



Vacuum Science and Technology in Accelerators

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Session 2

Basic Principles of Vacuum



Aims

- To present some of the results of the kinetic theory of gases and to understand how they affect our thinking about vacuum
- To understand the differences between gas flow regimes
- To understand why conductance is an important concept in vacuum



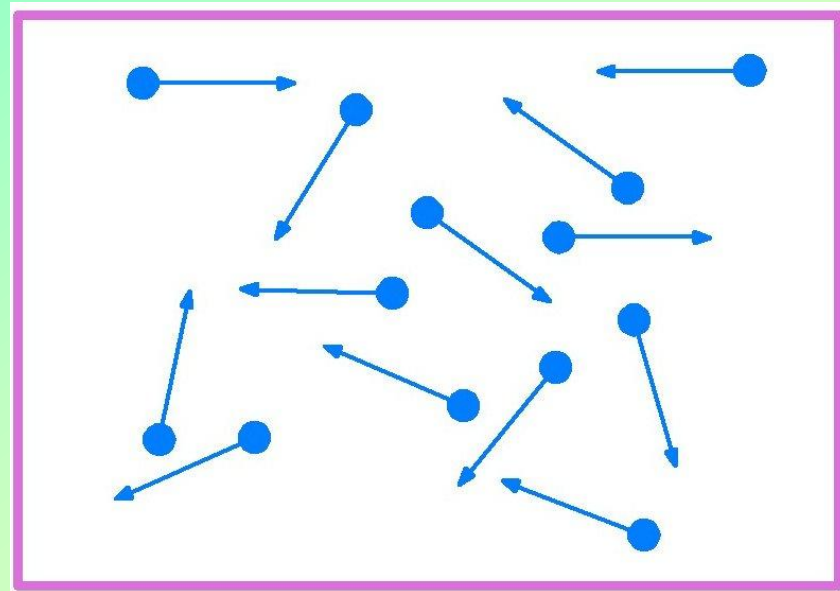
Kinetic Theory

- Consider gas as collection of independent small spheres in random motion, with average velocity \bar{v}
- All collisions are elastic

