Insertion Devices
Lecture 5
Additional Magnet Designs

Jim Clarke
ASTeC
Daresbury Laboratory
Introduction

We now have an understanding for how we can use Permanent Magnets to create the sinusoidal fields required by Insertion Devices

Next we will look at creating more complex field shapes, such as those required for variable polarisation

Later we will look at other technical issues such as the challenge of in-vacuum undulators, dealing with the large magnetic forces involved, correcting field errors, and also how and why we might cool undulators to ~150K

Finally, electromagnetic alternatives will be considered
Helical (or Elliptical) Undulators for Variable Polarisation

We need to include a finite horizontal field of the same period so the electron takes an elliptical path when it is viewed head on.

We want two orthogonal fields of equal period but of different amplitude and phase.

\[ B_x = B_{x_0} \sin \left( \frac{2\pi s}{\lambda_u} - \phi \right) \]
\[ B_y = B_{y_0} \sin \left( \frac{2\pi s}{\lambda_u} \right) . \]

Three independent variables are required for the arbitrary selection of any polarisation state.
Helical Undulators

Pure helical fields are suited to a **circular magnet geometry** (magnets surrounding the vacuum chamber) but these are not generally practical for a light source.

A planar geometry is better suited to light sources (all of the magnets in the plane above and below the axis).

- This allows the machine to have a narrow vertical gap and a wide horizontal gap – as required for injection.
- But, it is not so easy to generate H and V fields.
- It is not so easy to understand the fields either!
- Two degrees of freedom are needed to control the H & V fields independently, ideally three so you can control the phase as well.
- Hence two (or three) independent motion systems are needed.
The Helios Design

The first planar helical undulator design
The top array generates horizontal field, and the bottom array generates vertical field (on axis)
Each array can be independently adjusted vertically to control the field levels
Longitudinal movement of one array with respect to the other also gives phase control
Other Helical Planar Undulators

The later designs moved away from two independent arrays to more complex four array schemes.

Two above and two below the electron axis.

*Longitudinal* and *vertical* movement of the arrays is used to control the field levels.

The most popular design is called **APPLE-2**.

Many examples are in existence at several laboratories.

Gives relatively high field levels in circular mode.

The complexity is not too bad compared with other options.
The APPLE-2 Design

This undulator consists of **four standard** PPM arrays
Diagonally opposite arrays move longitudinally together
All the arrays also move vertically like a conventional undulator
The electron beam travels along the central axis of the magnet

Gap is altered also to tune magnet as usual
Phase shift, $D$

These two arrays move together
APPLE-2 Fields

Denote the bottom left and top right arrays as undulator \(a\)
Denote the bottom right and top left arrays as undulator \(b\)
The phase difference between \(a\) and \(b\) is \(\phi = \frac{2\pi D}{\lambda_u}\)
The fields on axis from \(a\) are

\[
B_{ax} = B_{x0} \sin \left( \frac{2\pi s}{\lambda_u} \right)
\]
\[
B_{ay} = B_{y0} \sin \left( \frac{2\pi s}{\lambda_u} \right)
\]

And for \(b\) are

\[
B_{bx} = -B_{x0} \sin \left( \frac{2\pi s}{\lambda_u} + \phi \right)
\]
\[
B_{by} = B_{y0} \sin \left( \frac{2\pi s}{\lambda_u} + \phi \right)
\]
The horizontal field from \(b\) is negative so cancels out when \(\phi = 0\)
APPLE-2 Fields

The total field on axis is just the sum of these contributions (superposition principle)

\[ B_x = B_{ax} + B_{bx} = B_{x0} \left( \sin \left( \frac{2\pi s}{\lambda_u} \right) - \sin \left( \frac{2\pi s}{\lambda_u} + \phi \right) \right) \]

\[ B_y = B_{ay} + B_{by} = B_{y0} \left( \sin \left( \frac{2\pi s}{\lambda_u} \right) + \sin \left( \frac{2\pi s}{\lambda_u} + \phi \right) \right) \]

These simplify to

\[ B_x = -2B_{x0} \sin \left( \frac{\phi}{2} \right) \cos \left( \frac{2\pi s}{\lambda_u} + \frac{\phi}{2} \right) \]

\[ B_y = 2B_{y0} \cos \left( \frac{\phi}{2} \right) \sin \left( \frac{2\pi s}{\lambda_u} + \frac{\phi}{2} \right) \]
APPLE-2 Phase

