Conventional Magnets for Accelerators

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Course Philosophy

1. Present a overview of magnet technology as used in particle accelerators, concerned only with room-temperature electro-magnets.

2. The special cases of a.c. and pulsed magnets (inc something on magnetic materials).

3. Power supply systems for cycling magnets.

4. Not covered:
   i) permanent magnet technology;
   ii) super-conducting technology.

Contents – first section.

1. DC Magnets-design and construction.

a) Introduction:

- Dipole magnets;
- Quadrupole magnets;
- Sextupole magnets;
- 'Higher order' magnets.

b) Magneto-statics in free space (no ferromagnetic materials or currents):

- Maxwell's 2 magneto-static equations;
- Solutions in two dimensions with scalar potential (no currents);
- Cylindrical harmonic in two dimensions (trigonometric formulation);
- Field lines and potential for dipole, quadrupole, sextupole;
- Significance of vector potential in 2D.

Contents (cont.)

c) Introduce ferromagnetic poles:

•Ideal pole shapes for dipole, quad and sextupole;

•Field harmonics-symmetry constraints and significance;

•'Forbidden' harmonics resulting from assembly asymmetries.

e) The introduction of currents and the magnetic circuit:

•Ampere-turns in dipole, quad and sextupole.

•Coil economic optimisation-capital/running costs.

•The magnetic circuit-steel requirements-permeability and coercivity.

•Backleg and coil geometry- 'C', 'H' and 'window frame' designs.

Contents (cont.)

f) Magnet design and f.e.a. software.

- FEA techniques - Modern codes- OPERA 2D; TOSCA.
- Judgement of magnet suitability in design.
- Classical solution to end and side geometries – the Rogowsky roll-off.
- Magnet ends-computation and design.

But first – nomenclature!

Magnetic Field: (the magneto-motive force produced by electric currents)

symbol is **H** (as a vector);

units are Amps/metre in S.I units (Oersteds in cgs);

Magnetic Induction or **Flux Density:** (the density of magnetic flux driven through a medium by the magnetic field)

symbol is **B** (as a vector);

units are Tesla (Webers/m^2 in mks, Gauss in cgs);

Note: induction is frequently referred to as "Magnetic Field".

Permeability of free space:

symbol is μ_0 ;

units are Henries/metre;

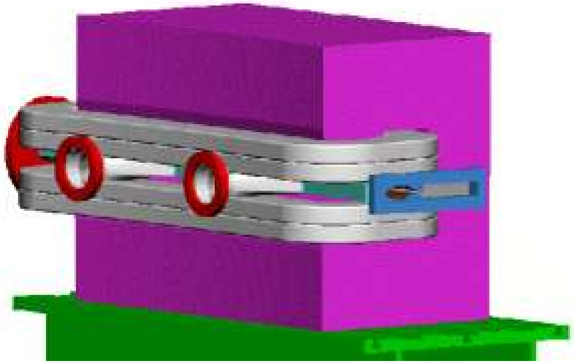
Permeability (abbreviation of **relative permeability**):

symbol is μ;

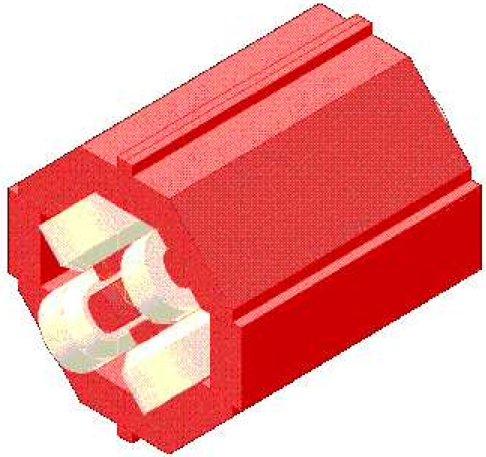
the quantity is dimensionless;

Magnets - introduction

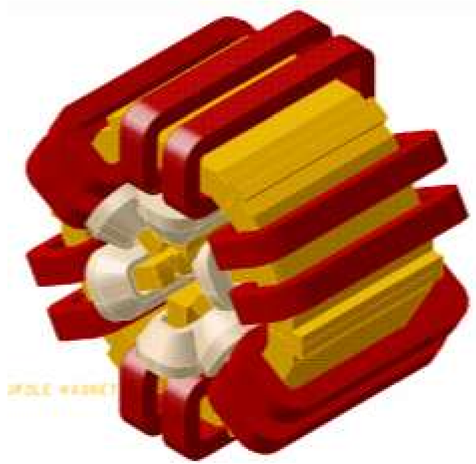
Dipoles to bend the beam:



Quadrupoles to focus it:



Sextupoles to correct chromaticity:

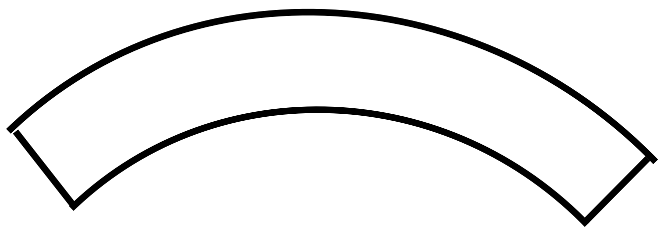


We shall establish a formal approach to describing these magnets.

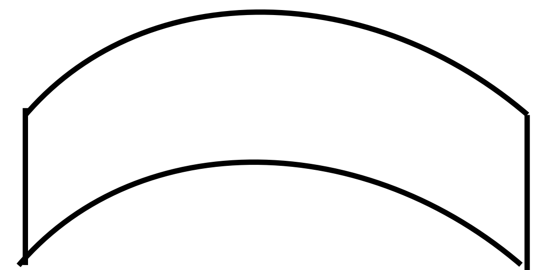
Magnets - dipoles

To bend the beam uniformly, dipoles need to produce a field that is constant across the aperture.

But at the ends they can be either:



Sector dipole

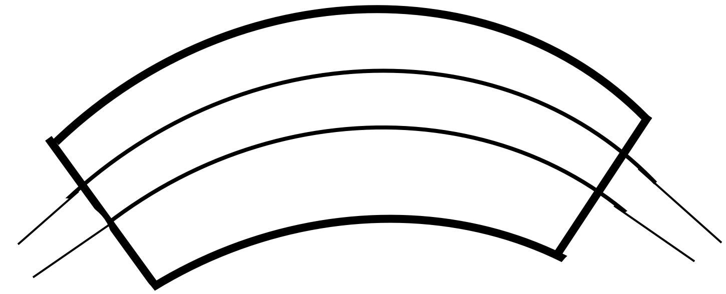


Parallel ended dipole.

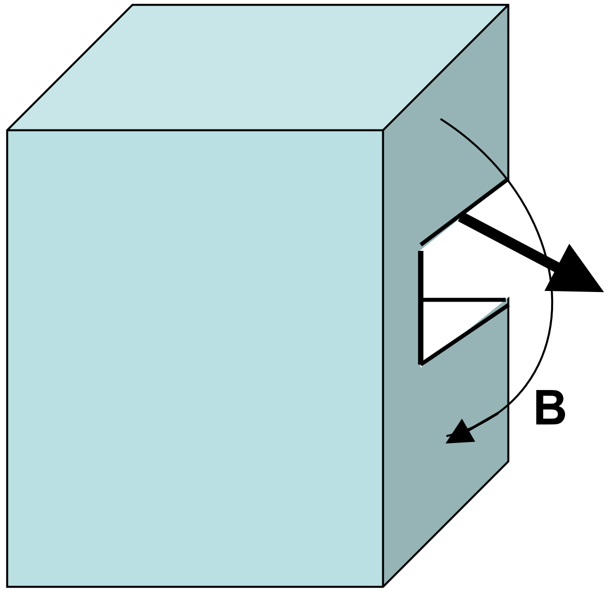
They have different focusing effect on the beam;
(their curved nature is to save material and has no effect on beam focusing).

Dipole end focusing

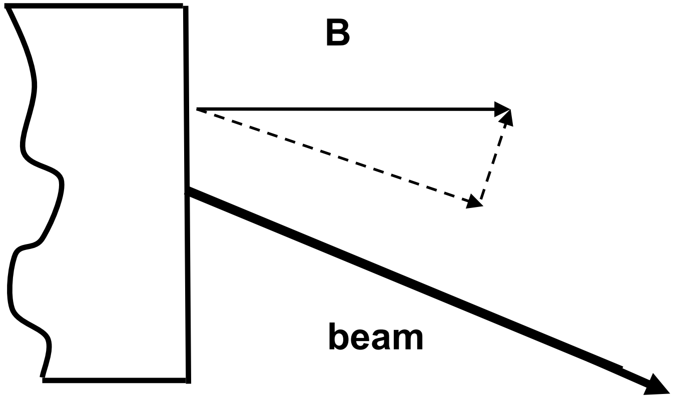
Sector dipoles focus horizontally :



The end field in a parallel ended dipole focuses vertically :



Off the vertical centre line, the field component normal to the beam direction produces a vertical focusing force.

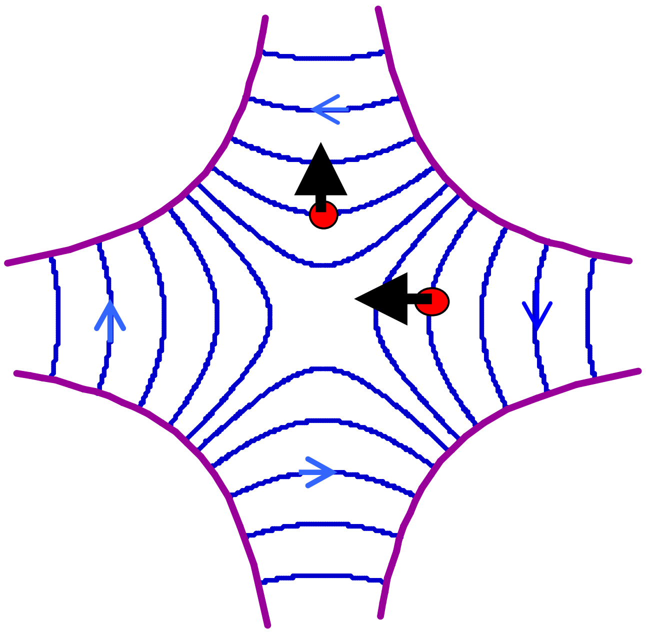


Magnets - quadrupoles

Quadrupoles produce a linear field variation across the beam.

Field is zero at the 'magnetic centre' so that 'on-axis' beam is not bent.

Note: beam that is radially focused is vertically defocused.



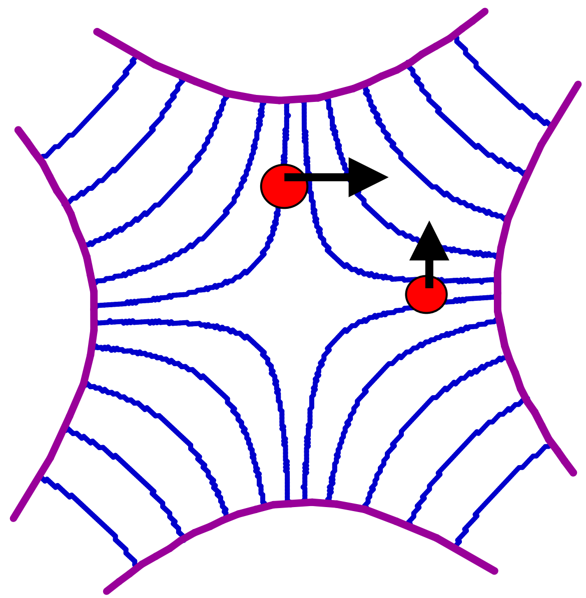
These are 'upright' quadrupoles.

‘Skew’ Quadrupoles.

Beam that has radial displacement (but not vertical) is deflected vertically;

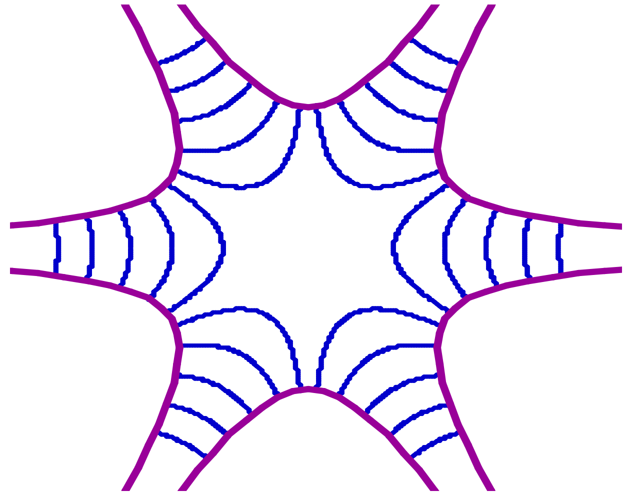
horizontally centred beam with vertical displacement is deflected radially;

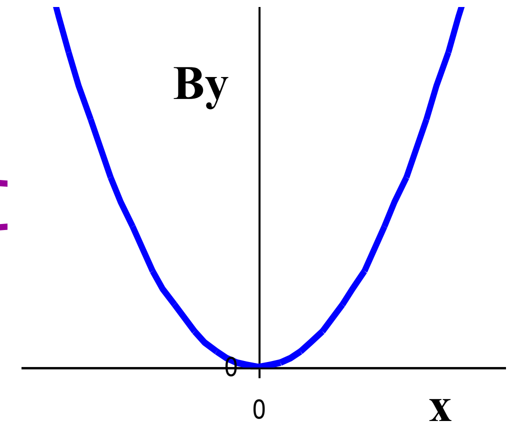
so skew quadrupoles couple horizontal and vertical transverse oscillations.



Magnets - sextupoles

In a sextupole, the field varies as the square of the displacement.





•off-momentum particles are incorrectly focused in quadrupoles (eg, high momentum particles with greater rigidity are under-focused), so transverse oscillation frequencies are modified - **chromaticity**;

•but off momentum particles circulate with a radial displacement (high momentum particles at larger x);

•so positive sextupole field corrects this effect – can reduce chromaticity to 0.

Magnets – 'higher orders'.

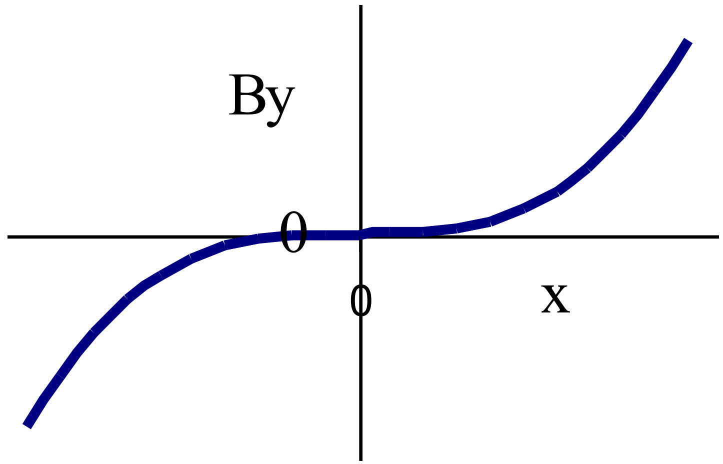
eg – Octupoles:

Effect?

$$B_y \propto x^3$$

Octupole field induces 'Landau damping' :

- introduces tune-spread as a function of oscillation amplitude;
- de-coheres the oscillations;
- reduces coupling.



No currents, no steel - Maxwell's static equations in free space:

$$\underline{\nabla}.\underline{\mathbf{B}} = 0 ;$$

$$\underline{\nabla} \wedge \underline{\mathbf{H}} = \underline{\mathbf{j}} ;$$

In the absence of currents: $\underline{\mathbf{j}} = 0.$

Then we can put: $\underline{\mathbf{B}} = - \underline{\nabla}\phi$

So that: $\underline{\nabla}^2\phi = 0$ (Laplace's equation).

Taking the two dimensional case (ie constant in the z direction) and solving for cylindrical coordinates (r,θ):

$$\phi = (E+F\,\theta)(G+H \ln r) + \Sigma_{n=1}^{\infty} (J_n\, r^{n} \cos n\theta + K_n\, r^{n} \sin n\theta + L_n\, r^{-n} \cos n\,\theta + M_n\, r^{-n} \sin n\,\theta)$$

In practical situations:

The scalar potential simplifies to:

$$\phi = \Sigma_n (J_n r^n \cos n\theta + K_n r^n \sin n\theta),$$

with n integral and J_n, K_n a function of geometry.

Giving components of flux density:

$$B_r = - \Sigma_n (n J_n r^{n-1} \cos n\theta + nK_n r^{n-1} \sin n\theta)$$
$$B_\theta = - \Sigma_n (-n J_n r^{n-1} \sin n\theta + nK_n r^{n-1} \cos n\theta)$$

Physical significance

This is an infinite series of cylindrical harmonics; they define the allowed distributions of **B** in 2 dimensions in the absence of currents within the domain of (r,θ).

Distributions not given by above are not physically realisable.

Coefficients J_n, K_n are determined by geometry (remote iron boundaries and current sources).

In Cartesian Coordinates

To obtain these equations in Cartesian coordinates, expand the equations for ϕ and differentiate to obtain flux densities;

$\cos 2\theta = \cos^2\theta - \sin^2\theta$; $\cos 3\theta = \cos^3\theta - 3\cos\theta \sin^2\theta$;

$\sin 2\theta = 2 \sin\theta \cos\theta$; $\sin 3\theta = 3\sin^2\theta - \sin^3\theta$;

$$\cos 4\theta = \cos^4\theta + \sin^4\theta - 6 \cos^2\theta \sin^2\theta;$$

$$\sin 4\theta = 4 \sin\theta \cos^3\theta - 4 \sin^3\theta \cos\theta;$$

etc (messy!);

$x = r \cos \theta$; $y = r \sin \theta$;

and $B_x = - \partial\phi/\partial x$; $B_y = - \partial\phi/\partial y$

n = 1 Dipole field!

Cylindrical:

$B_r = J_1 \cos\theta + K_1 \sin\theta;$

$B_\theta = -J_1 \sin\theta + K_1 \cos\theta;$

$\phi = J_1 r \cos\theta + K_1 r \sin\theta.$

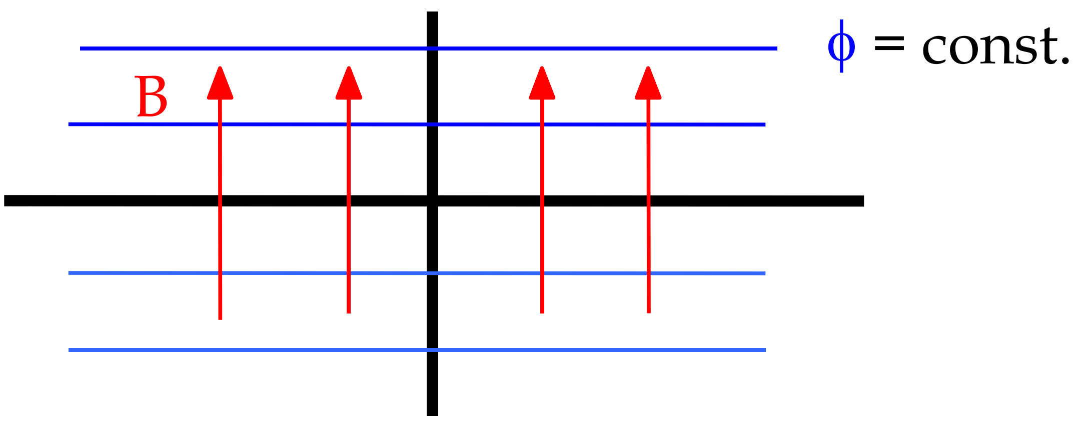
Cartesian:

$B_x = J_1$

$B_y = K_1$

$\phi = J_1 x + K_1 y$

So, $J_1 = 0$ gives vertical dipole field:



$K_1 = 0$ gives horizontal dipole field.

n = 2 Quadrupole field !