Volume of box = V

Number of molecules = N

Number density = $\frac{N}{V}$

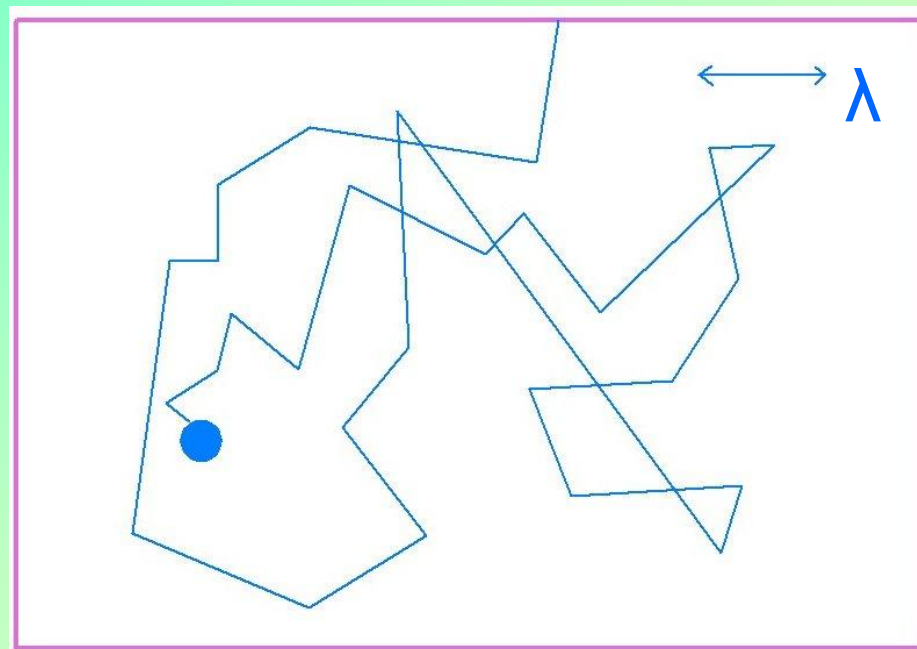




Kinetic Theory

- Molecules follow a random walk
- Mean free path λ

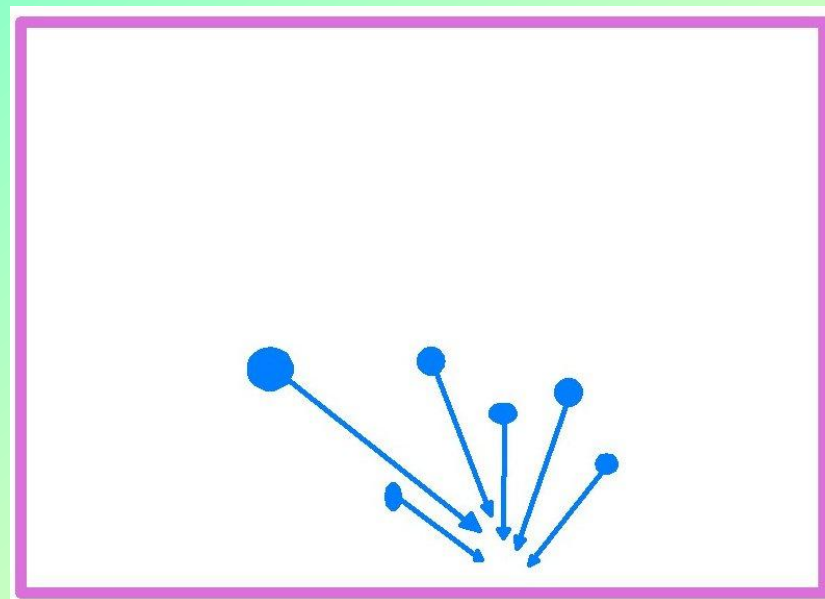
Pressure (mbar)	Mean free path (m)
10^3	6×10^{-8}
1	6×10^{-5}
10^{-3}	6×10^{-2}
10^{-6}	6
10^{-10}	6×10^5





Kinetic Theory

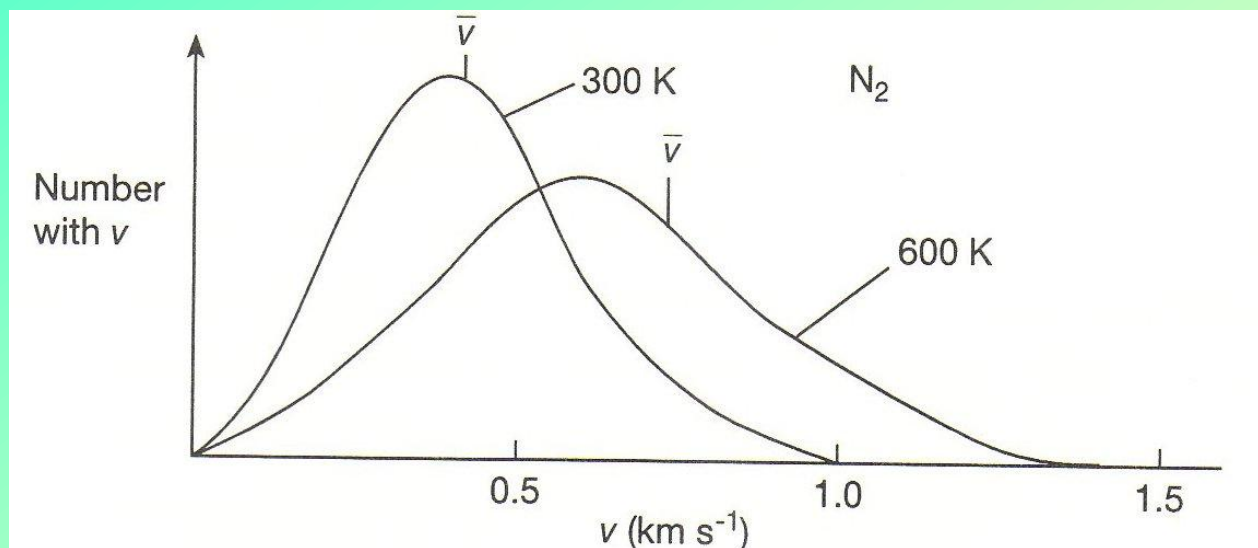
The pressure, p , exerted on the walls of the vessel depends on the molecular impingement rate or flux, J