From the field equations we can see that the two fields are always $\pi/2$ out of phase

This implies that the polarisation ellipse is always upright

The observer will see the electron describe an ellipse with principle axis always on the vertical axis

As the phase between the arrays changes from zero (standard PPM – pure vertical field, horizontal linear polarisation) the ellipse will become circular (circular polarisation), finally at $\phi = \pi$ the electron will just oscillate vertically (pure horizontal field, vertical linear polarisation)

Moving the phase in the opposite direction will again generate circular polarisation – **but of opposite helicity**. At $\phi = -\pi$ it will generate pure vertical polarisation as at $\phi = \pi$
APPLE-2 Achievable Fields

There is no analytical solution for the field level achieved for a particular APPLE-2 geometry.

An empirical equation has been generated based upon 3D models assuming square blocks with side length of $\lambda_u/2$.

The peak magnetic field in circular mode is

$$B_{circular} = 1.54 \exp\left(-4.46 \frac{g}{\lambda_u} + 0.43 \frac{g^2}{\lambda_u^2}\right)$$
Example Trajectories

Trajectories viewed head on – as the observer sees them
3 GeV electron beam
Undulator period is 50 mm
Magnet gap is 20 mm

Fields in circular mode
APPLE-2 in Opposing Mode

We can move the arrays longitudinally by equal amounts but in opposite directions to each other. In this case the fields are in phase at all times.

What polarisation will we see?

The vector sum of the two fields is no longer a constant (helical case) but has a sinusoidal field variation.
APPLE-2 in Opposing Mode

The device just behaves like a standard undulator that has been rotated about the electron axis.

Therefore linear polarisation orthogonal to the apparent undulator rotation angle is observed.

In this case the array phase changes the angle of the linear polarisation.
APPLE-2 Field Quality

The horizontal field changes rapidly with transverse position in the undulator (very undesirable!)

Hence small misalignments of the electron beam will affect the undulator performance

The impact of helical undulators on the electron beam are much more significant than standard ones – they can effectively stop a storage ring operating if these effects are not considered properly

Horizontal and vertical fields across the horizontal aperture on the vertical centre line (not plotted at the same longitudinal position)
APPLE-2 Examples

The typical block shape is square but with cut outs

The cut outs are used to mechanically clamp the blocks in place but they are kept well away from the electron beam axis and so they have a very small impact on the fields

End View

(Johannes Bahrdt, BESSY)
APPLE-2 Examples

Upper and lower beams – no arrays in place

SRS HU56
Fully assembled undulator being measured
Other APPLE-2 Examples

Diamond HU64

ALS
Alternative Helical Undulator Designs

The APPLE-3 is optimised for circular vacuum chambers (FELs)

Cutting away the block corners effectively allows for a smaller magnet gap

This enhances the field by ~40% in circular mode

Other designs have used extra arrays

In this case the field is enhanced by ~20%

Undulators with six arrays have also been built

(Johannes Bahrdt, BESSY)
Alternative Helical Undulator Designs

The Delta Undulator is an interesting new design
This generates even higher fields in circular mode (enhanced by ~70%)
It only allows for a circular beam pipe (FELs)

**No gap change is used**, instead the field levels are altered with just longitudinal motion

Another empirical field fit has been generated for the field in circular mode

\[
B_{\text{circular}} = 1.45 \exp \left( -1.28 \frac{g}{\lambda_u} - 2.24 \frac{g^2}{\lambda_u^2} \right)
\]

(A. B. Temnykh, PRSTAB 11, 120702, 2008)
Engineering demands are very high:
Very strong forces present during assembly and when complete
Must have high periodicity
Arrays must be parallel to μm precision and must stay parallel at all gaps

General design themes:
Blocks are held in individual holders – glued or clamped
Fastened to a backing beam
C shaped support frame
Very long magnets (>5m) are split into shorter modules (2 – 3m)
In-Vacuum Undulators

The minimum magnet gap limits the performance of an undulator.

The magnet gap is determined by the needs of the electron beam.

In practice this is set by the vacuum chamber.

For example:

- If an electron beam needs 10mm of vertical space.
- And the vacuum chamber walls are 2mm thick.
- With an allowance for mechanical tolerances of 1mm.
- The minimum magnet gap will be 15mm.