Cylindrical:

$B_r = 2\ J_2\ r\ \cos 2\theta + 2K_2\ r\ \sin 2\theta;$

$B_\theta = -2J_2\ r\ \sin 2\theta + 2K_2\ r\ \cos 2\theta;$

$\phi = J_2\ r\ 2 \cos 2\theta + K_2\ r\ 2 \sin 2\theta;$

Cartesian:

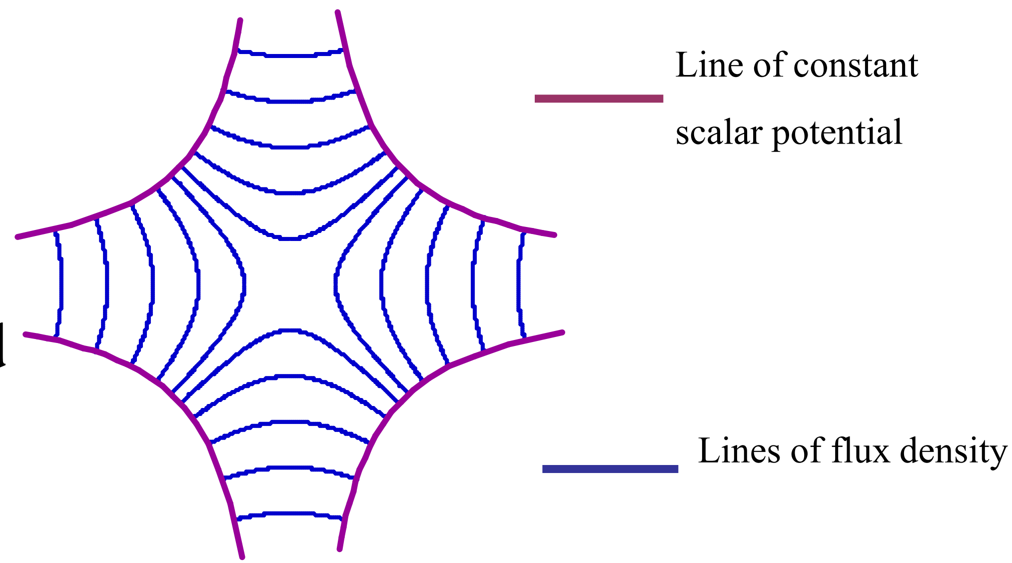
$Bx = 2\ (J_2\ x + K_2\ y)$

$B_y = 2\ (-J_2\ y + K_2\ x)$

$\phi = J_2\ (x^2 - y^2) + 2K_2\ xy$

$J_2 = 0$ gives 'normal' or 'right' quadrupole field.

$K_2 = 0$ gives 'skew' quad fields (above rotated by $\pi/4$).



n = 3 Sextupole field !

Cylindrical;

$B_r = 3\ J_3 r^2\ \cos 3\theta + 3K_3 r^2\ \sin 3\theta;$

$B_\theta = -3J_3\ r^2\ \sin 3\theta + 3K_3\ r^2 \cos 3\theta;$

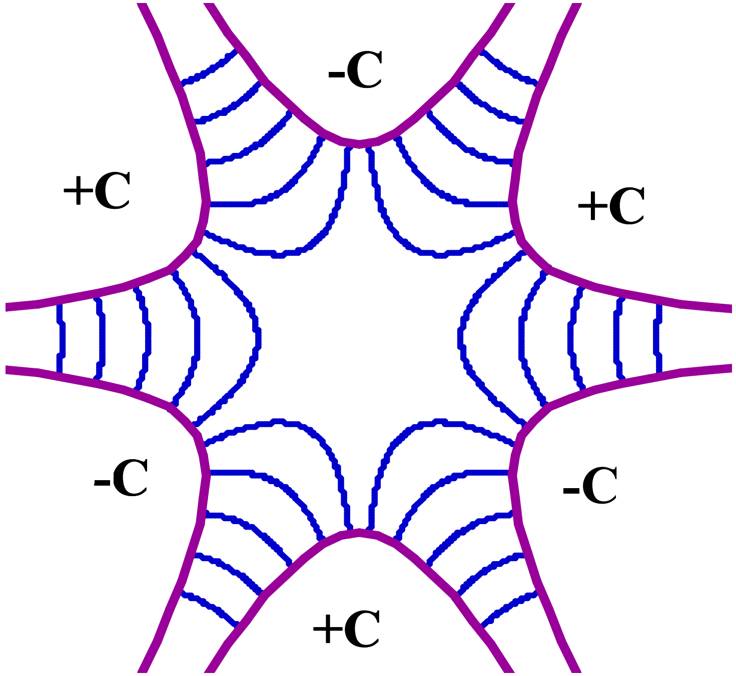
$\phi\ =\ J_3\ r^3 \cos 3\theta + K_3\ r^3 \sin 3\theta;$

Cartesian:

$B_x = 3\{J_3\ (x^2 - y^2) + 2K_3 yx\}$

$B_y = 3\{-2\ J_3\ xy + K_3(x^2 - y^2)\}$

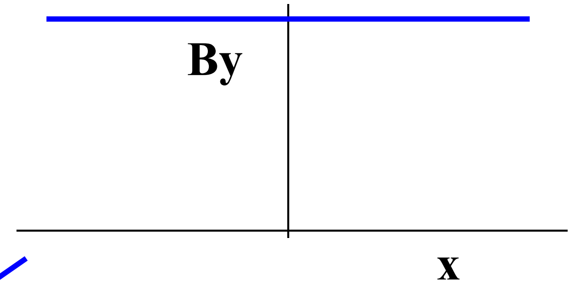
$\phi = J_3\ (x^3 - 3y^2 x) + K_3(3yx^2 - y^3)$



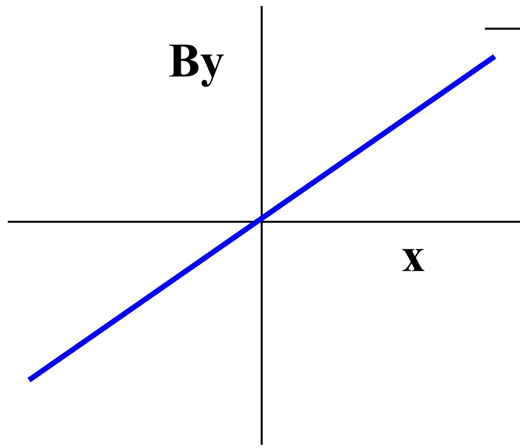
$J_3 = 0$ giving 'normal' or 'right' sextupole field.

Summary; variation of B_y on x axis

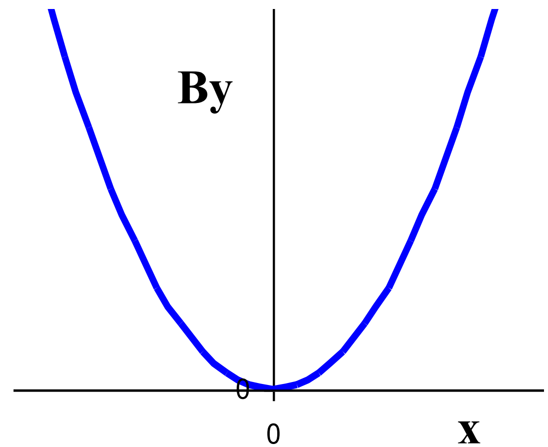
Dipole; constant field:



Quad; linear variation:



Sextupole: quadratic variation:



Alternative notification (Most lattice codes)

$$B(x) = B\rho \sum_{n=0}^{\infty} \frac{k_n x^n}{n!}$$

magnet strengths are specified by the value of k_n; (normalised to the beam rigidity);

order n of k is different to the 'standard' notation:

dipole is	$n = 0$;
quad is	$n = 1$; etc.

k has units:

k_0 (dipole)	m^{-1};
k_1 (quadrupole)	m^{-2}; etc.

Significance of vector potential in 2D.

We have: $\underline{\mathbf{B}} = \text{curl } \underline{\mathbf{A}}$ (**$\underline{\mathbf{A}}$** is vector potential);

and $\text{div } \underline{\mathbf{A}} = 0$

Expanding: $\underline{\mathbf{B}} = \text{curl } \underline{\mathbf{A}} =$

$(\partial A_z / \partial y - \partial A_y / \partial z)\,\mathbf{i} + (\partial A_x / \partial z - \partial A_z / \partial x)\,\mathbf{j} + (\partial A_y / \partial x - \partial A_x / \partial y)\,\mathbf{k}$**;**

where **i, j, k**, and unit vectors in x, y, z.

In 2 dimensions $B_z = 0$; $\partial / \partial z = 0$;

So $A_x = A_y = 0$;

and $\underline{\mathbf{B}} = (\partial A_z / \partial y)\,\mathbf{i} - (\partial A_z / \partial x)\,\mathbf{j}$

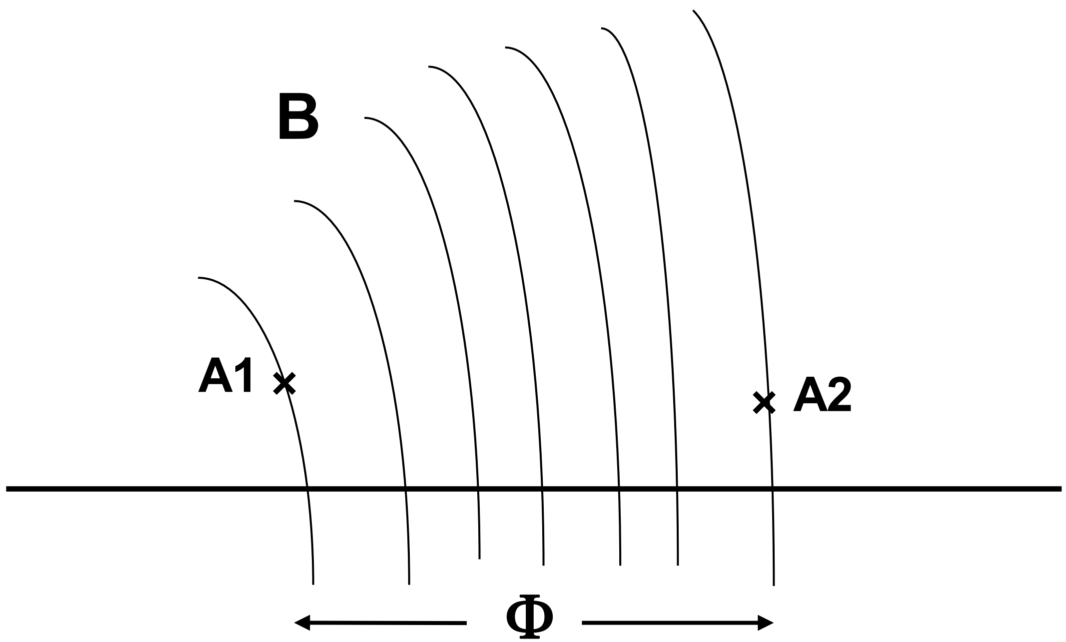
$\underline{\mathbf{A}}$ is in the z direction, normal to the 2 D problem.

Note: $\text{div } \underline{\mathbf{B}} = \partial^2 A_z / \partial x \partial y - \partial^2 A_z / \partial x \partial y = 0$;

Total flux between two points ∝ ΔA

In a two dimensional problem the magnetic flux between two points is proportional to the difference between the vector potentials at those points.

$$\Phi \propto (A_2 - A_1)$$



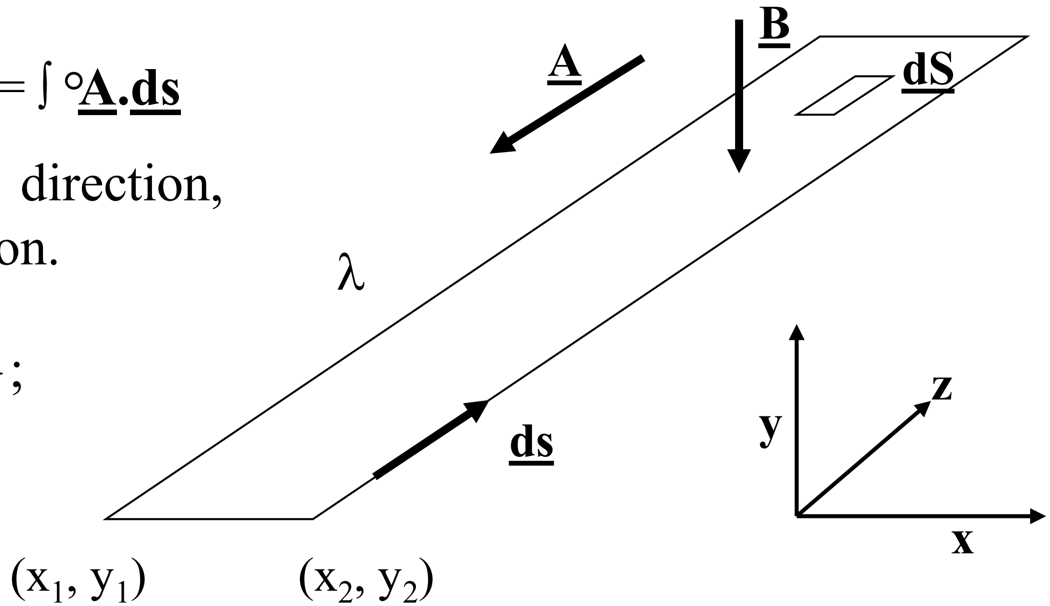
Proof on next slide.

Proof.

Consider a rectangular closed path, length λ in z direction at (x_1,y_1) and (x_2,y_2); apply Stokes' theorem:

$\Phi = \int\int \underline{\mathbf{B}}.\underline{\mathbf{dS}} = \int\int (\text{curl } \underline{\mathbf{A}}).\underline{\mathbf{dS}} = \oint \underline{\mathbf{A}}.\underline{\mathbf{ds}}$

But A is exclusively in the z direction, and is constant in this direction.
So:
$\int \mathbf{A}.\mathbf{ds} = \lambda \{ A(x_1,y_1) - A(x_2,y_2)\};$

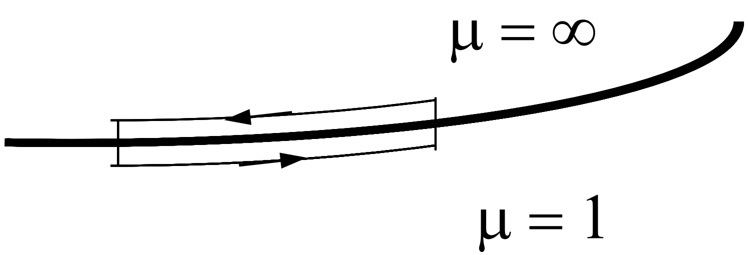


$$\Phi = \lambda \{ A(x_1,y_1) - A(x_2,y_2)\};$$

Introducing Iron Yokes

What is the ideal pole shape?

•Flux is normal to a ferromagnetic surface with infinite μ:



curl H = 0

therefore ∫ H.ds = 0;

in steel H = 0;

therefore parallel H air = 0

therefore B is normal to surface.

•Flux is normal to lines of scalar potential, ($\underline{\mathbf{B}} = -\underline{\nabla}\phi$);

•So the lines of scalar potential are the ideal pole shapes!

(but these are infinitely long!)

Equations for the ideal pole

Equations for Ideal (infinite) poles;

(J_n = 0) for **normal** (ie not skew) fields:

Dipole:

$y = \pm g/2;$

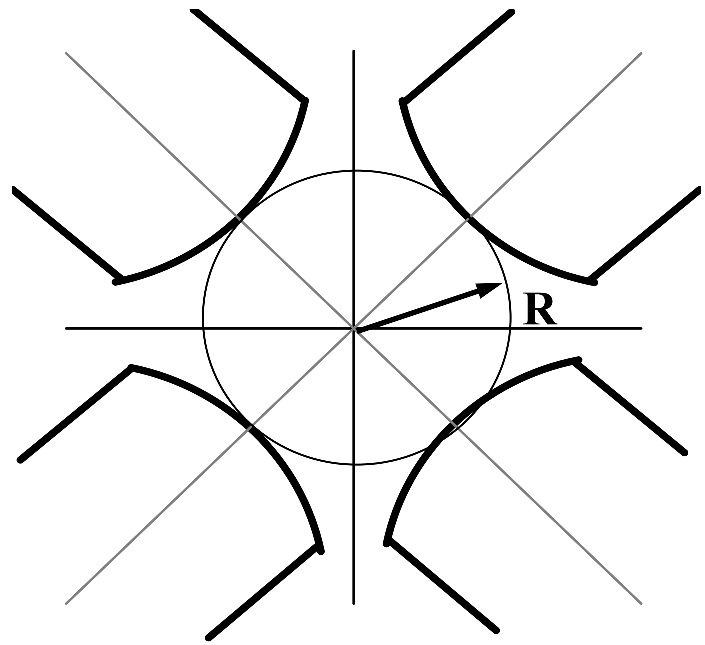
(g is interpole gap).

Quadrupole:

$xy = \pm R^2/2;$

Sextupole:

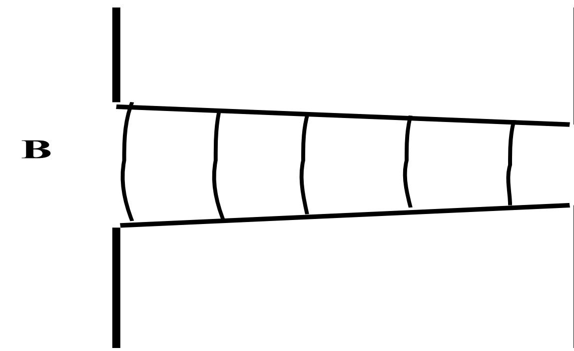
$3x^2y - y^3 = \pm R^3;$



Combined function (c.f.) magnets

'Combined Function Magnets' - often dipole and quadrupole field combined (but see next-but-one slide):

A quadrupole magnet with physical centre shifted from magnetic centre.