Basic Principles of Vacuum

Maxwell-Boltzmann Distribution





Some results from Kinetic Theory

Average kinetic energy

$$\frac{1}{2}m\bar{v}^2 = \frac{3}{2}kT$$

Average velocity

$$\bar{v} = \sqrt{\frac{8kT}{\pi m}} = \sqrt{\frac{8RT}{\pi M}} = 145\sqrt{\frac{T}{M}}$$

Pressure

$$p = nkT$$

Mean free path

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 n}$$

Impingement Rate

$$J = \frac{p}{\sqrt{2\pi mkT}} = \frac{pN_A}{\sqrt{2\pi MRT}}$$



The Gas Laws

Boyle's Law

$$pV = NkT = n_M RT$$

Avogadro's Number 6.02×10^{23}

$V_M = 22.4$ l at 273 K and 1.103 Pa

Dalton's Law

$$p = \sum_i p_i$$



A Useful Exercise

From the equation for impingement rate, if we assume that every gas molecule which impinges on a surface sticks, prove that the time, τ , to form a monolayer of gas at a pressure p mbar on a surface (i.e. where there is one gas atom for each atom in the surface) is given by

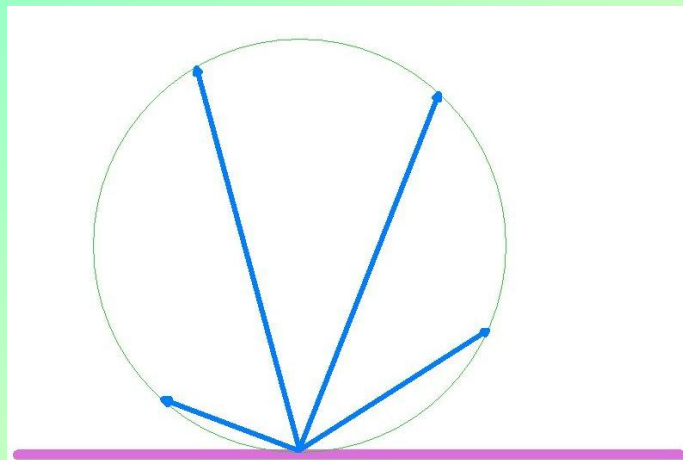
$$\tau \approx \frac{10^{-6}}{p}$$



Basic Principles of Vacuum

Gas scattering at a surface

- Knudsen's cosine law
 - When a gas molecule strikes a surface it remains on the surface sufficiently long to be fully accommodated
 - Therefore when it leaves the surface, the distribution of velocities follows a cosine law





Gas Flow

- There are several so-called gas flow regimes
 - Continuum flow
 - Fluid flow
 - Short mean free path
 - Molecule-molecule collisions are dominant
 - Transitional flow
 - Molecular flow
 - Long mean free path
 - No molecule-molecule collisions



Gas Flow

Knudsen Number, Kn

$$Kn = \frac{\lambda}{d}$$

λ is the mean free path

d is a characteristic dimension of the flow system

Continuum flow

$$Kn < 0.01$$

Transition flow

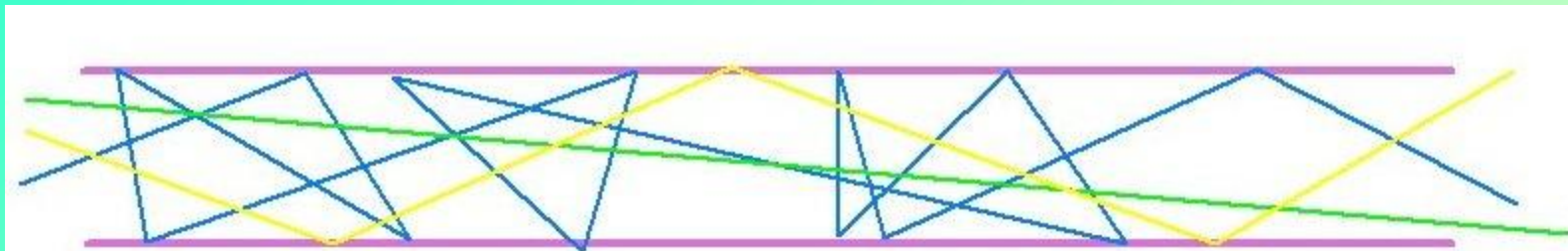
$$0.01 < Kn < 1$$

Molecular flow

$$Kn > 1$$



Molecular flow through a cylindrical pipe



$$Q = C \Delta p$$

For a long pipe

$$C = \frac{D^3}{6L} \sqrt{\frac{2\pi RT}{M}} = 12.4 \frac{D^3}{L} \quad \text{l sec}^{-1} \text{ (for N}_2 \text{ at 295K)}$$