One option is to put the undulator inside the vacuum chamber, in this example the magnet gap reduces by ~30%.
In-Vacuum Undulators

Magnet blocks are not very good for use within a vacuum system
They must be **coated** to prevent outgassing (TiN or Ni)
They also must be **baked** for UHV – this affects the magnet performance and can cause irreversible losses
Generally only bake at ~130 °C
The surface **resistance** of the blocks is high – need a **sheet of copper** on top of each array to provide low resistance path for the electron’s image current
Magnet measurements are only possible before full assembly

*Flexible vacuum chambers are an alternative solution*
In-Vacuum Examples

Diamond U23
In-vacuum undulator
In-Vacuum Examples

Diamond U23
In-vacuum undulator
In-Vacuum Examples

ALS in-vacuum undulator
In-Vacuum Examples

Flexible transition with integrated water cooling between fixed vacuum chamber and variable gap undulator arrays
Forces due to the Magnetic Field

A permanent magnet cannot be switched off!
Therefore magnetic forces are always present
These forces increase rapidly as the gap between the two arrays decreases
Mechanical designs must take full account of the forces between individual blocks and also between arrays
When two magnets with the same poles are brought together there is a strong repulsion
The energy we exert in bringing them together is stored by the magnetic field
If opposite poles face each other there will be an attractive force and energy will be removed from the field and do work on the system
Forces due to the Magnetic Field

To calculate the force we need to know how the energy stored by the field changes with unit distance.

The energy stored in an inductor is \( U = \frac{1}{2}LI^2 \).

It can be shown that for a solenoid the magnetic energy per unit volume is

\[
\frac{dE}{dV} = \frac{d^3E}{dx dy dz} = \frac{B^2}{2\mu_0}
\]

This is a **general result** for the **magnetic energy density** in a vacuum and with non-magnetic materials.
Forces due to the Magnetic Field

Since force is energy or work done per unit distance, \( F = \frac{dE}{dy} \)

The force between two magnets is

\[
F = \int \int \frac{dE}{dV} \, dx \, ds = \int \int \frac{B^2}{2\mu_0} \, dx \, ds
\]

So, in a region of **uniform magnetic field** over an area of the \( x \)-s plane equal to \( A \) the force would be

\[
F = \frac{B^2 A}{2\mu_0}
\]

This example is a fair approximation to a dipole magnet with pole area \( A \)
Forces due to the Magnetic Field

For an undulator with sinusoidal magnetic field then the integral along the longitudinal axis is

\[ \int_{-L/2}^{L/2} B_y^2 ds = B_{y_0}^2 \int_{-L/2}^{L/2} \sin^2 \left( \frac{2\pi s}{\lambda_u} \right) ds = \frac{B_{y_0}^2 L}{2} \]

So the force between the two arrays is

\[ F = \frac{\int B_{y_0}^2 L dx}{4\mu_0} \]

If we assume the vertical field is constant in x over a width W, and then falls to zero (top hat shape), then the total force between the two arrays is

\[ F = \frac{B_{y_0}^2 LW}{4\mu_0} \]
Assume a 50 mm period and 20mm gap with a permanent magnet remanent field of 1.1T. A typical magnet field width in x is ~60mm.

The force between the two arrays is ~3500N per meter of length.

Note that this force changes rapidly with gap as the field changes exponentially.
Measuring Magnet Field Quality – Phase Error

There are various quantitative ways of measuring and comparing undulator magnetic field quality.

Here we just consider one which has the largest impact on the photon output quality.

Ideally the electron will advance by $2\pi$ from one period to the next at the first harmonic wavelength to maintain the interference condition.

In practice, the magnet will not be perfect, and the phase advance across a period will average $2\pi$ but will have some statistical spread about that point.

This spread is called the RMS phase error $\sigma \Phi$. 
Undulator Phase Error

Here is an example phase error for a 20 period device.

In this case, $\sigma_\Phi$ is 8°

The impact of the phase error is to reduce the output intensity – the harmonic width broadens.

The phase error scales with the harmonic number so the impact is greater for the higher harmonics.
**Shimming** is a general term which means making small modifications to the magnet so as to optimise the magnet performance (minimising the phase error, for example).

Magnet block positions may be slightly adjusted by ~0.1mm displacements.

Small pieces of thin iron sheet (shims) can be placed on top of the arrays to slightly modify the field.

The exact dimensions of the iron pieces can be selected to have the required effect upon the field level.

Although shimming can be a time consuming task, it is worthwhile in general, since the phase (and other) errors can be significantly reduced.
Cryo-Undulators

These are a relatively new idea that take advantage of the variation of remanent field with temperature. If the undulator can operate at ~150K then there will be a significant field increase.