Characterised by 'field index' n, +ve or -ve depending on direction of gradient; do not confuse with harmonic n!

$$n = -\left(\frac{\rho}{B_0}\right)\left(\frac{\partial B}{\partial x}\right),$$

ρ is radius of curvature of the beam;

B_o is central dipole field

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Typical combined dipole/quadrupole

SRS Booster c.f. dipole

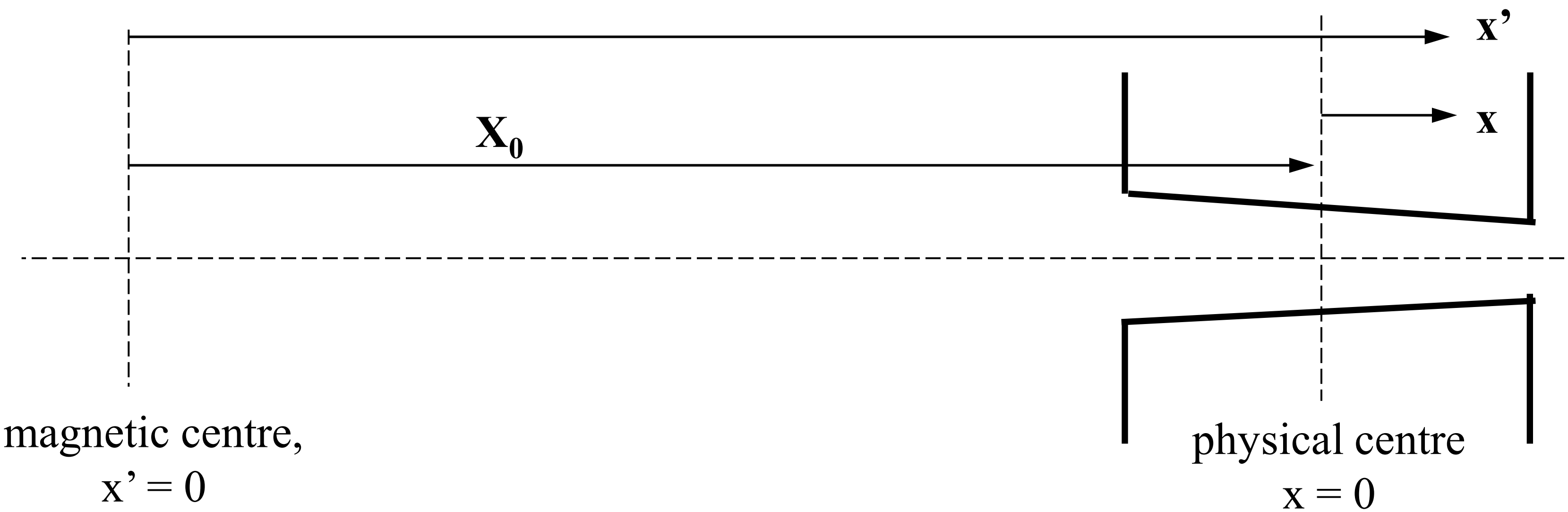
‘D’ type +ve n.

‘F’ type
-ve n

Combined function geometry

Combined function (dipole & quadrupole) magnet:

- beam is at physical centre
- flux density at beam = B_0;
- gradient at beam = $\partial B/\partial x$;
- magnetic centre is at B = 0.
- separation magnetic to physical centre = X_0



Pole of a c.f. dip.& quad. magnet

If physical and magnetic centres are separated by X_0

Then $$B_0 = \left(\frac{\partial B}{\partial x}\right) X_0;$$

therefore $$X_0 = -\rho / n;$$

in a quadrupole $$x' y = \pm R^2 / 2$$

where x' is measured from the true quad centre;

Put $$x' = x + X_0$$

So pole equation is $$y = \pm \frac{R^2}{2} \frac{n}{\rho} \left(1 - \frac{nx}{\rho}\right)^{-1}$$

or $$\boxed{y = \pm g \left(1 - \frac{nx}{\rho}\right)^{-1}}$$

where g is the half gap at the physical centre of the magnet

Other combined function magnets.

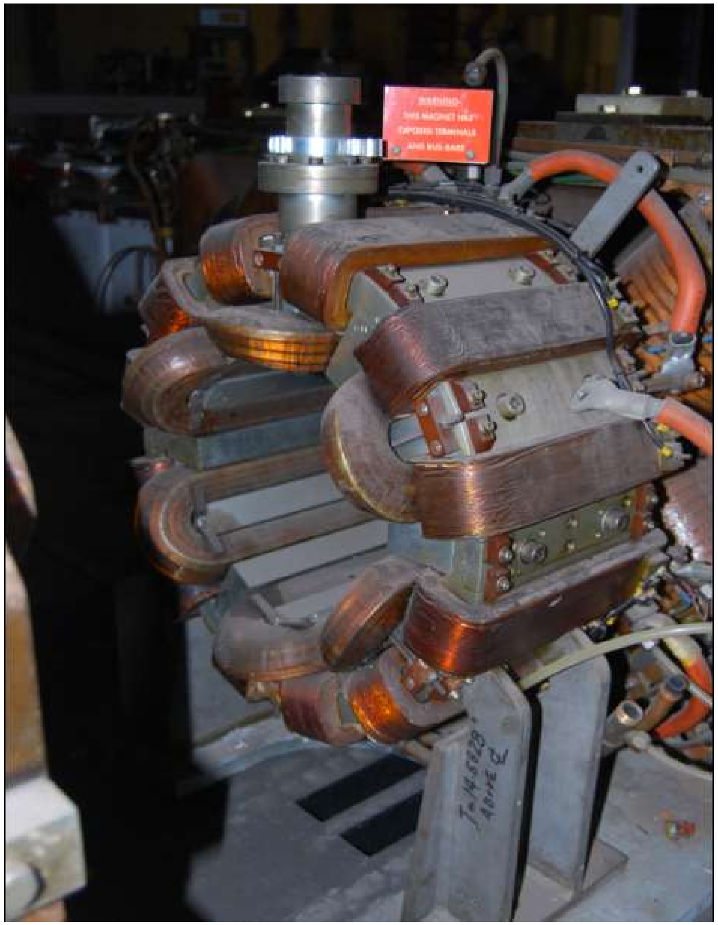
Other combinations:

- •dipole, quadrupole and sextupole;
- •dipole & sextupole (for chromaticity control);
- •dipole, skew quad, sextupole, octupole (at DL)

Generated by

- •pole shapes given by sum of correct scalar potentials
  - amplitudes built into pole geometry – not variable.
- •multiple coils mounted on the yoke
  - amplitudes independently varied by coil currents.

The SRS multipole magnet.



Could develop:

- vertical dipole
- horizontal dipole;
- upright quad;
- skew quad;
- sextupole;
- octupole;
- others.

See tutorial question.

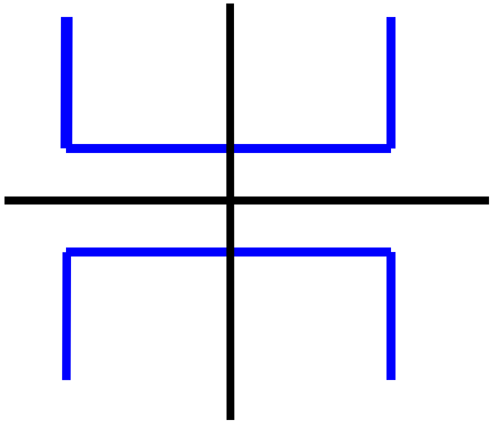
The Practical Pole

Practically, poles are finite, **introducing errors**;

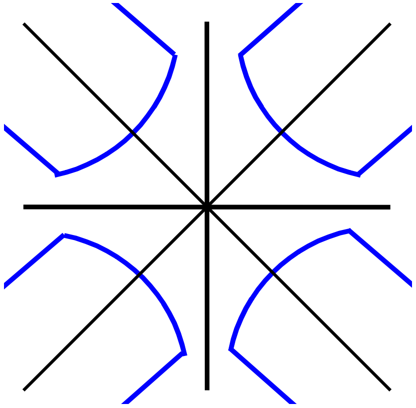
these appear as higher harmonics which degrade the field distribution.

However, the iron geometries have certain symmetries that **restrict** the nature of these errors.

Dipole:



Quadrupole:



Possible symmetries

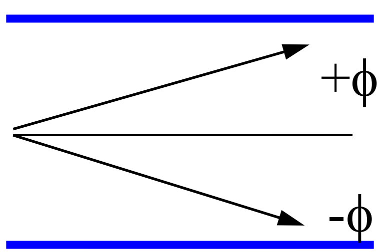
Lines of symmetry:

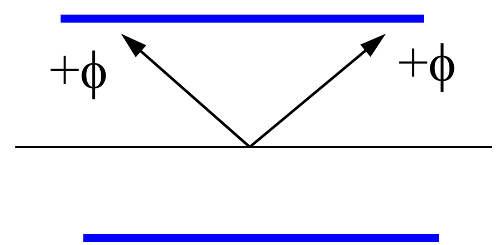
	Dipole:	Quad
Pole orientation determines whether pole is normal or skew.	$y = 0;$	$x = 0;\ y = 0$
Additional symmetry imposed by pole edges.	$x = 0;$	$y = \pm x$

The additional constraints imposed by the symmetrical pole edges limits the values of n that have non zero coefficients

Dipole symmetries

Type	Symmetry	Constraint
Pole orientation	$\phi(\theta) = -\phi(-\theta)$	all $J_n = 0$;
Pole edges	$\phi(\theta) = \phi(\pi - \theta)$	K_n non-zero only for: $n = 1, 3, 5$, etc;





So, for a fully symmetric dipole, only 6, 10, 14 etc pole errors can be present.

Quadrupole symmetries

Type	Symmetry	Constraint
Pole orientation	$\phi(\theta) = -\phi(-\theta)$	All $J_n = 0$;
	$\phi(\theta) = -\phi(\pi - \theta)$	$K_n = 0$ all odd n;
Pole edges	$\phi(\theta) = \phi(\pi/2 - \theta)$	K_n non-zero only for: n = 2, 6, 10, etc;

So, a fully symmetric quadrupole, only 12, 20, 28 etc pole errors can be present.

Sextupole symmetries

Type	Symmetry	Constraint
Pole orientation	$\phi(\theta) = -\phi(-\theta)$ $\phi(\theta) = -\phi(2\pi/3 - \theta)$ $\phi(\theta) = -\phi(4\pi/3 - \theta)$	All $J_n = 0$; $K_n = 0$ for all n not multiples of 3;
Pole edges	$\phi(\theta) = \phi(\pi/3 - \theta)$	K_n non-zero only for: n = 3, 9, 15, etc.

So, a fully symmetric sextupole, only 18, 30, 42 etc pole errors can be present.

Summary - 'Allowed' Harmonics

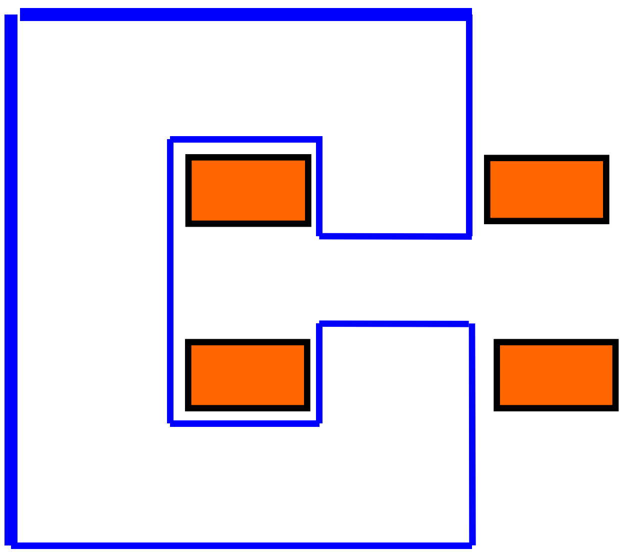
Summary of 'allowed harmonics' in fully symmetric magnets:

Fundamental geometry	'Allowed' harmonics
Dipole, n = 1	**n = 3, 5, 7, ...** **(6 pole, 10 pole, etc.)**
Quadrupole, n = 2	**n = 6, 10, 14, ...** **(12 pole, 20 pole, etc.)**
Sextupole, n = 3	**n = 9, 15, 21, ...** **(18 pole, 30 pole, etc.)**
Octupole, n = 4	**n = 12, 20, 28, ...** **(24 pole, 40 pole, etc.)**

Asymmetries generating harmonics (i)

Two sources of asymmetry generate 'forbidden' harmonics:

i) magnetic asymmetries - significant at low permeability:

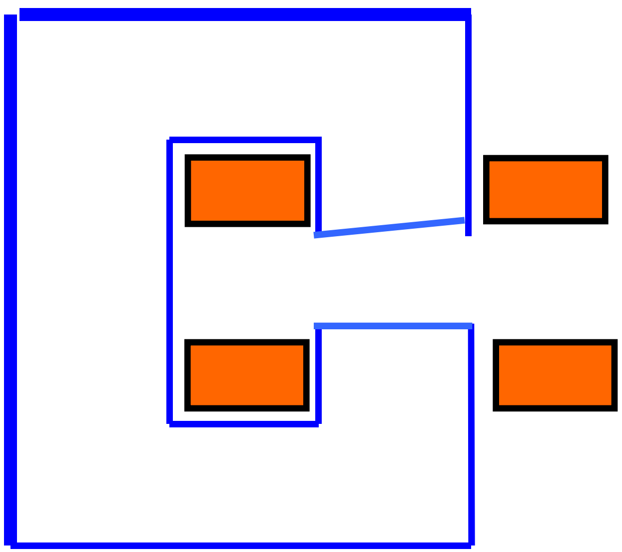


eg, C core dipole not completely symmetrical about pole centre, but negligible effect with high permeability.

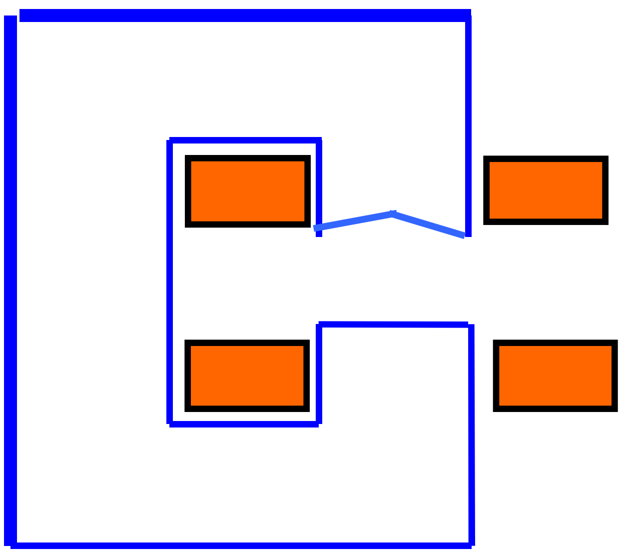
Generates n = 2,4,6, etc.

Asymmetries generating harmonics (ii)

ii) asymmetries due to small manufacturing errors in dipoles:



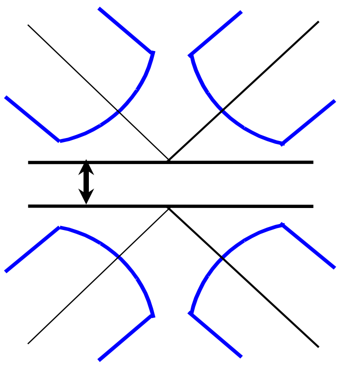
n = 2, 4, 6 etc.



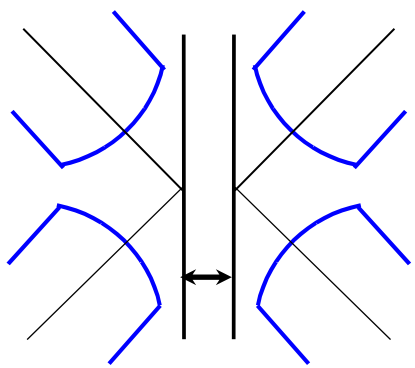
n = 3, 6, 9, etc.

Asymmetries generating harmonics (iii)

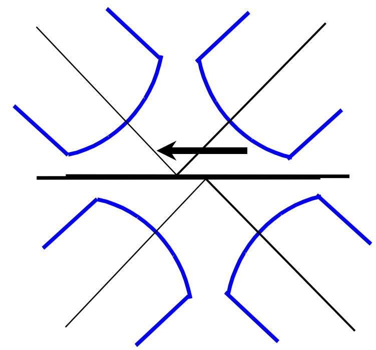
ii) asymmetries due to small manufacturing errors in quadrupoles:



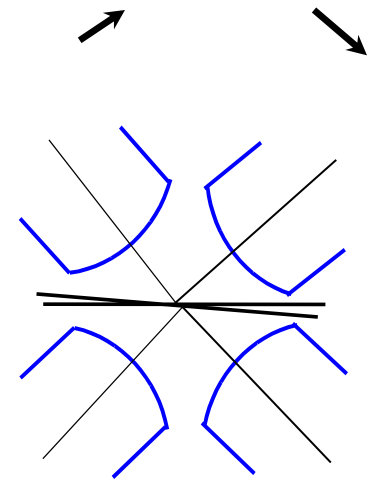
n = 4 - ve



n = 4 + ve



n = 3;



n = 2 (skew)

n = 3;

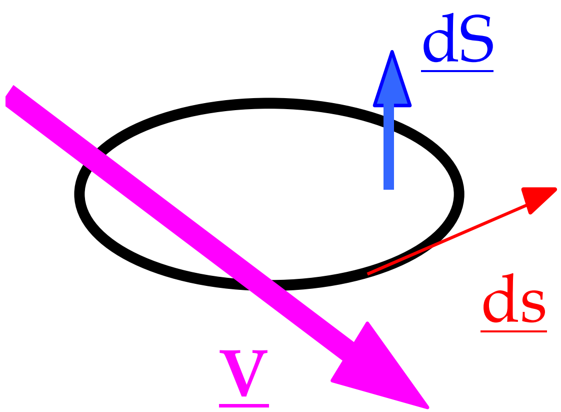
These errors are bigger than the finite μ type, can seriously affect machine behaviour and must be controlled.

Introduction of currents

Now for $\underline{j} \neq 0$ $\qquad \underline{\nabla} \wedge \underline{\mathbf{H}} = \underline{\mathbf{j}}$;

To expand, use Stoke's Theorum:
for any vector $\underline{\mathbf{V}}$ and a closed curve s
:

$$\int \underline{\mathbf{V}}.\underline{\mathbf{ds}} = \iint \mathbf{curl}\ \underline{\mathbf{V}}.\underline{\mathbf{dS}}$$



Apply this to: $\quad \mathbf{curl\ H} = \underline{\mathbf{j}}$;

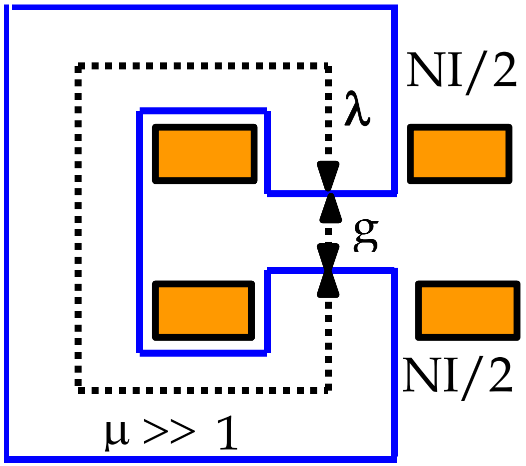
then in a magnetic circuit:

$$\int \underline{\mathbf{H}}.\underline{\mathbf{ds}} = \mathrm{N\ I};$$

N I (Ampere-turns) is total current cutting $\underline{\mathbf{S}}$

Excitation current in a dipole

B is approx constant round the loop made up of λ and g, (but see below);



But in iron, $\mu \gg 1$,

and $H_{iron} = H_{air} / \mu$;

So

$$B_{air} = \mu_0 NI / (g + \lambda/\mu);$$

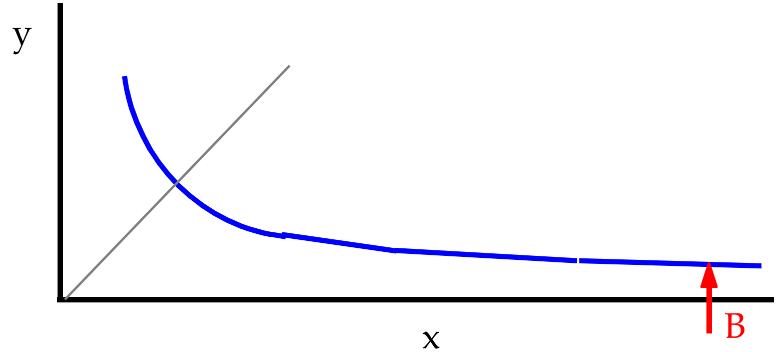
g, and λ/μ are the 'reluctance' of the gap and iron.