D, L in cm

For a short pipe

$$C = 12.4 \frac{D^3/L}{1 + 4D/3L}$$



Molecular flow through a thin aperture

$$C_A = A \sqrt{\frac{RT}{2\pi M}} = 11.8A$$

l sec⁻¹ (for N₂ at 295K)
A in cm²



Transmission probability

Define transmission probability, α , of a duct as the ratio of the flux of gas molecules at the exit aperture to the flux at the inlet aperture

$$\text{i.e.} \quad \alpha = \frac{J_{out}}{J_{in}}$$

Then, in general, the conductance, C , of the duct is given by

$$C = \alpha C_A$$

Where C_A is the conductance of the entrance aperture.



Basic Principles of Vacuum

Transmission probability

α is independent of the dimensions of the duct and depends only on the ratio of length to transverse dimension and shape of the cross section of the duct.

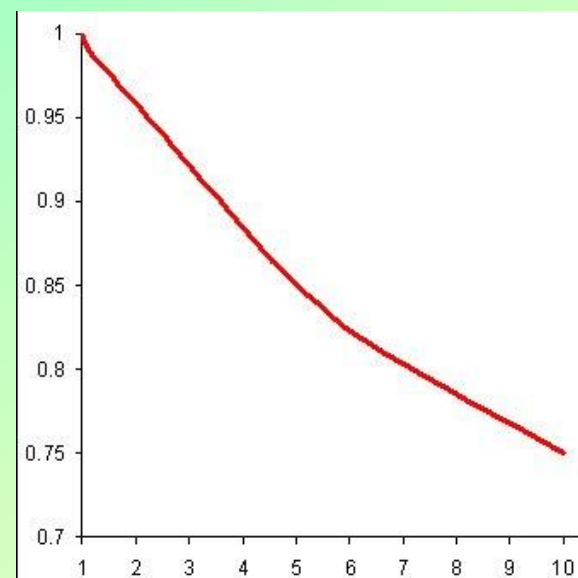
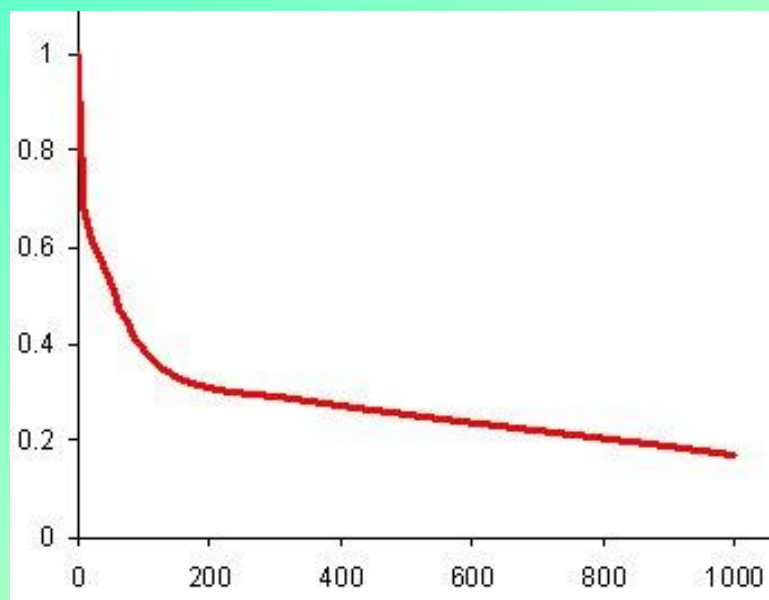
For a cylindrical pipe,

L/D	α
0	1
0.5	0.67
1	0.51
10	0.11
50	0.25



Non cylindrical ducts

For ducts of non circular cross section (e.g. ellipses or rectangles) an empirical correction factor can be applied to the transmission coefficient