In vacuum undulators are being adapted to try out this novel idea.

The intrinsic coercivity increases also which helps with radiation resistance and allows selection of stronger grades.

H Kitamura, Spring-8
Cryo-Undulators

Prototype installed in ESRF in Jan 2008

Temperature varies with electron beam fill pattern – power from beam larger than expected (HOMs suspected)

Second prototype with higher remanent field under construction
Electromagnetic Devices

Given that virtually all magnets in particle accelerators are electromagnets (dipoles, quadrupoles, sextupoles, …) it seems surprising that relatively few electromagnetic undulators and wigglers are built.

One niche area where electromagnets are used is when there is a requirement for the fields to be changed quickly.

This is generally because of the need to rapidly change polarisation states – it is much easier to switch the direction of a current quickly than to physically move magnet arrays.

Superconductors, of course, have always been used in very high field applications.
Electromagnetic Devices

Let’s consider a basic conceptual layout of an electromagnetic insertion device

All the coils are connected in series

One individual coil is highlighted in red

The magnetic field is varied by changing the current in the coils

There is no need to move the arrays

Electromagnetic insertion devices generally have a much lower capital cost but a much higher operating cost
Simple Electromagnetic Analysis

Consider the design concept as a **series of dipoles** of alternating polarity

The (approximate) field produced by a dipole with gap, \( g \), driven by \( NI \) Ampere-turns is

\[
B = \frac{2\mu_0 NI}{g}
\]

So the undulator **K parameter** will be

\[
K = 2.35 \times 10^{-4} NI \frac{\lambda_u}{g}
\]

Note that to reach \( K \sim 1 \) we will require \( NI \sim 1000 \) A-t

If the gap is fixed and we want to reduce \( \lambda_u \) then \( NI \) will have to increase to maintain \( K \)

But, as the period reduces the space for the coil shrinks as well so the current density increases very rapidly

At some period the resistive losses will be so high that cooling will not be practical
More Realistic Electromagnetic Model

By examining the fields in a 2D simulation of an undulator a better model has been derived

\[ B_y = \frac{32 \mu_0 N I}{\sqrt{2\pi} \lambda_u} \left[ \frac{\cosh(ky)}{\sinh(kg/2)} - \frac{\cosh(3ky)}{3 \sinh(3kg/2)} \right] \quad k = \frac{2\pi}{\lambda_u} \]

The number of Ampere turns for a given \( K \) as a function of period to gap ratio for the two electromagnetic models.

The simple dipole model holds until the period to gap < 3, this is when the poles no longer act independently

The 2D model shows that the situation is much worse at short periods than the simple model predicted
If we assume a coil cross section of $\frac{\lambda_u}{8} \times \frac{\lambda_u}{2}$
And a maximum current density of 10 A/mm$^2$
Then we can plot the results for a 20mm gap

$K \sim 1$ at 55mm period
$K \sim 10$ at 110mm period – but note that this does not include any iron saturation effects
Comparison with Permanent Magnets

Plot of field level in a PPM and a hybrid insertion device (including iron saturation!)

\[ K \approx 3 \text{ @ 55mm period} \]

\[ K \approx 14 \text{ @ 110mm period} \]
Electromagnet Examples

An elliptical wiggler is installed on Elettra
The period is 212 mm and the peak fields are 0.5 T vertically and 0.1 T horizontally
The horizontal field can switch at up to 100Hz
The rapid change in polarisation between left and right circular can be utilised by the experiment to increase the signal to noise level
Electromagnet Examples

This example from SuperACO in France is called Ophelie
It has a period of 250 mm and generates 0.11 T in both planes

Note that only two of the four arrays are shown
Summary

The **APPLE-2** is the most popular undulator design for generating variable polarisation states.

Other designs give higher fields but they have not been implemented yet.

All helical undulators have a much bigger impact on the electron beam behaviour than standard undulators.

The **forces** in permanent magnet insertion devices can be very high and increase rapidly as the gap closes.

In-vacuum solutions allow for smaller magnet gaps at the expense of more complex engineering.

One of the key indicators of undulator field quality is the **phase error**.

Undulator field quality can be improved before installation by **shimming**.

Cryogenic undulators at ~150K will generate higher field levels and these are starting to be introduced now.

Normal conducting electromagnets cannot compete with permanent magnets, especially at short periods but they do allow **fast switching** of fields.