Approximation ignoring iron reluctance ($\lambda/\mu \ll g$):

$$NI = B g / \mu_0$$

Excitation current in quad & sextupole

For quadrupoles and sextupoles, the required excitation can be calculated by considering fields and gap at large x. For example: **Quadrupole:**



Pole equation: $xy = R^2 / 2$

On x axes $B_Y = gx$;

where g is gradient (T/m).

At large x (to give vertical lines of B):

$$N\,I = (gx)\,(R^2 / 2x) / \mu_0$$

ie

$$N\,I = g\,R^2 / 2\,\mu_0 \text{ (per pole).}$$

The same method for a **Sextupole**, (coefficient g_S,), gives:

$$N\,I = g_S\,R^3 / 3\,\mu_0 \text{ (per pole)}$$

General solution for magnets order n

In air (remote currents!), $\mathbf{B} = \mu_0 \mathbf{H}$

$$\underline{\mathbf{B}} = - \underline{\nabla}\phi$$

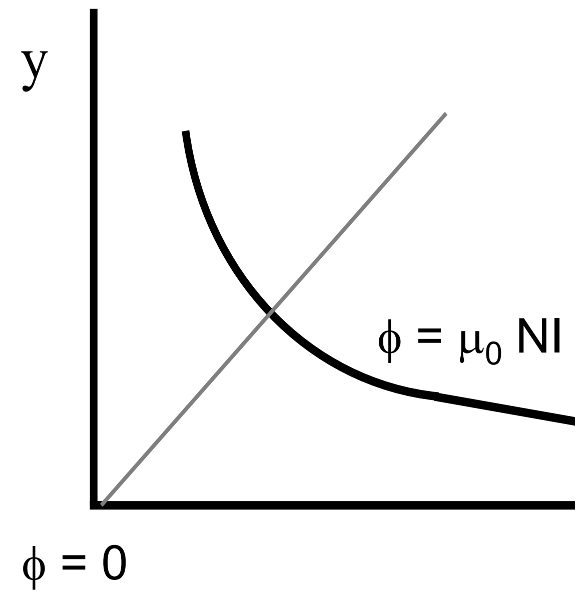
Integrating over a limited path (not circular) in air: $N I = (\phi_1 - \phi_2)/\mu_o$

ϕ_1, ϕ_2 are the scalar potentials at two points in air.

Define $\phi = 0$ at magnet centre;

then potential at the pole is:

$$\mu_o NI$$



Apply the general equations for magnetic field harmonic order n for non-skew magnets (all Jn = 0) giving:

$$N I = (1/n)\,(1/\mu_0)\,\{B_r/R^{(n-1)}\}\,R^n$$

Where:

NI is excitation per pole;

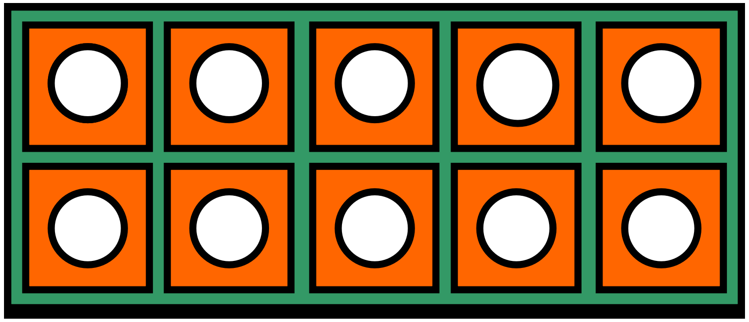
R is the inscribed radius (or half gap in a dipole);

term in brackets {} is magnet strength in $T/m^{(n-1)}$.

Coil geometry

Standard design is rectangular copper (or aluminium) conductor, with cooling water tube. Insulation is glass cloth and epoxy resin.

Amp-turns (NI) are determined, but total copper area (A_{copper}) and number of turns (N) are two degrees of freedom and need to be decided.



Heat generated in the coil is a function of the RMS current density:

$j_{rms} = NI_{rms}/A_{copper}$

Optimum j_{rms} determined from **economic** criteria.

I_{rms} depends on current waveform

Heat generated in coil $$W = R\int\{I(t)\}^2\,dt = R\,I_{rms}^2$$

- In a DC magnet the $I_{rms} = I_{dc}$
- For a pure sin-wave $I_{rms} = (1/\sqrt{2})I_{peak}$
- A typical waveform

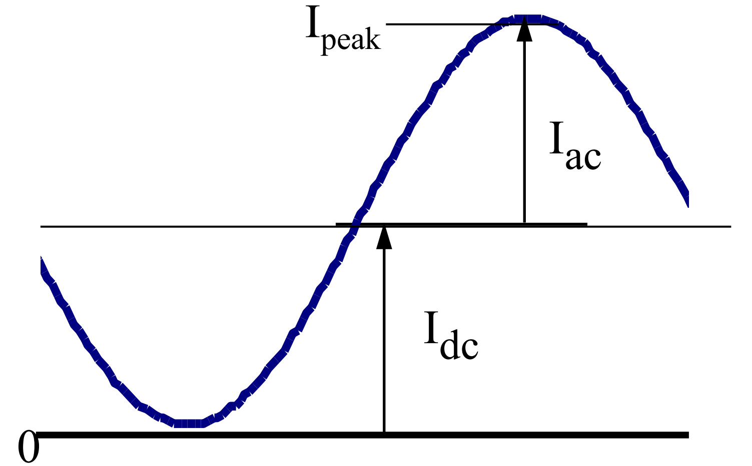
for a booster synchrotron is

a biased sinwave:

$I_{rms} = \sqrt{\{I_{dc}^2+(1/2)\,I_{ac}^2\}}$

If $I_{dc} = I_{ac} = (1/2)\,I_{peak}$

$I_{rms} = I_{dc}\,(\sqrt{(3/2)} = I_{peak}\,(1/2)(\sqrt{(3/2)}$



Current density (j_{rms})- optimisation

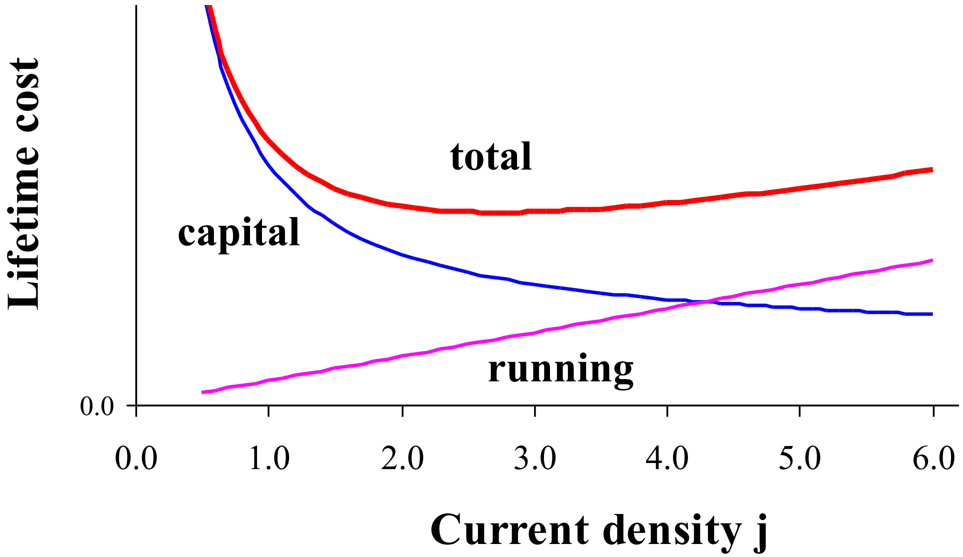
Advantages of low j_{rms}:

- **lower power loss** – power bill is decreased;
- **lower power loss** – power converter size is decreased;
- **less heat** dissipated into magnet tunnel.

Advantages of high j:

- **smaller coils;**
- **lower capital cost;**
- **smaller magnets.**

Chosen value of j_{rms} is an optimisation of magnet capital against power costs.



Number of turns, N

The value of number of turns (N) is chosen to match power supply and interconnection impedances.

Factors determining choice of N:

Large N (low current)	Small N (high current)
Small, neat terminals.	Large, bulky terminals
Thin interconnections- low cost and flexible.	Thick, expensive connections.
More insulation in coil; larger coil volume, increased assembly costs.	High percentage of copper in coil; more efficient use of available space;
High voltage power supply safety problems.	High current power supply. --greater losses.

Examples of typical turns/current

From the Diamond 3 GeV synchrotron source:

Dipole:

N (per magnet):	40;	
I max	1500	A;
Volts (circuit):	500	V.

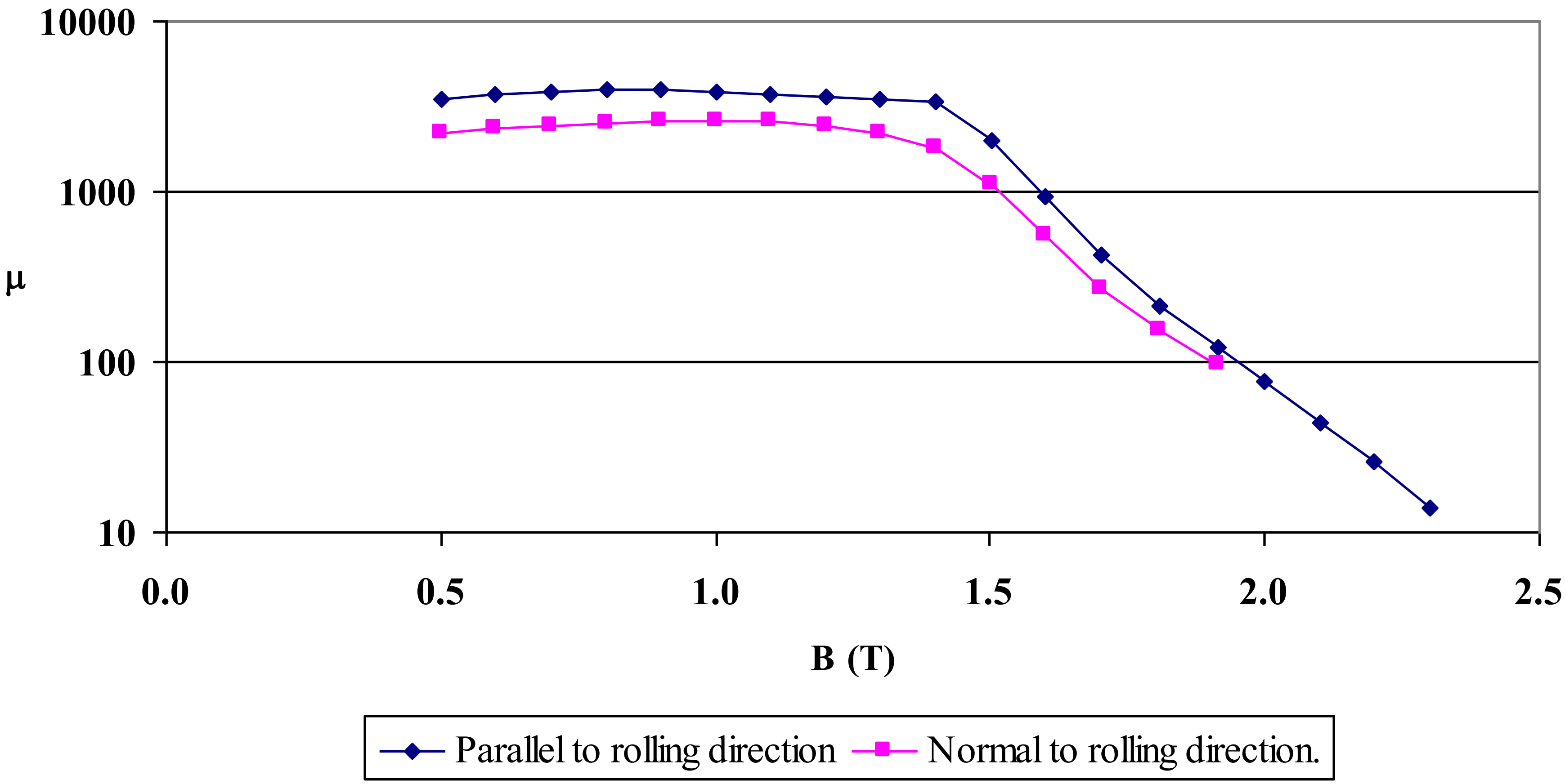
Quadrupole:

N (per pole)	54;	
I max	200	A;
Volts (per magnet):	25	V.

Sextupole:

N (per pole)	48;	
I max	100	A;
Volts (per magnet)	25	V.

Yoke - Permeability of low silicon steel



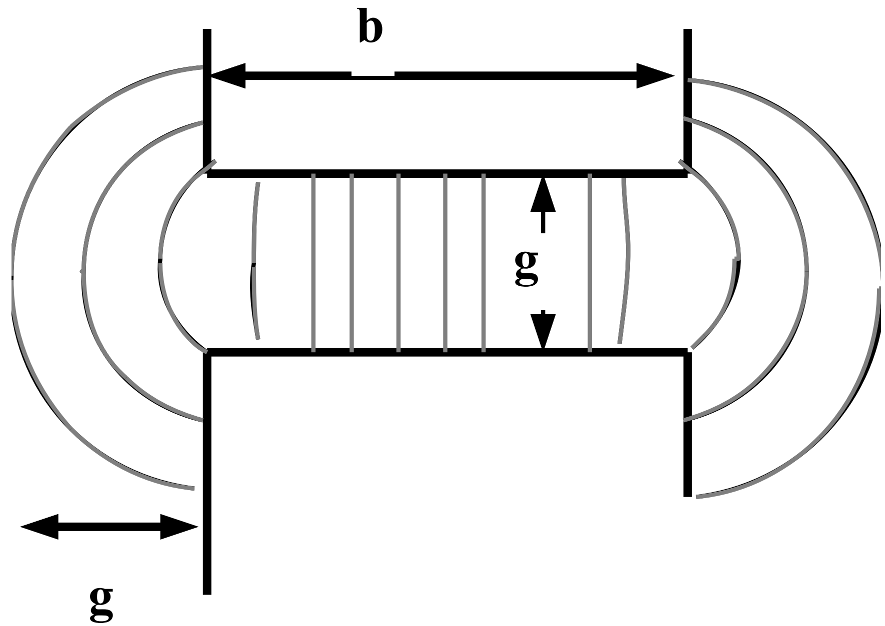
Iron Design/Geometry.

Flux in the yoke includes the gap flux and stray flux, which extends (approx) one gap width on either side of the gap.

To calculate total flux in the back-leg of magnet length λ:

$$\Phi = B_{gap} (b + 2g) \lambda.$$

Width of backleg is chosen to limit B_{yoke} and hence maintain high μ.



Note – fea codes give values of vector potential (Az); hence values of total flux can be obtained.

'Residual' fields

Residual field - the flux density in a gap at I = 0;

Remnant field B_R - value of B at H = 0;

Coercive force H_C - negative value of field at B = 0;

I = 0: $\int H.ds = 0$;

So: $(H_{steel})\lambda + (H_{gap})g = 0$;

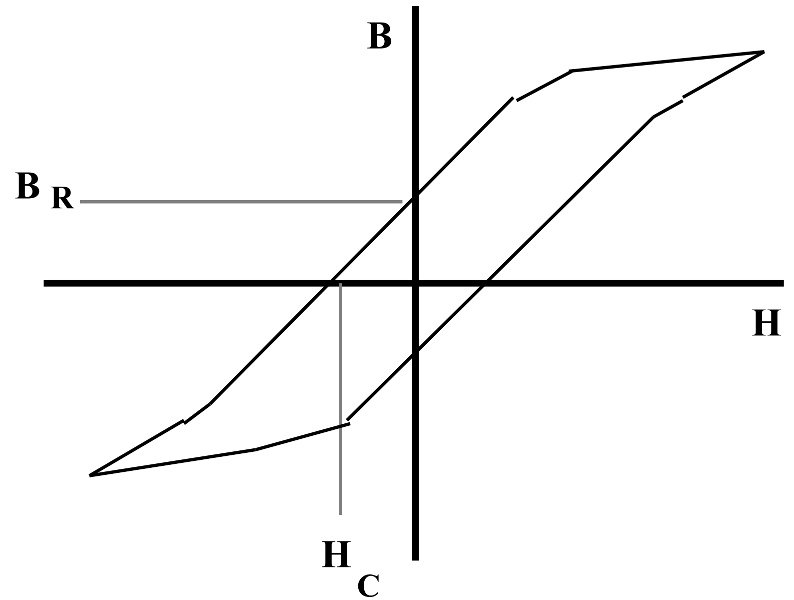
$B_{gap} = (\mu_0)(-H_{steel})(\lambda/g)$;

$B_{gap} \approx (\mu_0)(H_C)(\lambda/g)$;

Where: λ is path length in steel;

g is gap height.

Because of presence of gap, residual field is determined by coercive force H_C (A/m) and not remnant flux density B_R (Tesla).



