved iteratively

Pressure Distributions – Method of Angular Coefficients

The advantage of this method is that it can run much faster than a Monte Carlo simulation and therefore optimisation of a vacuum layout is a more tractable problem.

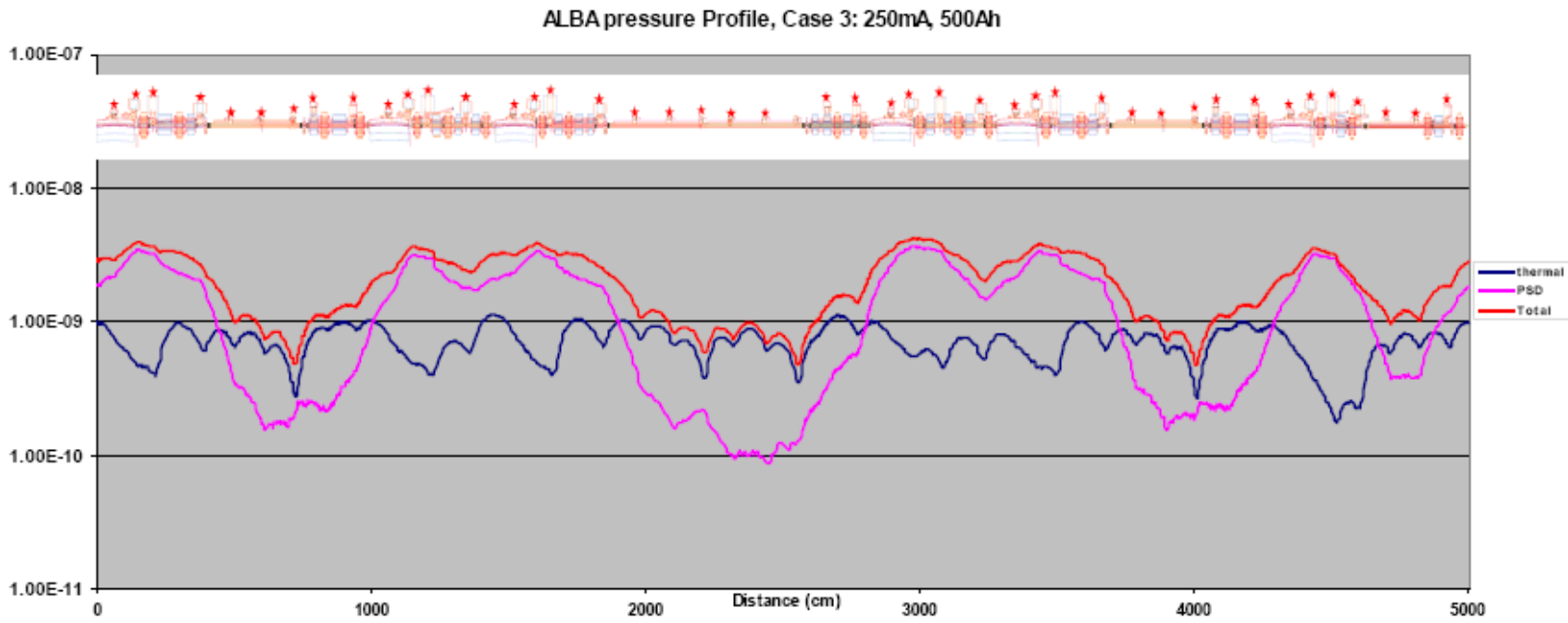
It can be set up to run on MathCad or MatLab.

For quasi one dimensional layouts, this method has been benchmarked against Monte Carlo.

Calculations to Support the Design

Pressure Distributions

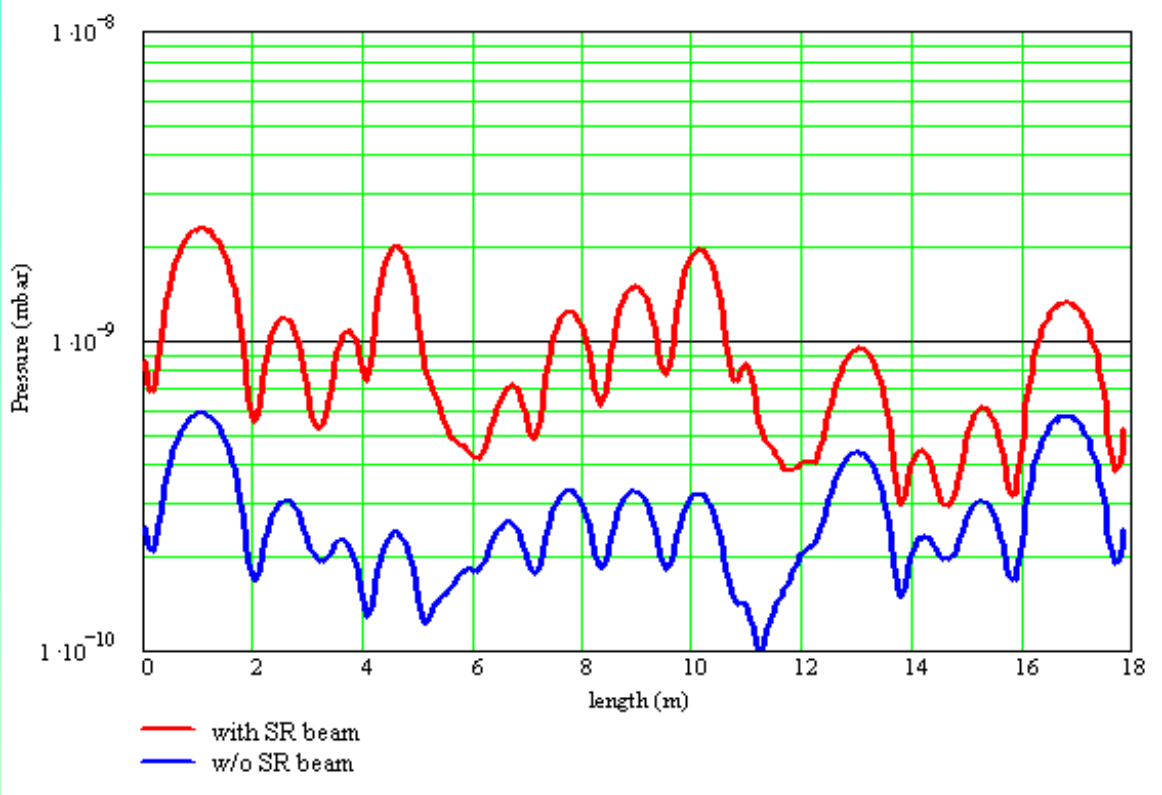
This is the result of a Molflow simulation



Calculations to Support the Design

Pressure Distributions

This is the result of a calculation using the method of angular coefficients



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Method of Angular Coefficients

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