Basic Principles of Vacuum

Conductance of complex structures

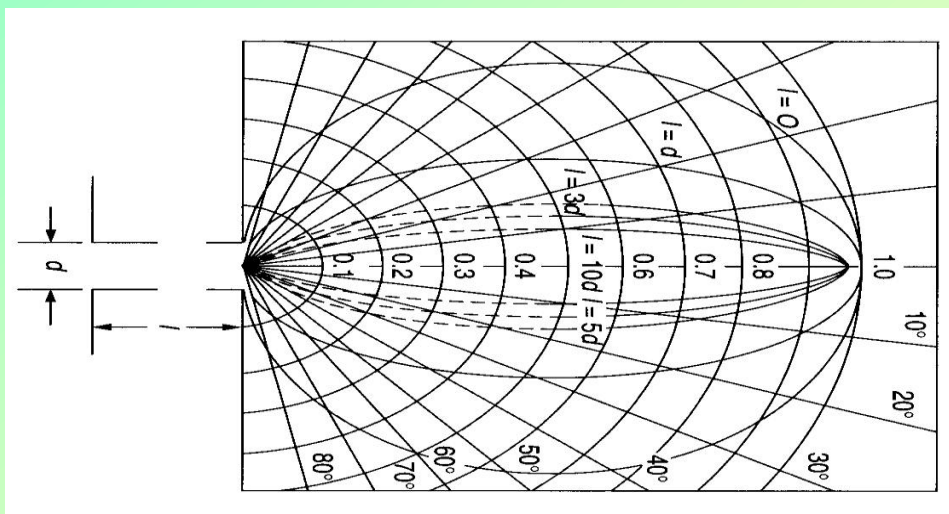
Conductances in parallel

$$C = \sum_i C_i$$

Conductances in series

$$\frac{1}{C} = \sum_i \frac{1}{C_i}$$

But this ignores
beaming





Conductance of complex structures

For complex structures, e.g. bent pipes and vessel strings of varying cross section, transmission coefficients are most accurately computed by methods such as Monte-Carlo simulation



Gas flow: Throughput and Pumping Speed

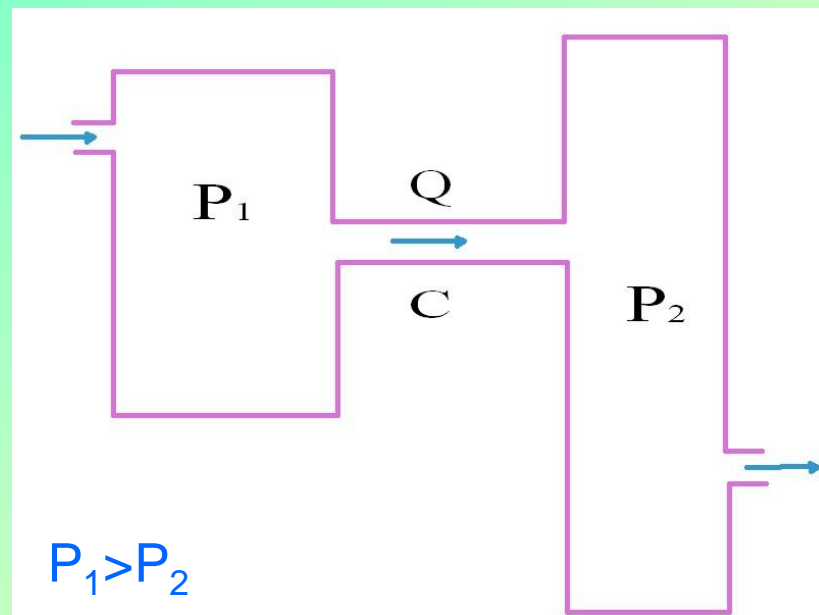
Consider gas flowing through the conductance, C . The quantity of gas entering in unit time must be the same as that leaving.

Upstream, this mass occupies a volume V_1 and downstream V_2

$$\text{So } P_1 V_1 = P_2 V_2$$

Volumetric flow rate is $\frac{\partial V}{\partial t}$

$$\text{Throughput is } Q = P \frac{\partial V}{\partial t}$$





Gas flow: Throughput and Pumping Speed

Volumetric flow rate is often referred to as pumping speed, S , and has units of litre sec^{-1} .