Magnet geometry

Dipoles can be ‘C core’ ‘H core’ or ‘Window frame’

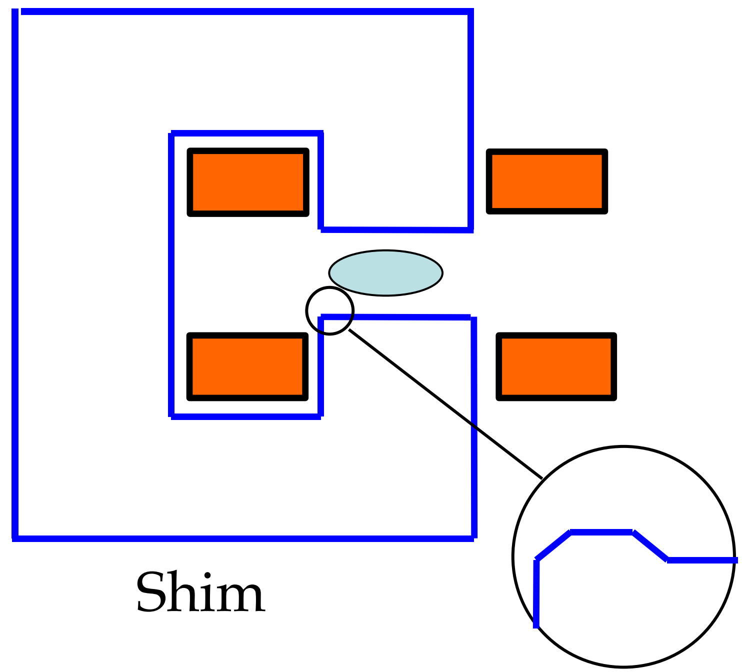
"C' Core:

Advantages:

- Easy access;
- Classic design;

Disadvantages:

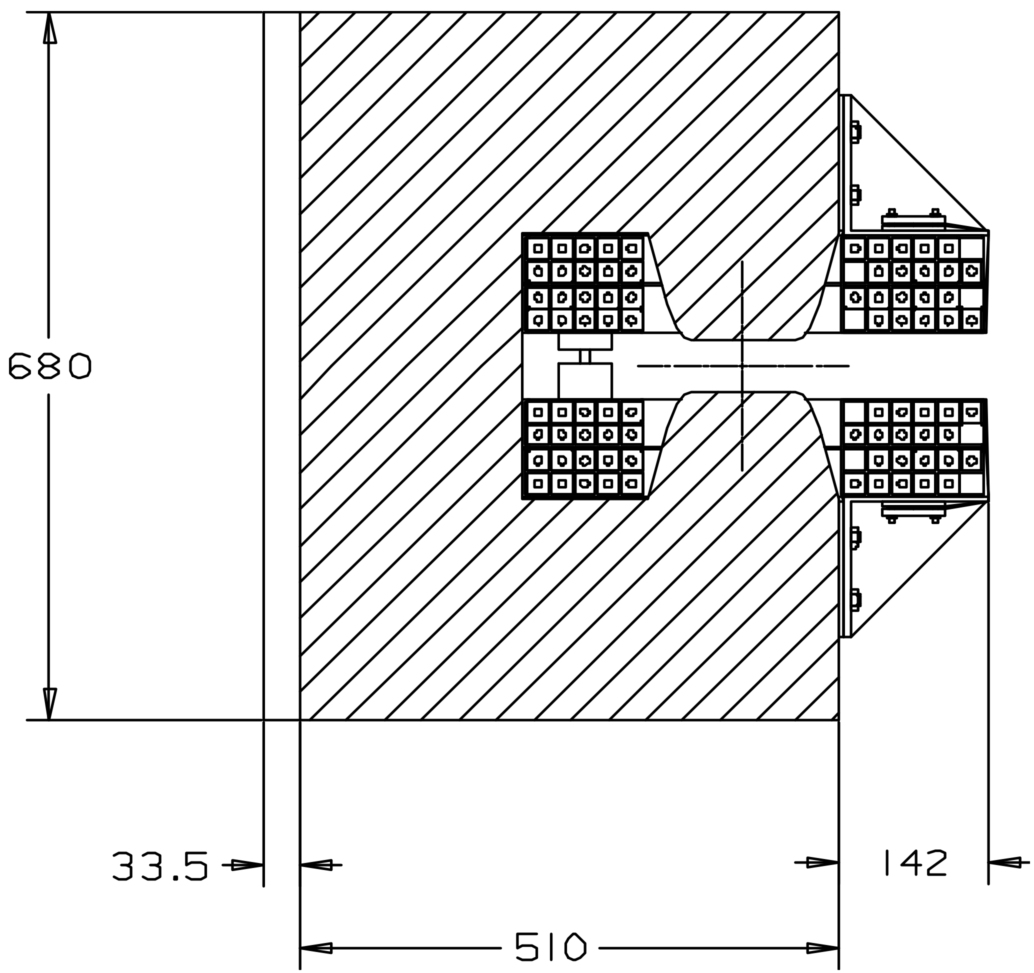
- Pole shims needed;
- Asymmetric (small);
- Less rigid;



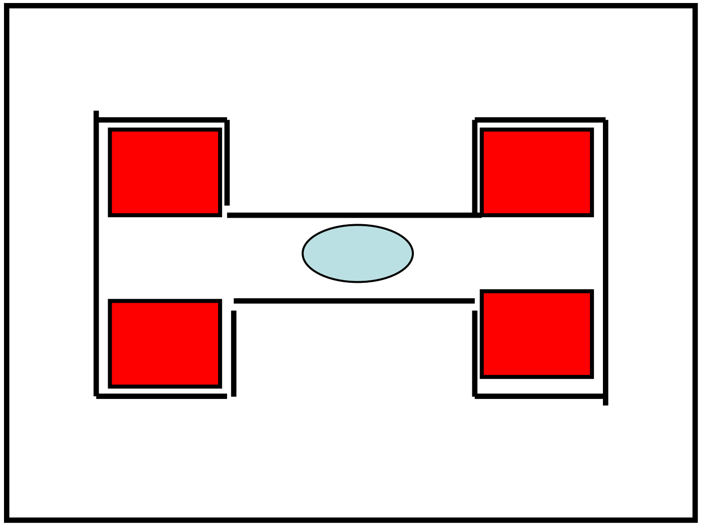
The ‘shim’ is a small, additional piece of ferro-magnetic material added on each side of the two poles – it compensates for the finite cut-off of the pole, and is optimised to reduce the 6, 10, 14...... pole error harmonics.

A typical 'C' cored Dipole

Cross section of the Diamond storage ring dipole.



H core and window-frame magnets



‘H core’:

Advantages:

- Symmetric;
- More rigid;

Disadvantages:

- Still needs shims;
- Access problems.

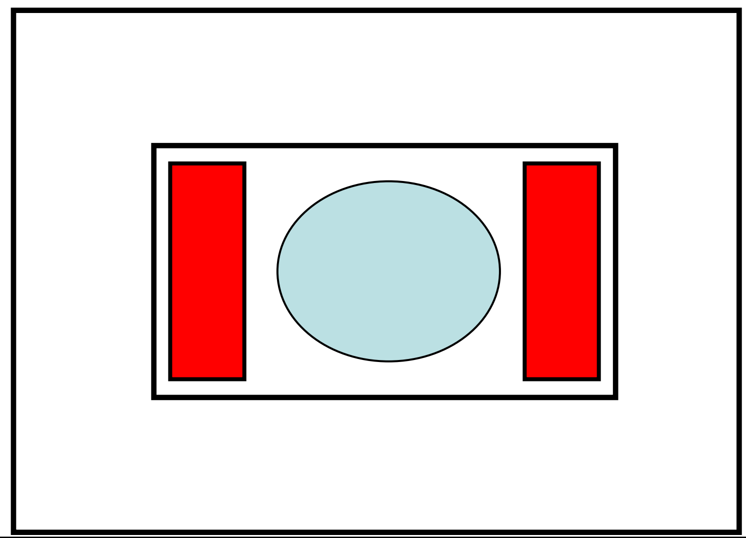
"Window Frame'

Advantages:

- High quality field;
- No pole shim;
- Symmetric & rigid;

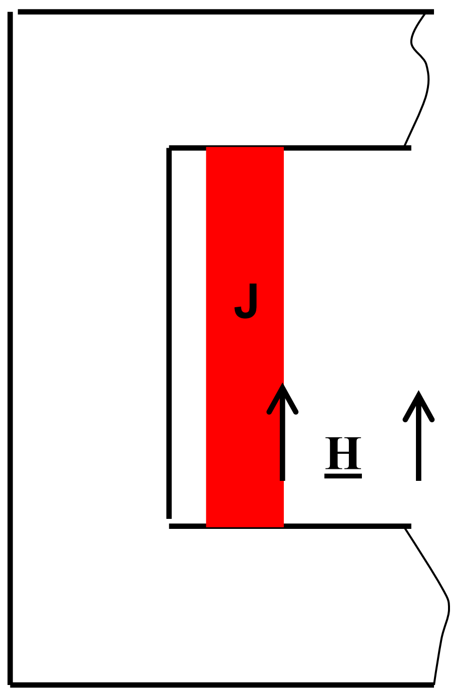
Disadvantages:

- Major access problems;
- Insulation thickness



Window frame dipole

Providing the conductor is continuous to the steel 'window frame' surfaces (impossible because coil must be electrically insulated), and the steel has infinite μ, this magnet generates perfect dipole field.

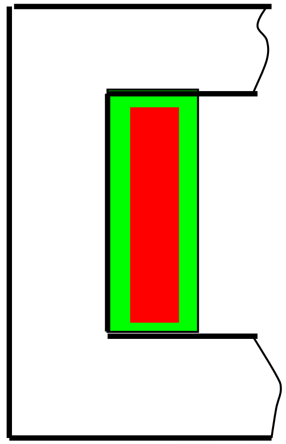


Providing current density J is uniform in conductor:

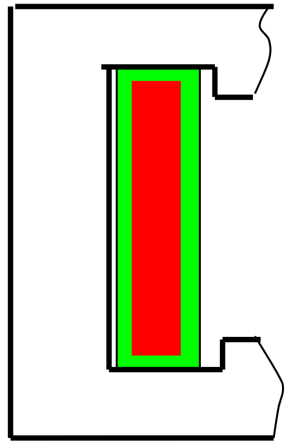
- **H** is uniform and vertical up outer face of conductor;
- **H** is uniform, vertical and with same value in the middle of the gap;
- → perfect dipole field.

The practical window frame dipole.

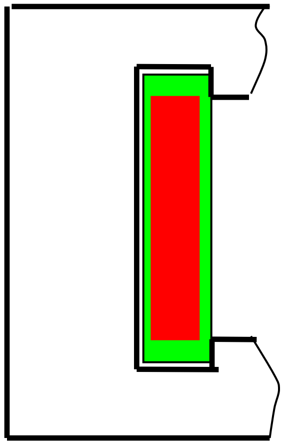
Insulation added to coil:



B increases close to coil insulation surface



B decrease close to coil insulation surface



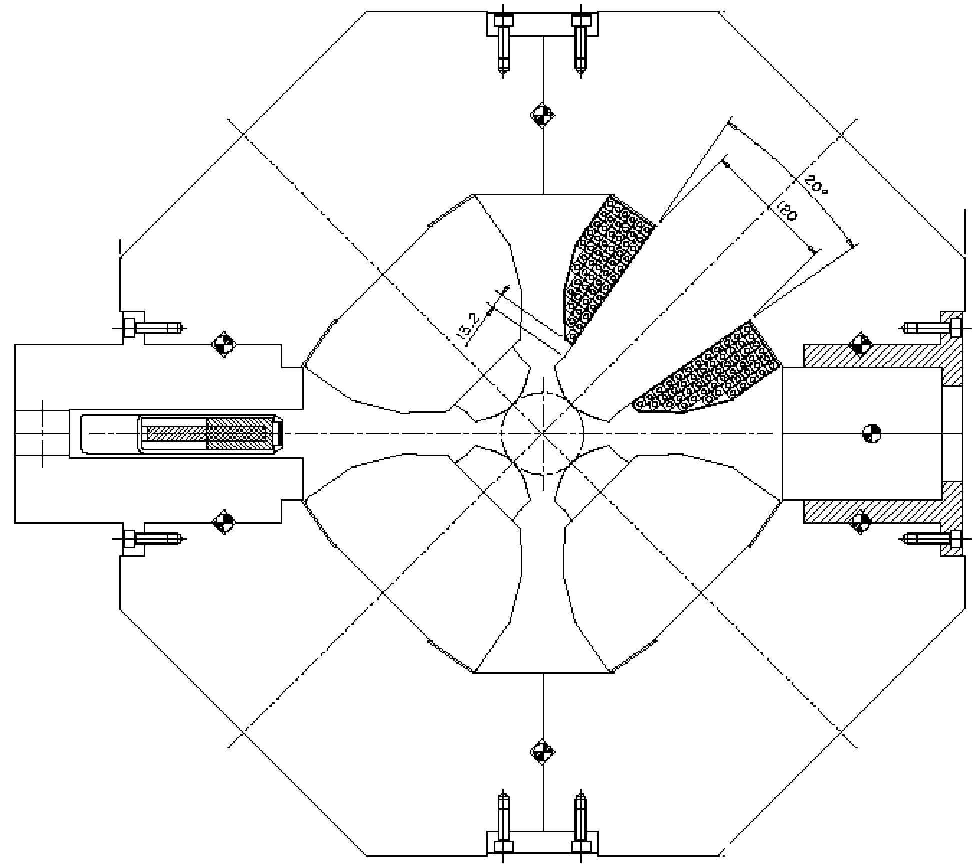
best compromise

An open-sided Quadrupole.

'Diamond' storage ring quadrupole.

The yoke support pieces in the horizontal plane need to provide space for beam-lines and are not ferro-magnetic.

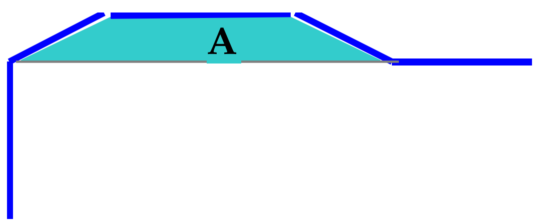
Error harmonics include n = 4 (octupole) a finite permeability error.



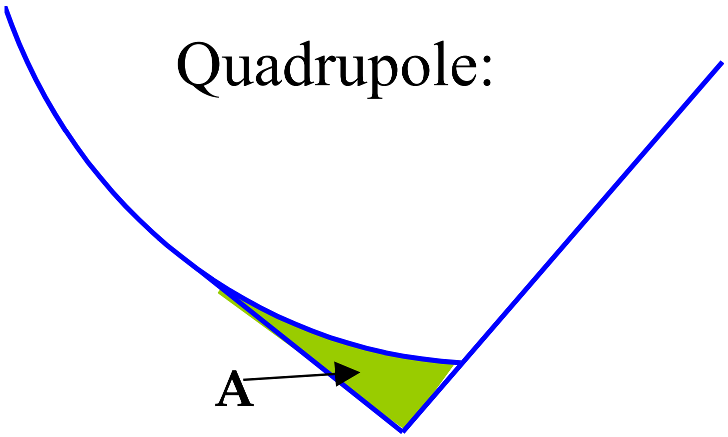
Typical pole designs

To compensate for the non-infinite pole, shims are added at the pole edges. The area and shape of the shims determine the amplitude of error harmonics which will be present.

Dipole:



Quadrupole:



The designer optimises the pole by 'predicting' the field resulting from a given pole geometry and then adjusting it to give the required quality.

When high fields are present, chamfer angles must be small, and tapering of poles may be necessary

Assessing pole design

A first assessment can be made by just examining $B_y(x)$ within the required 'good field' region.

Note that the expansion of $B_y(x)_{\ y=0}$ is a Taylor series:

$$B_y(x) = \sum_{n=1}^{\infty} \{b_n x^{(n-1)}\}$$
$$= b_1 + b_2x + b_3x^2 + \ldots\ldots\ldots$$

dipole → b_1, quad → b_2x, sextupole → b_3x^2

Also note:

$$\partial B_y(x) / \partial x = b_2 + 2\, b_3 x + \ldots\ldots..$$

So quad gradient $\quad g \equiv b_2 = \partial B_y(x) / \partial x$ in a quad

But sext. gradient $\quad g_s \equiv b_3 = 2\, \partial^2 B_y(x) / \partial x^2$ in a sext.

So coefficients are not equal to differentials for n = 3 etc.

Assessing an adequate design.

A <u>simple</u> judgement of field quality is given by plotting:

•**Dipole:**	$\{B_y(x) - B_y(0)\}/B_Y(0)$	$(\Delta B(x)/B(0))$
•**Quad:**	$dB_y(x)/dx$	$(\Delta g(x)/g(0))$
•**6poles:**	$d^2B_y(x)/dx^2$	$(\Delta g_2(x)/g_2(0))$

'Typical' acceptable variation inside 'good field' region:

$$\Delta B(x)/B(0) \leq 0.01\%$$
$$\Delta g(x)/g(0) \leq 0.1\%$$
$$\Delta g_2(x)/g_2(0) \leq 1.0\%$$

Design by computer codes.

Computer codes are now used; eg the Vector Fields codes -'OPERA 2D' and 'TOSCA' (3D).

These have:

- finite elements with variable triangular mesh;
- multiple iterations to simulate steel non-linearity;
- extensive pre and post processors;
- compatibility with many platforms and P.C. o.s.

Technique is iterative:

- calculate flux generated by a defined geometry;
- adjust the geometry until required distribution is acheived.

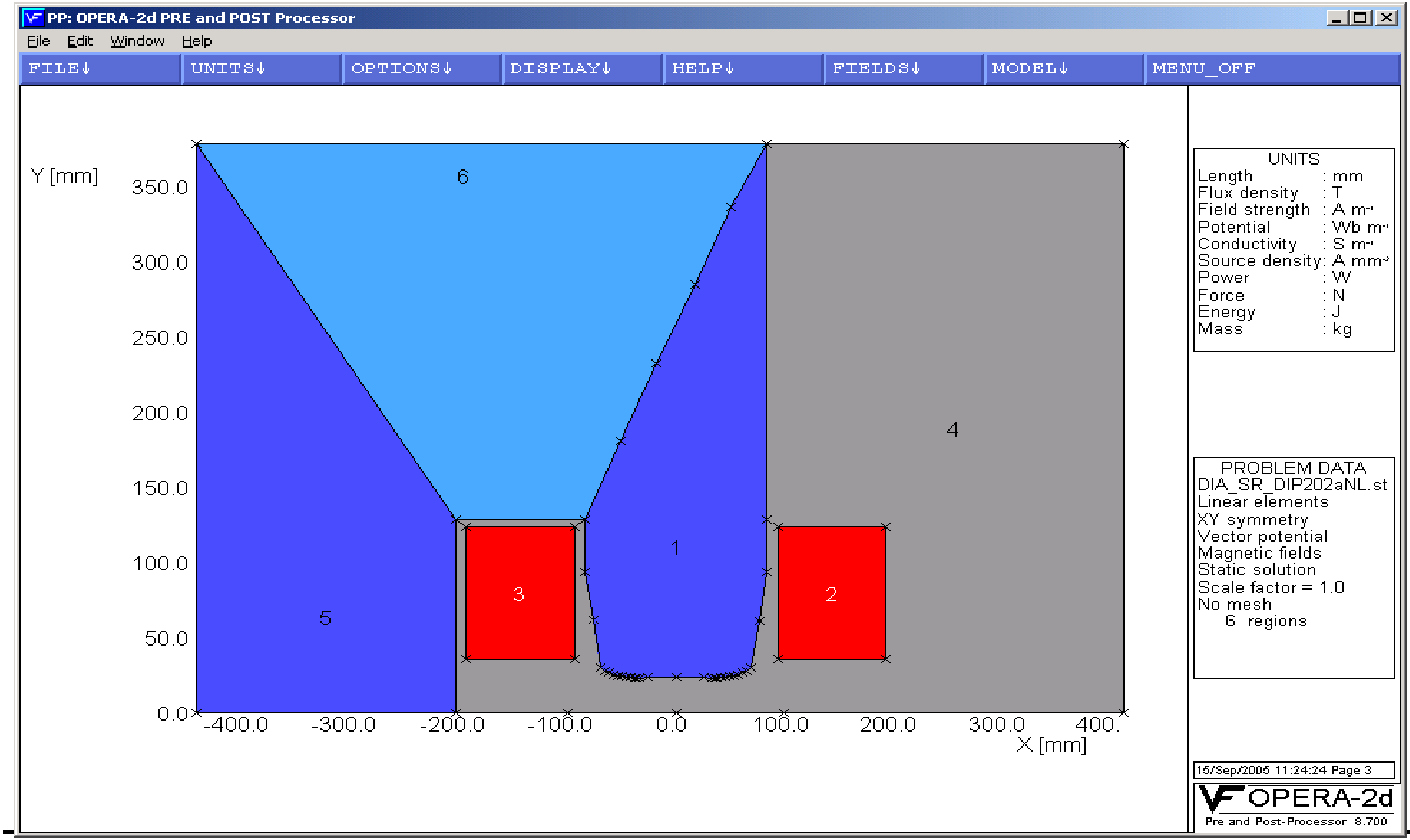
Design Procedures – OPERA 2D.

Pre-processor:

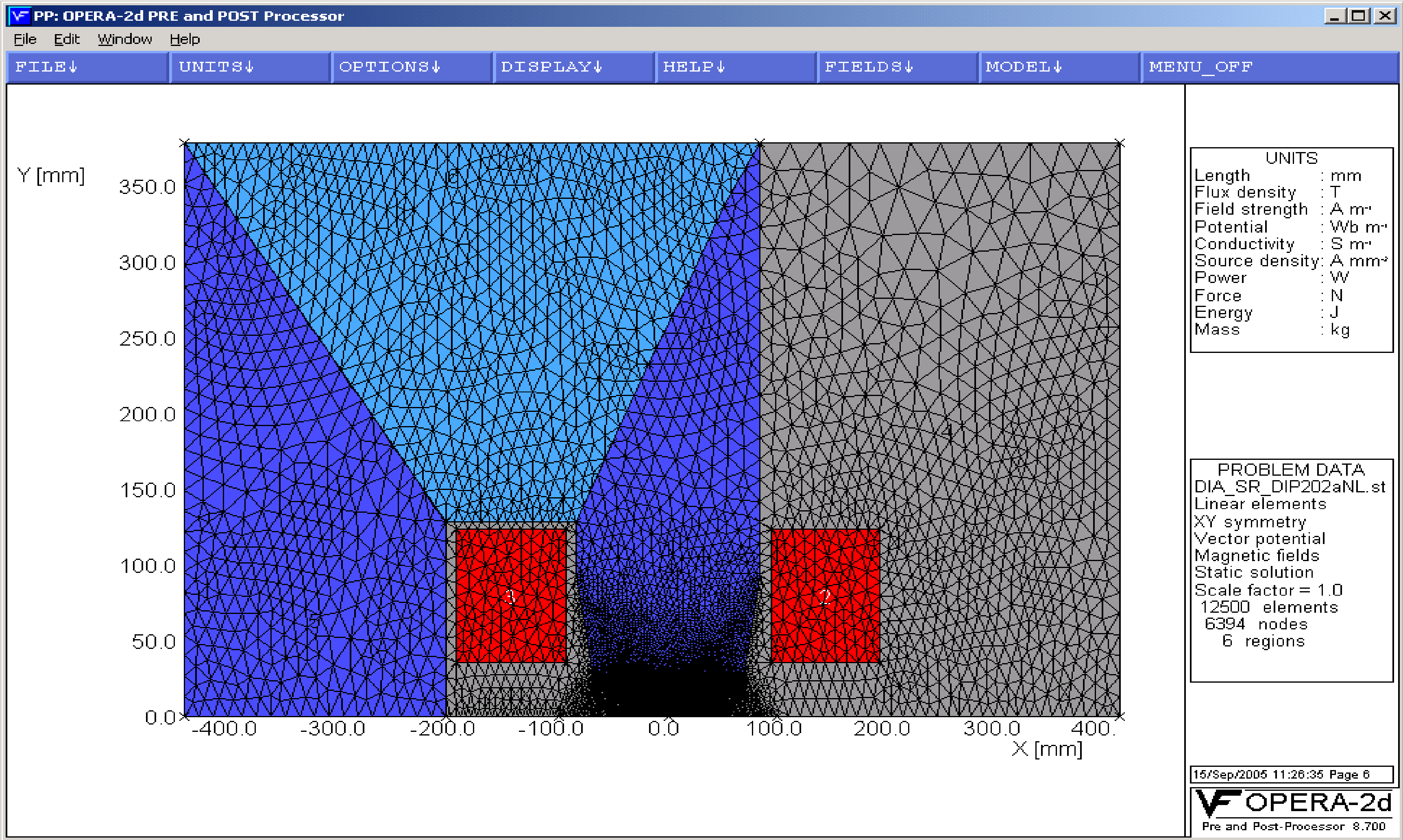
The model is set-up in 2D using a GUI (graphics user's interface) to define 'regions':

- steel regions;
- coils (including current density);
- a 'background' region which defines the physical extent of the model;
- the symmetry constraints on the boundaries;
- the permeability for the steel (or use the pre-programmed curve);
- mesh is generated and data saved.

Model of Diamond s.r. dipole

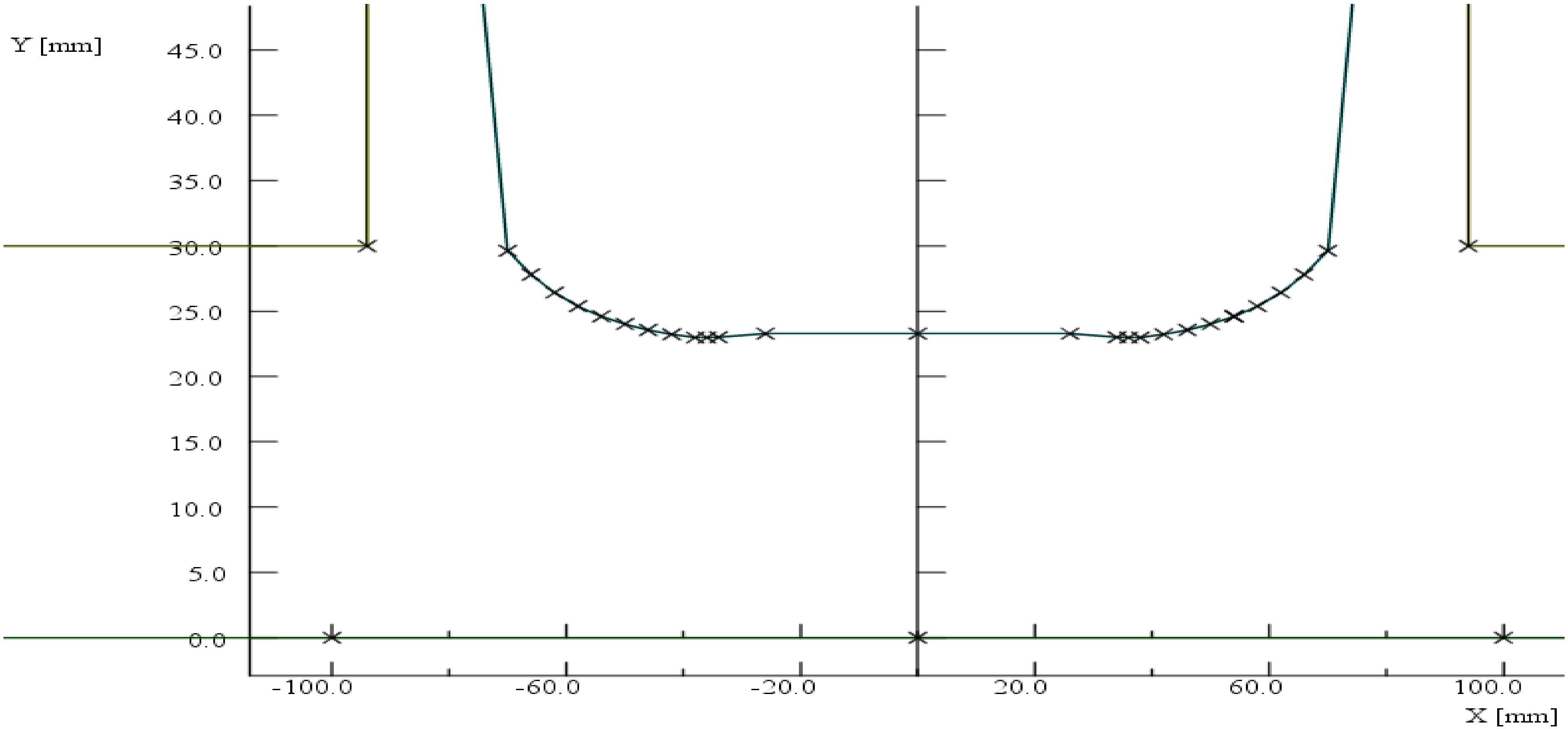


With mesh added



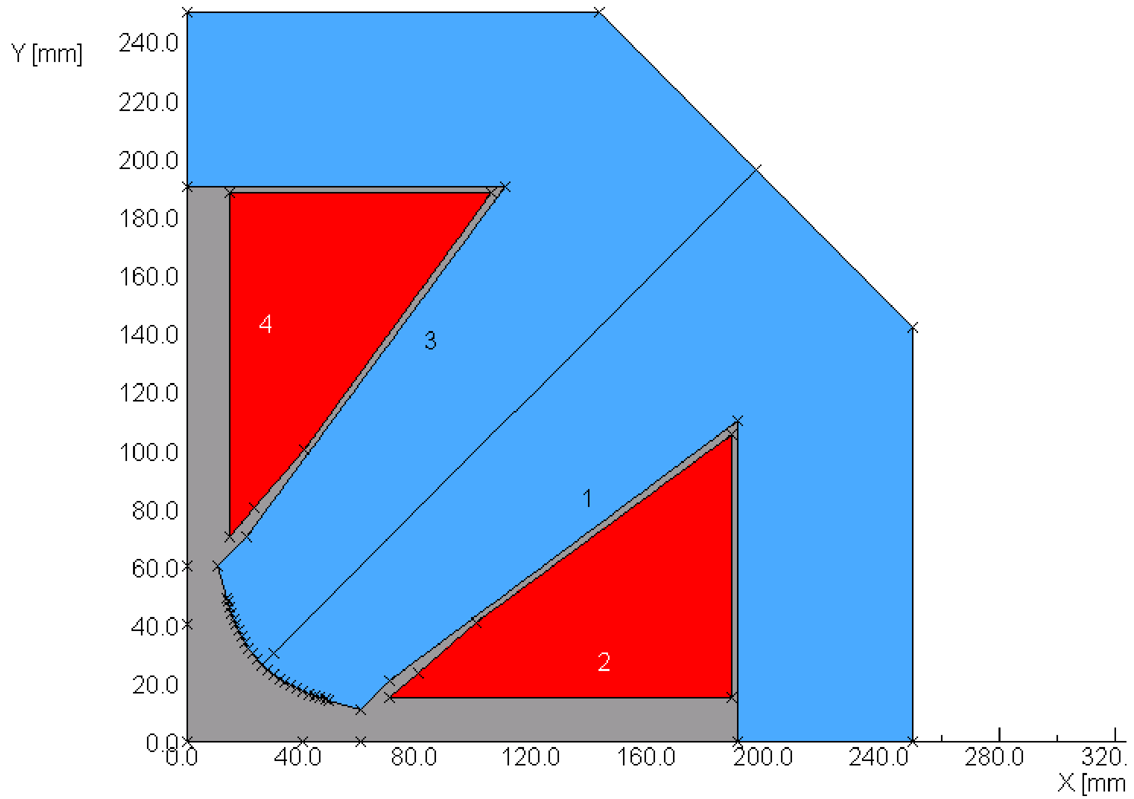
'Close-up' of pole region.

Pole profile, showing shim and 'Rogowski side roll-off' for Diamond 1.4 T dipole.:



Diamond quadrupole model

Very preliminary model – fully symmetric around 4 axies.



Note – one eighth of quadrupole <u>could</u> be used with opposite symmetries defined on horizontal and y = x axis.

Calculation.

‘Solver’:

either:

- linear which uses a predefined constant permeability for a single calculation, or

- non-linear, which is iterative with steel permeability set according to B in steel calculated on previous iteration.

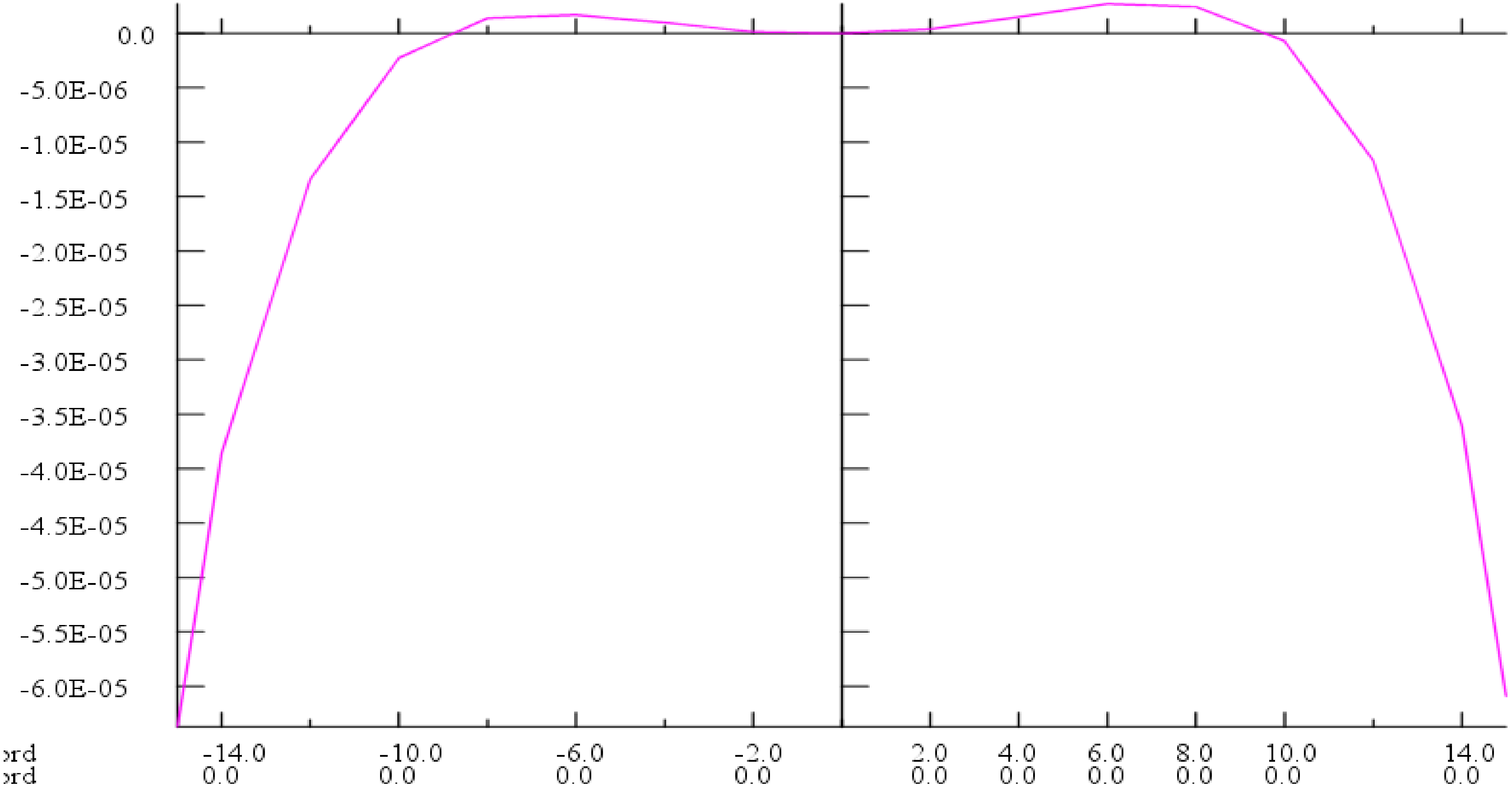
Data Display – OPERA 2D.

Post-processor:

uses pre-processor model for many options for displaying field amplitude and quality:

- field lines;
- graphs;
- contours;
- gradients;
- harmonics (from a Fourier analysis around a pre-defined circle).

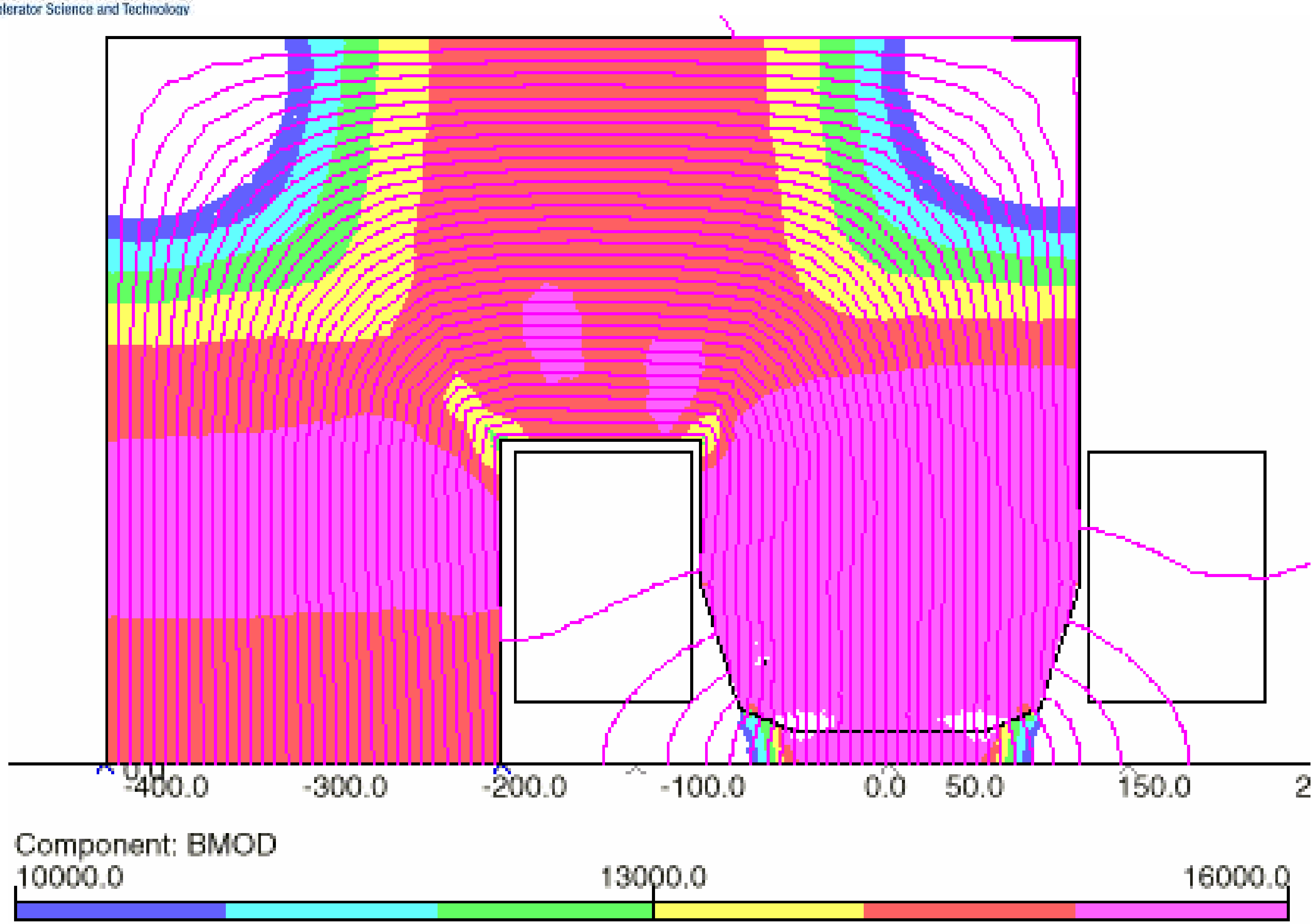
2 D Dipole field homogeneity on x axis



Diamond s.r. dipole: $\Delta B/B = \{By(x) - B(0,0)\}/B(0,0)$;

typically $\pm$ 1:10^4 within the 'good field region' of -12mm $\leq x \leq$ +12 mm..