Thus $Q = SP$

Conductance, C , is also given by $Q = C(P_1 - P_2) = C\Delta p$

C also has the units of litre sec^{-1} .



Pumping in the molecular flow regime

The mechanism of pumping is that gas molecules find their way by means of a random walk into a “pump” where they are either trapped, ejected from the vacuum system or return to the vacuum system.

We can define the capture coefficient, σ , of a pump as the probability of a molecule entering the pump being retained. Then the effective pumping speed of the pump, S_e , is given by

$$S_e = \sigma C_E$$

where C_E is the conductance of the entrance aperture of the pump.



Pumping in the molecular flow regime

In general a pump will be attached to the vessel which we wish to pump with a tube of some sort. If this tube has a conductance C , then the net pumping speed at the vessel will be given by

$$\frac{1}{S_{eff}} = \frac{1}{C} + \frac{1}{S_0} \quad \text{or} \quad S_{eff} = \frac{CS_0}{C + S_0}$$

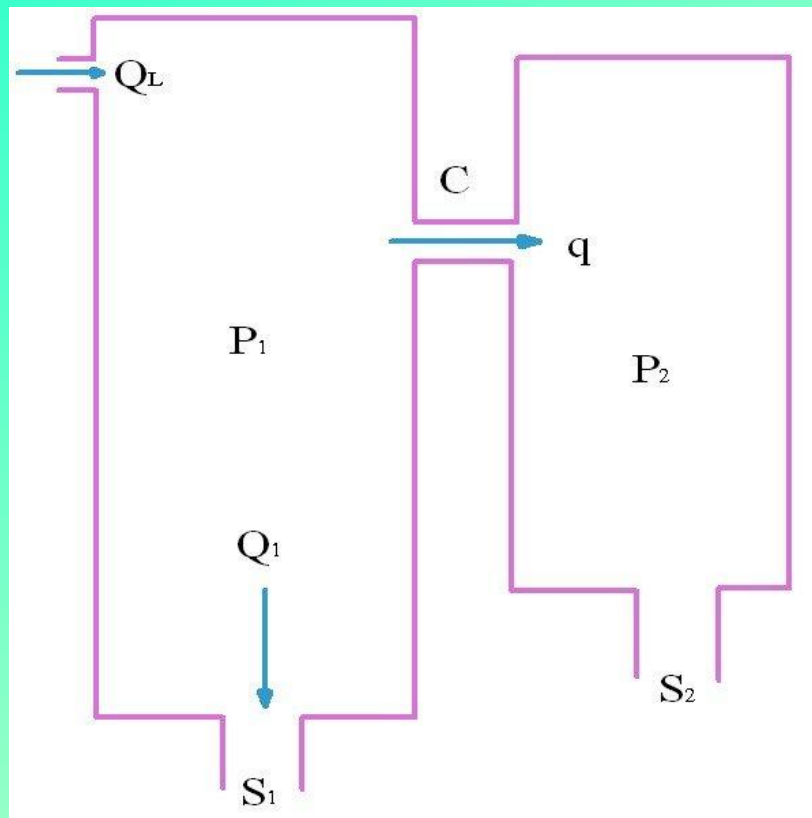
The pumpdown will be given by

$$P = P_0 \exp\left(-\frac{S_{eff}}{V} t\right)$$



Basic Principles of Vacuum

Differential Pumping



A common requirement is to maintain part of a system at a relatively low pressure while another part is at a relatively high pressure (e.g. an ion gun and a target chamber). We need to calculate the pumping speed S_2 required to maintain the pressure P_2

Assume C is small, so

$$P_1 \gg P_2$$

$$\text{then } S_2 = \frac{CP_1}{P_2} = \frac{CQ_L}{P_2S_1}$$



Pressure Regimes

Rough Vacuum	Atmos – 10^{-3} mbar
Medium Vacuum	10^{-3} – 10^{-6} mbar
High Vacuum (HV)	10^{-6} – 10^{-9} mbar
Ultra High Vacuum (UHV)	10^{-9} – 10^{-11} mbar
Extreme High Vacuum (XHV)	$< 10^{-11}$ mbar