2 D Flux density distribution in a dipole.

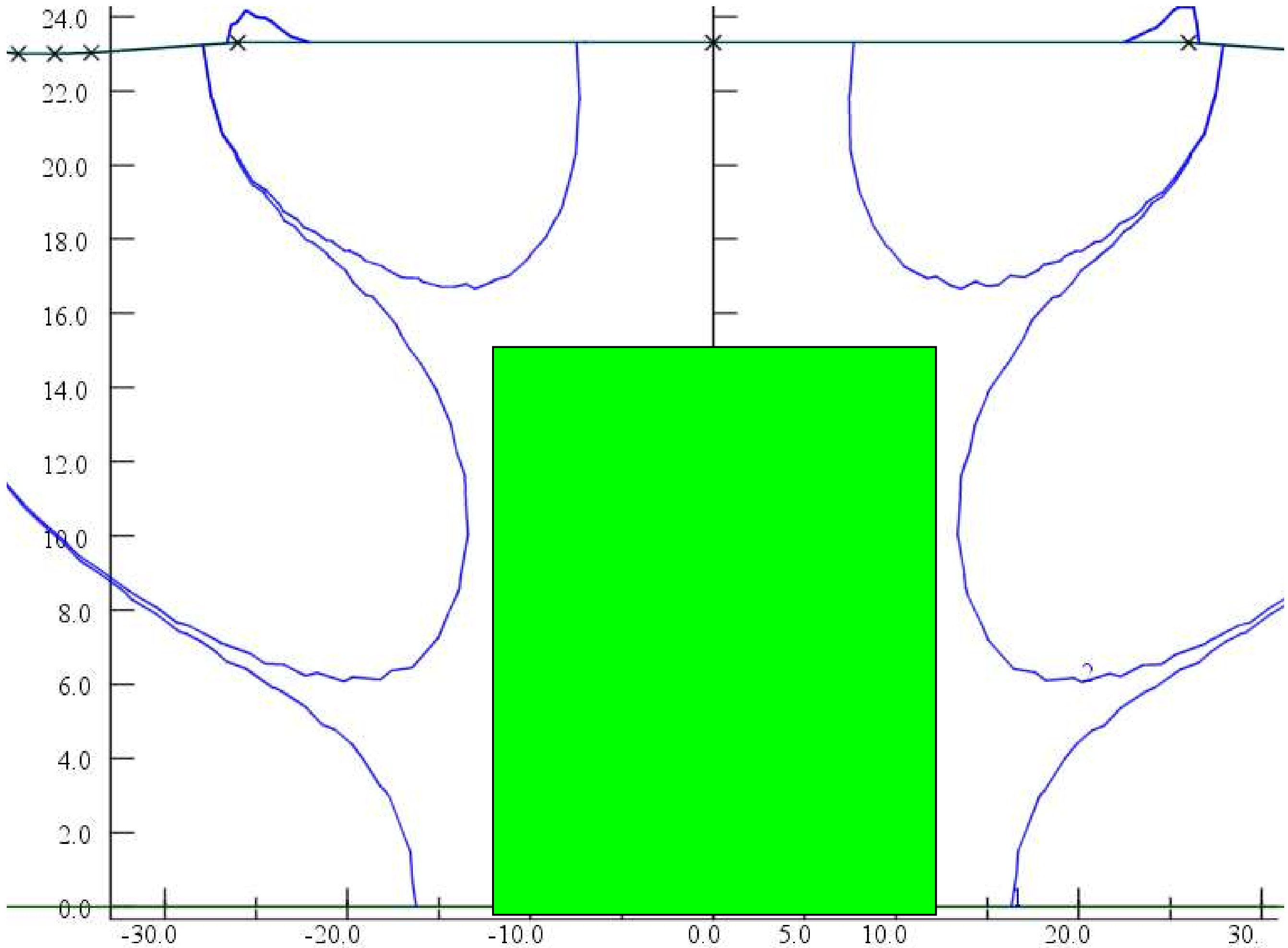


2 D Dipole field homogeneity in gap

Transverse (x,y) plane in Diamond s.r. dipole;

contours are ±0.01%

required good field region:

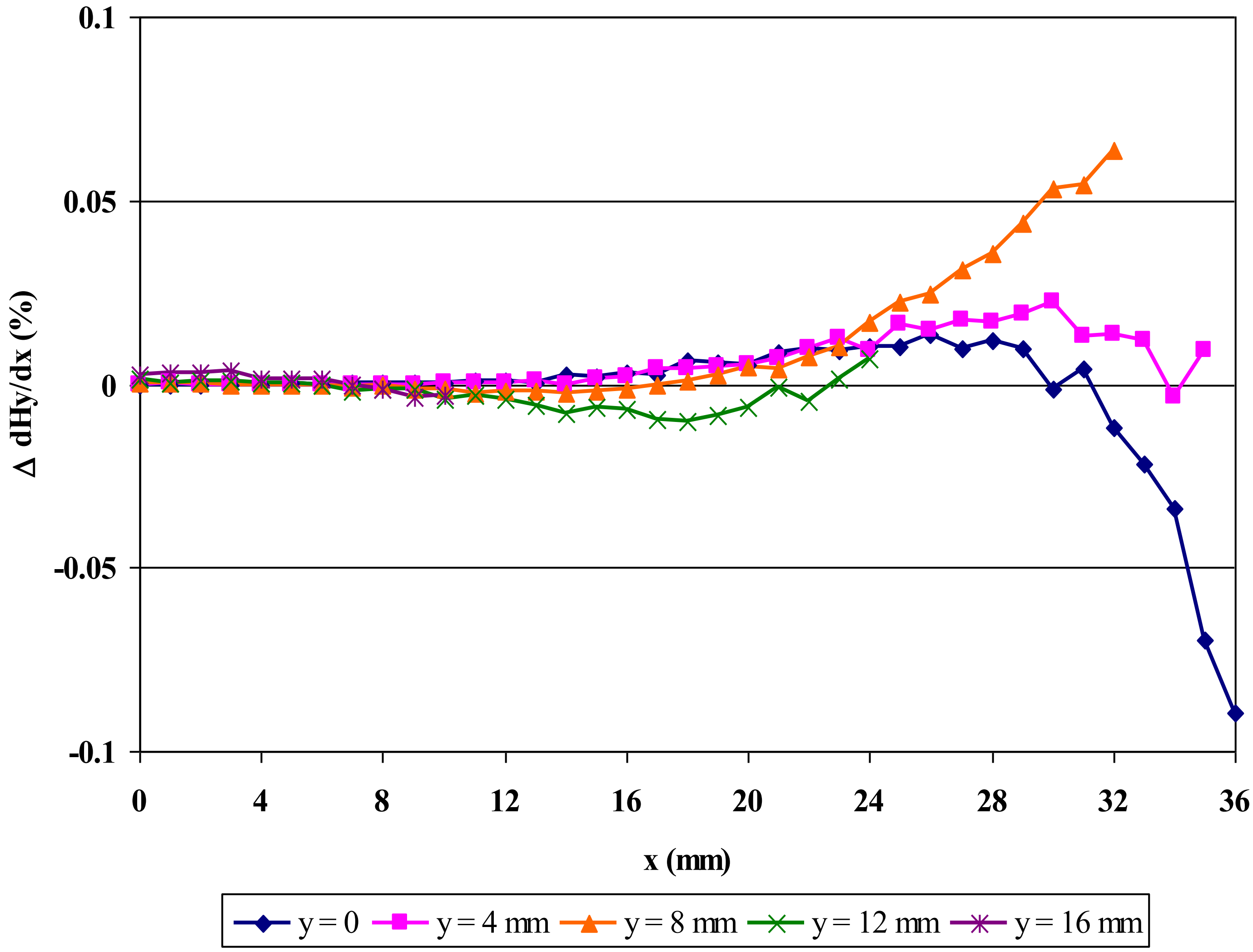


2 D Assessment of quad gradient.

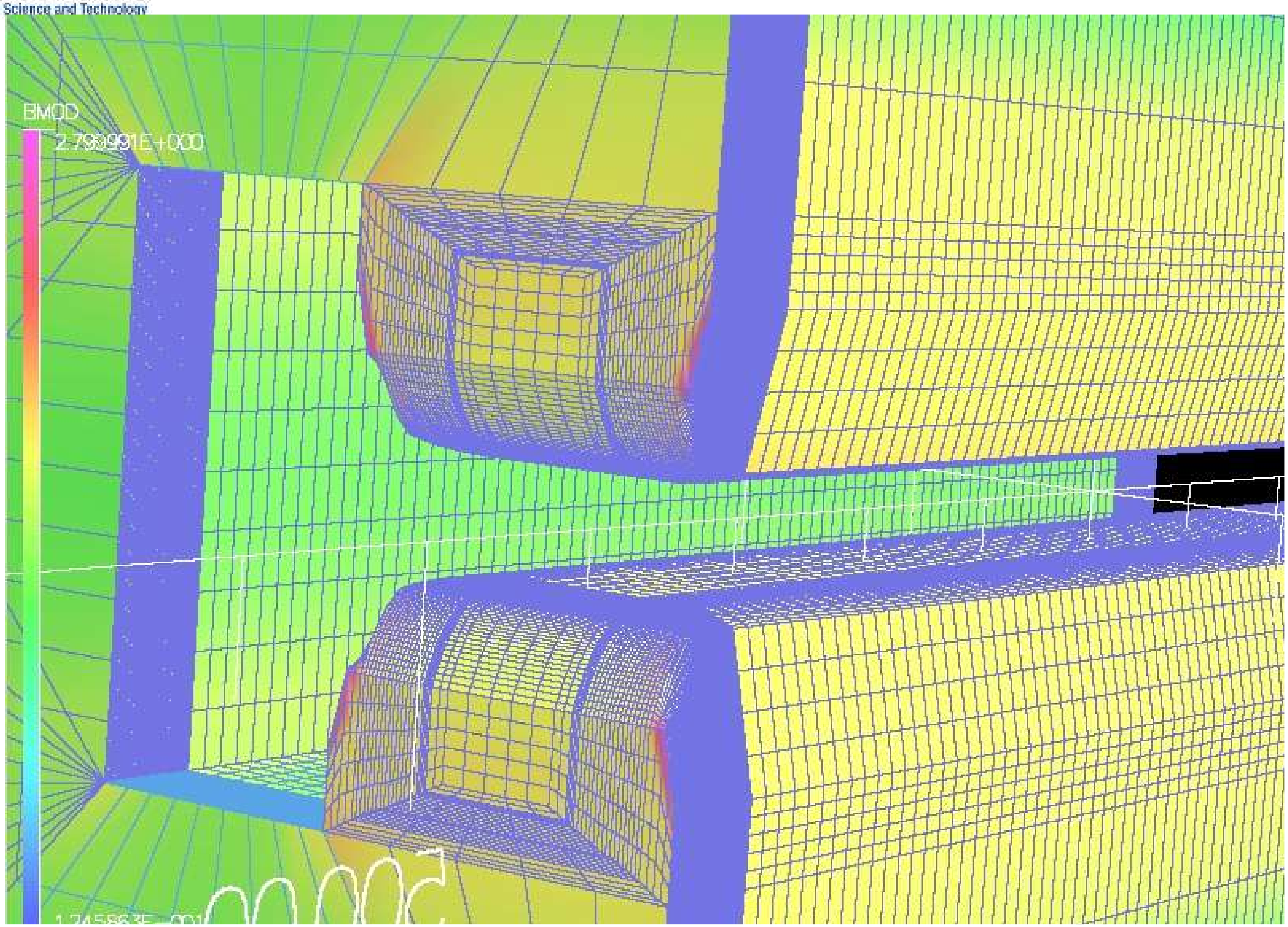
Diamond WM quadrupole:

graph is <u>percentage</u> variation in dBy/dx vs x at different values of y.

Gradient quality is to be ± 0.1 % or better to x = 36 mm.



OPERA 3D model of Diamond dipole.



Harmonics indicate magnet quality

The amplitude and phase of the harmonic components in a magnet provide an assessment:

- when accelerator physicists are calculating beam behaviour in a lattice;
- when designs are judged for suitability;
- when the manufactured magnet is measured;
- to judge acceptability of a manufactured magnet.

Measurement of a magnet after manufacture will be discussed in the section on measurements.

Classical end or side solution

The 'Rogowski' roll-off:

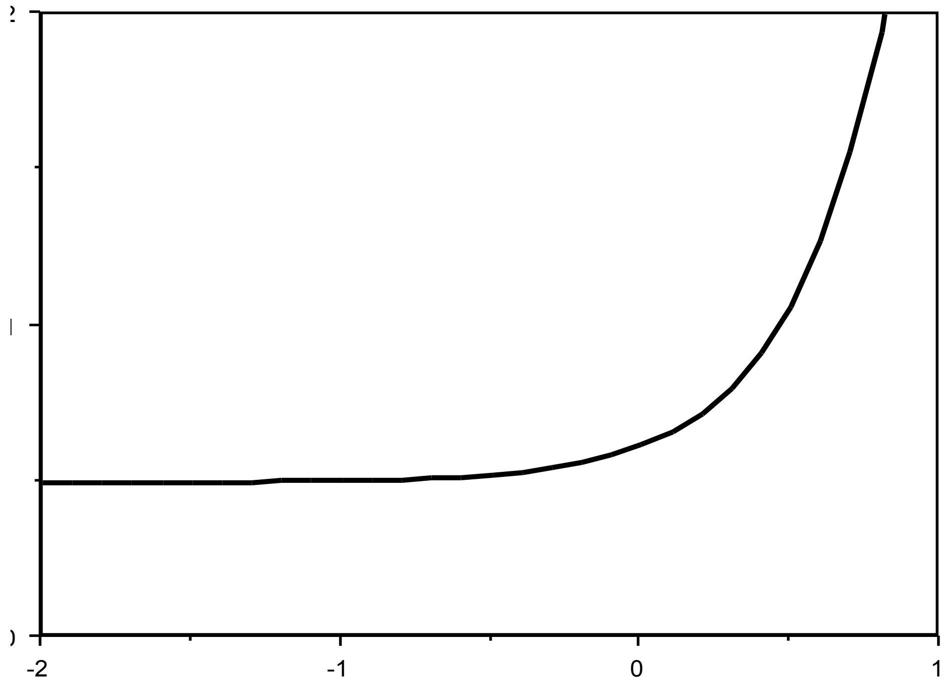
Equation: $y = g/2 + (g/\pi\alpha)\,[\exp(\alpha\pi x/g) - 1];$

g/2 is dipole half gap;

y = 0 is centre line of gap;

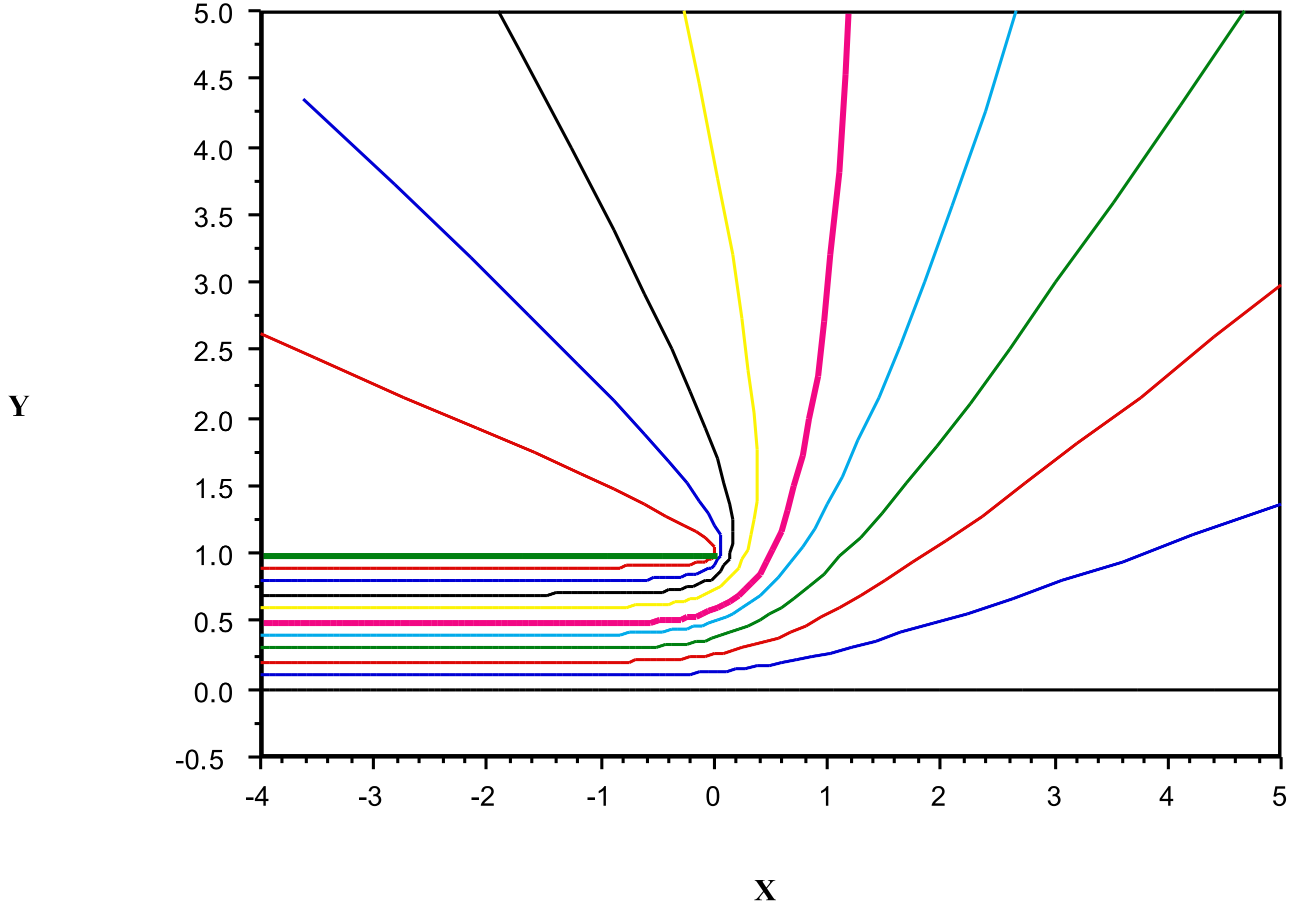
α is a parameter controlling gradient at x = 0 (~1).

This profile provides the maximum rate of **increase** in gap with a monotonic **decrease** in flux density at the surface ie no saturation

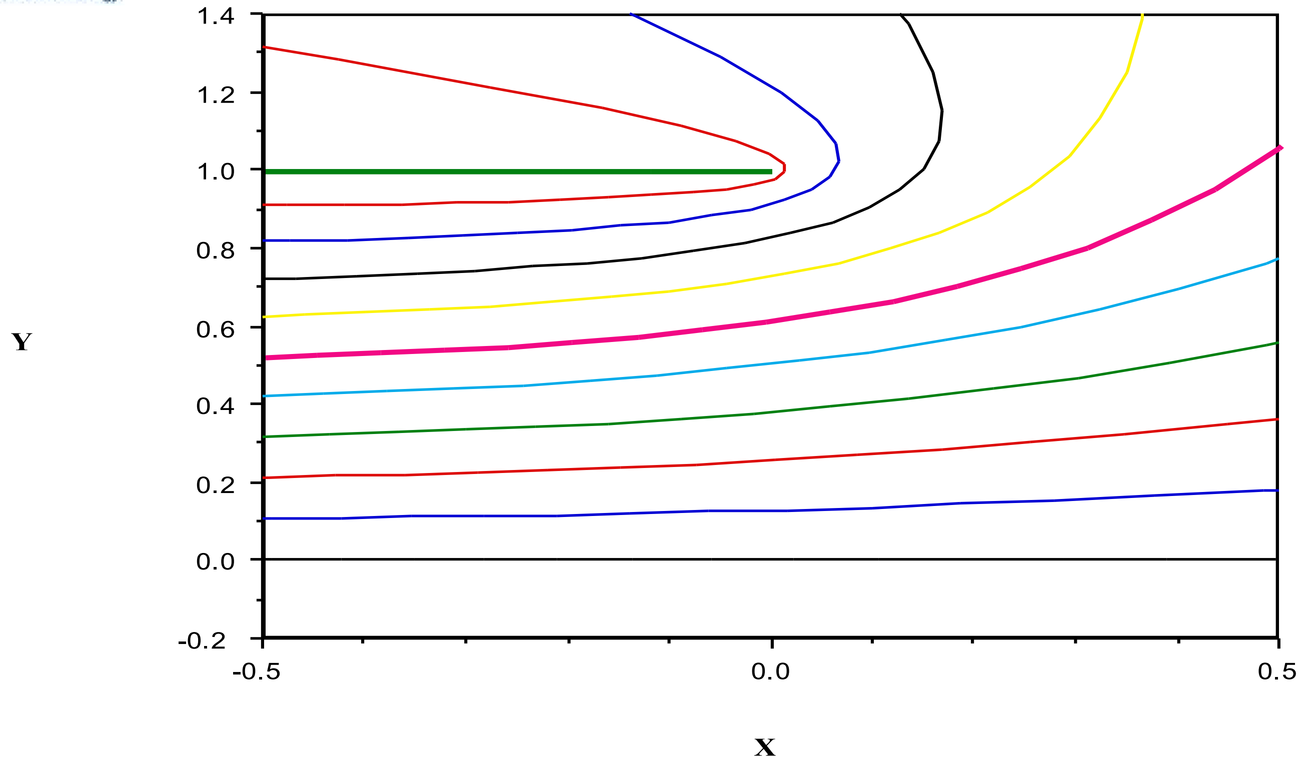


How derived?

Rogowski calculated electric potential lines around a flat capacitor plate:



Blown-up version

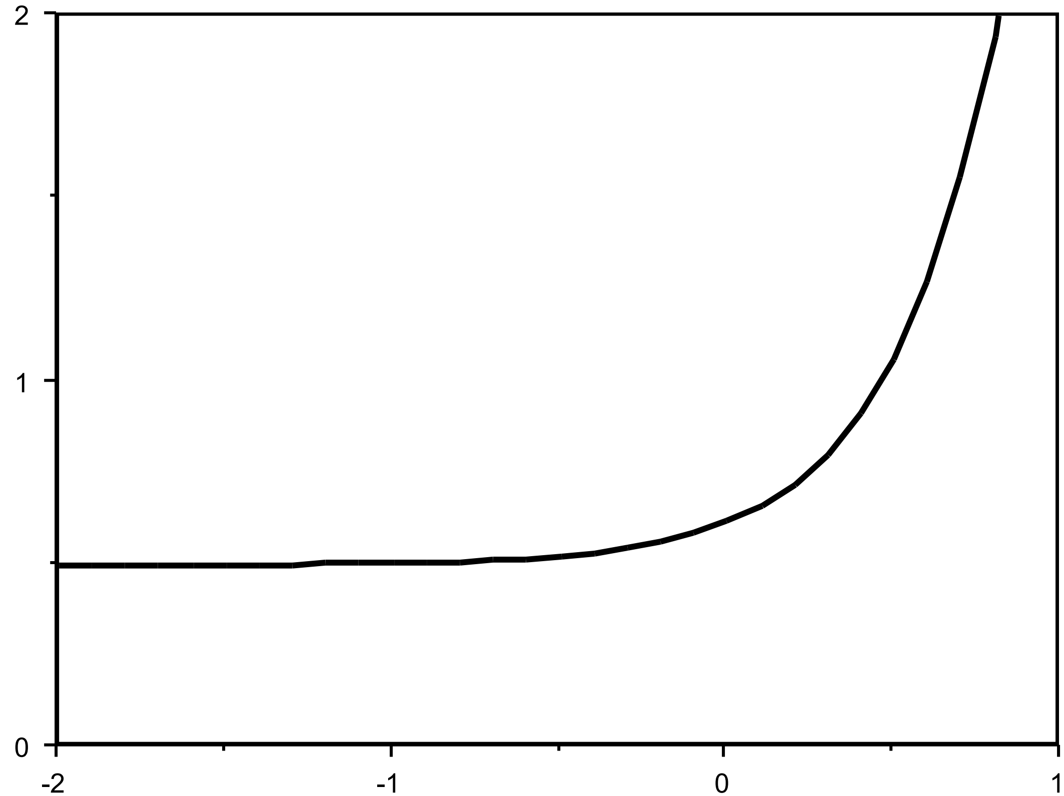


The central heavy line is for Pot = 0.5.

Then Rogowski showed that this was the fastest changing line along which the field intensity (grad potential) was **<u>monotonically decreasing</u>.**

Then applied to magnet ends

Conclusion: Recall that a high μ steel surface is a line of constant scalar potential. Hence, a magnet pole end using the $\phi = 0.5$ potential line will see a **monotonically decreasing** flux density normal to the steel; at some point where B is much lower, this can break to a vertical end line.



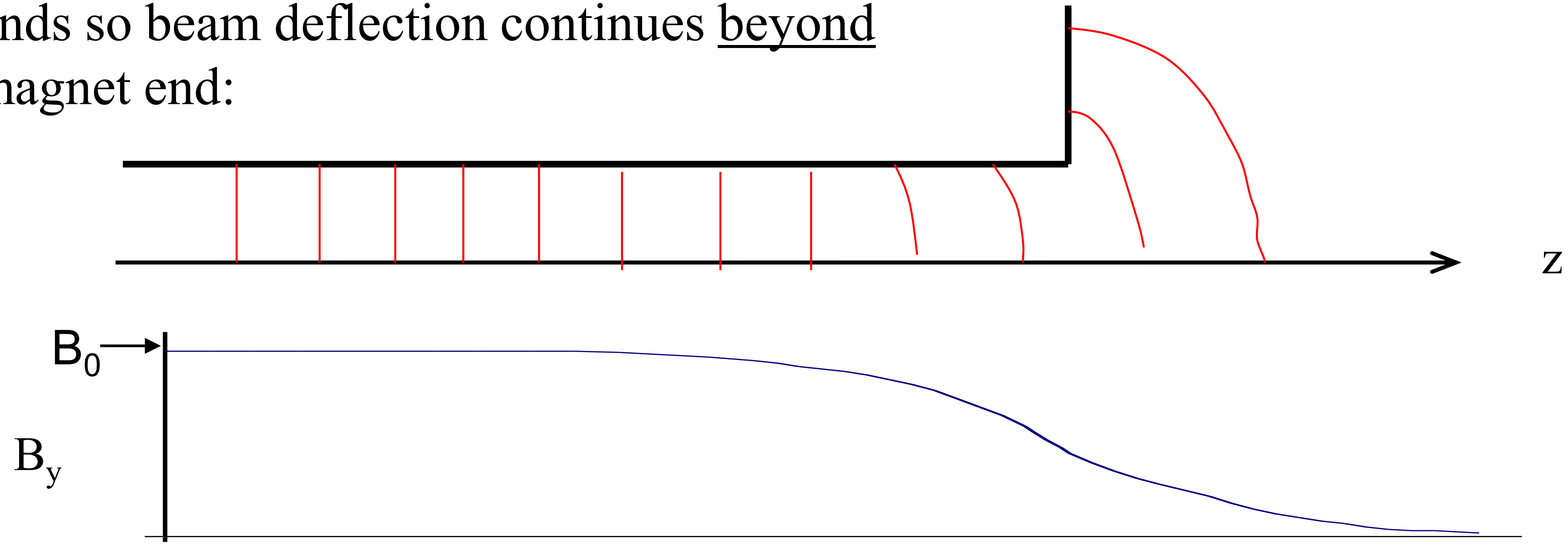
Magnet half gap height $= g/2$;

Centre line of gap is $y = 0$;

Equation is: $y = g/2 + (g/\pi) \exp((\pi x/g)-1)$

The third dimension – magnet ends.

Fringe flux will be present at the magnet ends so beam deflection continues beyond magnet end:



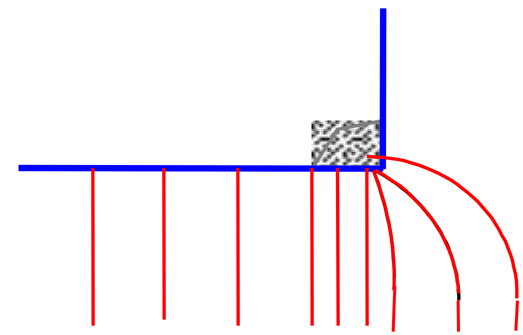
The magnet's strength is given by $\int By\,(z)\,dz$ along the magnet , the integration including the fringe field at each end;

The **'magnetic length'** is defined as **$(1/B_0)(\int By\,(z)\,dz\,)$** over the same integration path, where B_0 is the field at the azimuthal centre.

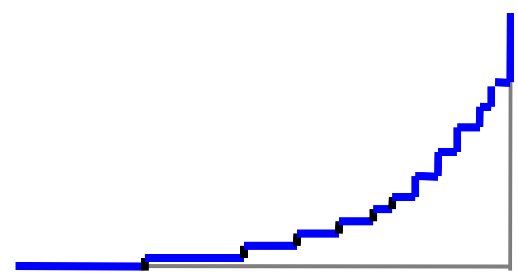
Magnet End Fields and Geometry.

It is necessary to terminate the magnet in a controlled way:

- to define the length (strength);
- to prevent saturation in a sharp corner (see diagram);
- to maintain length constant with x, y;
- to prevent flux entering normal to lamination (ac).

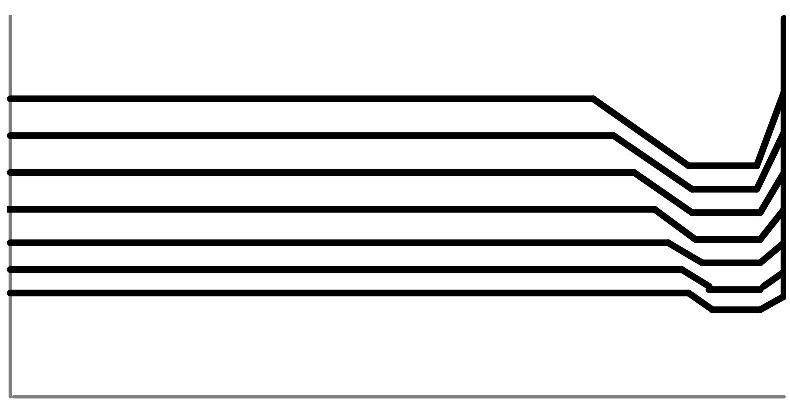


The end of the magnet is therefore 'chamfered' to give increasing gap (or inscribed radius) and lower fields as the end is approached

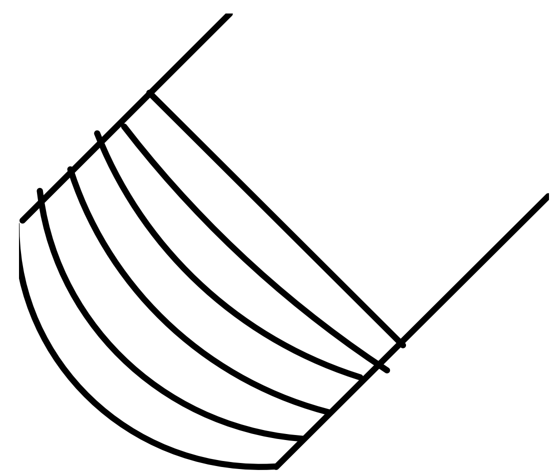


Pole profile adjustment

As the gap is increased, the size (area) of the shim is increased, to give *some* control of the field quality at the lower field. This is far from perfect!



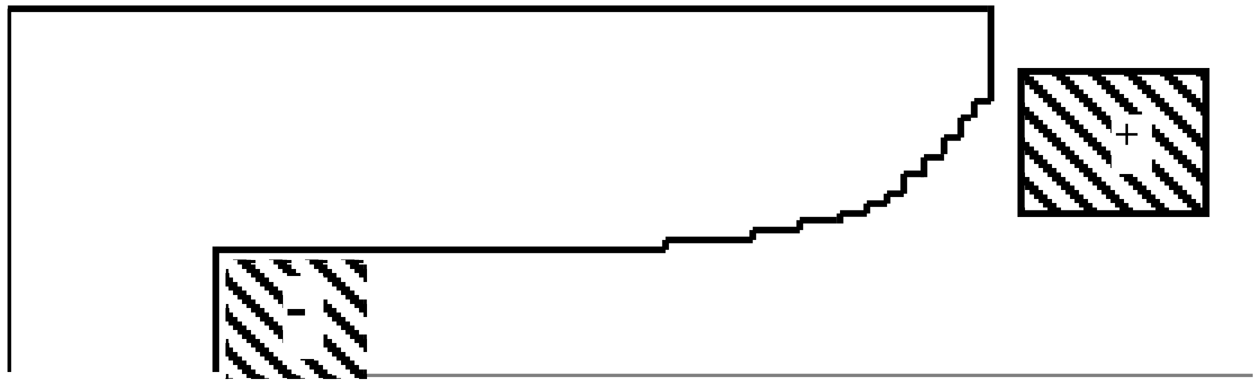
Transverse adjustment at end of dipole



Transverse adjustment at end of quadrupole

Calculation of end effects with 2D codes.

FEA model in longitudinal plane, with correct end geometry (including coil), but 'idealised' return yoke:



This will establish the end distribution; a numerical integration will give the 'B' length.

Provided dBY/dz is not too large, single 'slices' in the transverse plane can be used to calculated the radial distribution as the gap increases. Again, numerical integration will give $\int$ B.dl as a function of x.

This technique is less satisfactory with a quadrupole, but end effects are less critical with a quad.

End geometries - dipole

Simpler geometries can be used in some cases.

The Diamond dipoles have a Rogawski roll-off at the ends (as well as Rogawski roll-offs at each side of the pole).

See photographs to follow.

This give small negative sextupole field in the ends which will be compensated by adjustments of the strengths in adjacent sextupole magnets – this is possible because each sextupole will have int own individual power supply

Diamond Dipole

Diamond dipole ends

Diamond Dipole end

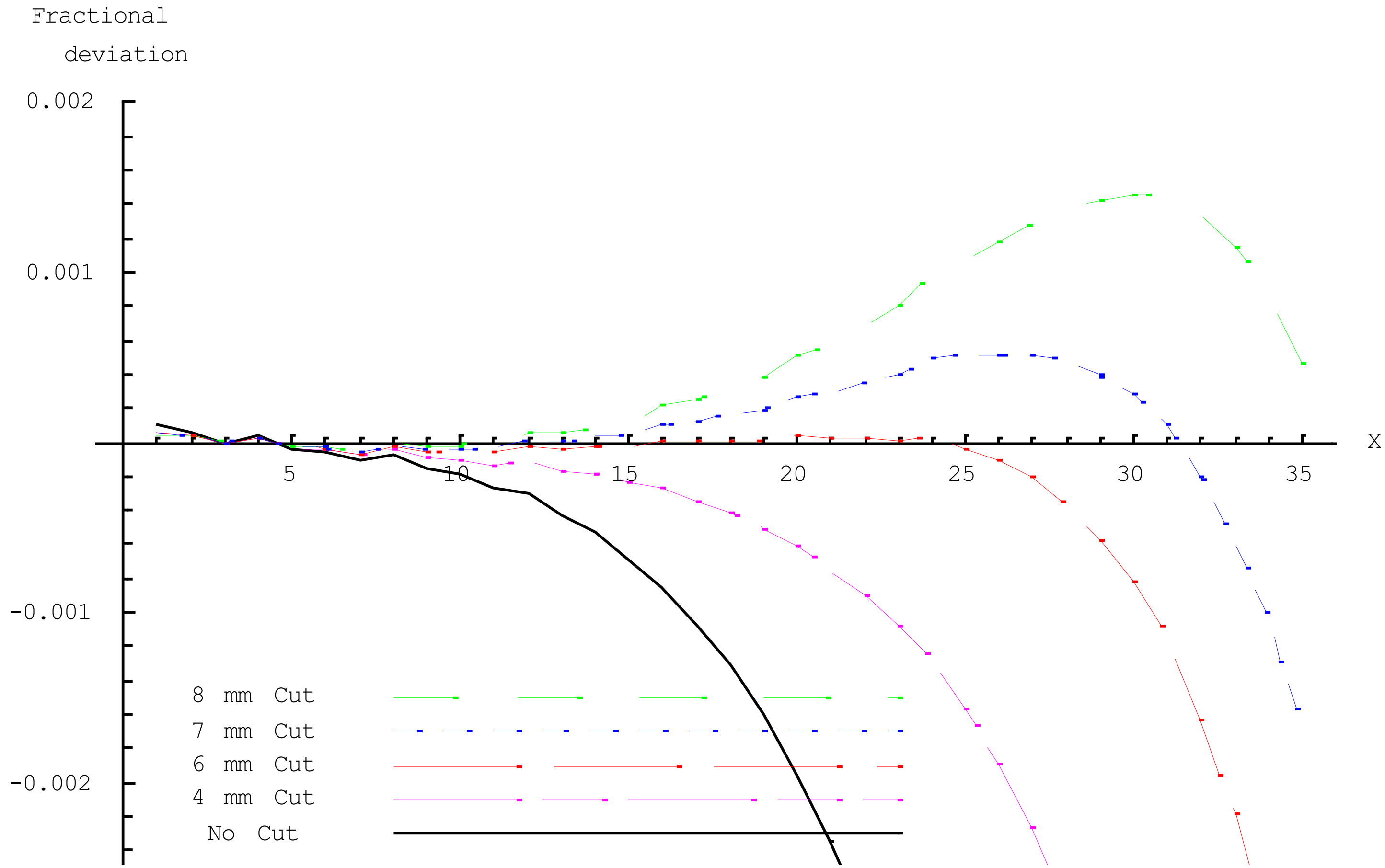
Simplified end geometries - quadrupole

Diamond quadrupoles have an angular cut at the end; depth and angle were adjusted using 3D codes to give optimum integrated gradient.

Diamond W quad end

End chamfering - Diamond 'W' quad

Tosca results - different depths 45° end chamfers on $\Delta g/g_0$ integrated through magnet and end fringe field (0.4 m long WM quad).



Thanks to Chris Bailey (DLS) who performed this working using OPERA 3D.

Sextupole ends

It is not usually necessary to chamfer sextupole ends (in a d.c. magnet). Diamond sextupole end: