Insertion Devices
Lecture 4
Permanent Magnet Undulators

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Introduction to Lecture 4

So far we have discussed at length what the properties of SR are, when it is generated, and how it can be tailored to suit our needs (wavelength, polarisation, flux, etc)

But, how do we know what magnetic fields are actually achievable?

In this lecture we will look at how periodic fields are generated and what the limitations are

For example, can we have a period of 1 mm and field strength of 20 T?

Later we will look at the present state of the art and some future possibilities
What are the possibilities?

To generate magnetic field we can use:

Electromagnets
- Normal conducting or
- Superconducting

Permanent Magnets

Both types can also include iron if required
Permanent Magnet Basics

I will give a brief introduction only

The magnetic properties of materials is a big topic!

Further reading:
P Campbell, Permanent magnet materials and their applications, CUP 1994

also

What is a Permanent Magnet?

**Definition:**

A magnet is said to be **Permanent** (or **Hard**) if it will independently support a useful flux in the air gap of a device.

A material is magnetically **Soft** if it can only support such a flux with the help of an external circuit.

A PM can be considered as a passive device analogous to a spring (which stores mechanical energy).

An electron in a microscopic orbit has a magnetic dipole moment – can be modelled as a current flowing in a loop.

In a Permanent Magnet these ‘molecular’ currents can be identified with atoms with unfilled inner shells.

eg 3d metals (Fe, Co, Ni) or 4f rare earths (Ce to Yb)
Permanent Magnet materials are manufactured so that their magnetic properties are enhanced along a preferred axis. To do this, advantage is taken of crystal lattices. The direction of alignment is called the easy axis.

When a magnetic field, $B$, is applied to a magnetic material, each dipole moment tries to align itself with the field direction. When $B$ is strong enough (at saturation) all of the moments are aligned, overcoming other atomic forces which resist this. A Permanent Magnet must be able to maintain this alignment after $B$ is removed.
An Ideal Permanent Magnet

The characteristics of a Permanent Magnet are determined by its behaviour under an external magnetization force $H$

1. $H$ large, material saturated, all moments aligned.

2. $H$ reducing, moments stay aligned.

3. $H$ large and negative, material flips – now aligned with opposite direction.

Magnetization is magnetic dipole moment per unit volume, $B = \mu_0 M$.
The Ideal BH Curve

Magnetic Flux Density
\[ B = \mu_0(H+M) \]

Remanent Field
\[ B_r \]

Coercivity
\[ H_c \]

In 2\(^{nd}\) quadrant the ideal PM is linear

Gradient = \( \mu_0 \)

Magnetizing Force
The BH Product

Permanent Magnets are operated in the 2nd quadrant – no external fields are present, moments are aligned along the easy axis.

The product BH represents the energy density of the material. Examining the peak BH value in the 2nd quadrant is a good way of comparing the strength of different materials.
The Working Point

The position on the B-H curve at which the Permanent Magnet is operated is called the **Working Point**

If the working point is at $\text{BH}_{\text{max}}$ then the maximum potential energy available is being utilised

It is good practice to design Permanent Magnet systems that work near this point

*If Permanent Magnets are operating well away from this point then they are unlikely to be doing anything particularly useful*

The line through the origin to the Working Point is called the load line
Current Sheet Equivalent Materials (CSEM)

An ideal Permanent Magnet is uniformly magnetized (homogeneous)

The equivalent current model is a sheet of current flowing on the surface with no internal (volume) currents

The relative permeability is ~1 so we can consider the bulk material to be vacuum

This CSEM model implies that the contributions from different magnets can be added linearly (just like adding contributions from currents)

Analytical calculations then become fairly simple because we can calculate the field at a point from every block independently and just add all the individual contributions up
Current Sheet Equivalent Materials (CSEM)

Lines of flux for an **ideal** Permanent Magnet

Lines of flux for a CSEM model
Real Permanent Magnet Materials

Not all materials have a linear BH curve in the $2^{\text{nd}}$ quadrant. This can be a problem if the working point drops into the non-linear region. This might happen when closing an undulator gap or even transiently during assembly of the undulator.

If the working point moves from $a$ to $b$ then when $B$ increases again the magnet will operate at $c$ instead – this is an irreversible effect.
Temperature Effects

At higher temperatures the materials become more non-linear
So long as the working point stays in the linear region this is a reversible effect
Note that the remanent field drops with increasing temperature – the reverse is also true, cold magnets have a higher remanent field

Day to day temperature variations are important and must be controlled (minimised)
Many undulators are operated in air conditioned environments to keep their output more stable
Available Materials

Two types of permanent magnet are generally used – Samarium Cobalt (SmCo) and Neodymium Iron Boron (NdFeB)

<table>
<thead>
<tr>
<th></th>
<th>SmCo</th>
<th>NdFeB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Remanent Field</td>
<td>0.85 to 1.05 T</td>
<td>1.1 to 1.4 T</td>
</tr>
<tr>
<td>Coercivity</td>
<td>600 to 800 kA/m</td>
<td>750 to 1000 kA/m</td>
</tr>
<tr>
<td>Relative Permeability</td>
<td>1.01 parallel, 1.04 perp</td>
<td>1.05, 1.15</td>
</tr>
<tr>
<td>Temperature Coefficient</td>
<td>-0.04 %/°C</td>
<td>-0.11 %/°C</td>
</tr>
<tr>
<td>Max Energy Density</td>
<td>150 to 200 kJ/m³</td>
<td>200 to 350 kJ/m³</td>
</tr>
<tr>
<td>Max operating temperature</td>
<td>~300°C</td>
<td>~100°C</td>
</tr>
<tr>
<td>Comment</td>
<td>Brittle, easily damaged, better intrinsic radiation resistance, expensive</td>
<td>Less brittle but still liable to chip, easier to machine, expensive</td>
</tr>
</tbody>
</table>
How are Permanent Magnets Manufactured?

Melting of the alloy under vacuum with inductive heating

Crushing of polycrystalline cast ingots

Milling of the coarse powder

Pre-alignment in a magnetic field

(courtesy of Vacuumschmelze)
How are Permanent Magnets Manufactured?

Pressing of magnets

Die pressed parts are sintered in ovens (heating a powder below its melting point until its particles adhere to each other)
The result is a ceramic like object

Now final machining can take place & possibly a protective coating applied

The final stage is magnetization – usually in a strong pulsed magnetic field

(courtesy of Vacuumschmelze)
An Example Material

**Vacodym 633 HR**

M-H and B-H curves are shown (2\textsuperscript{nd} quadrant)
The material is linear at 20°C but non-linear above about 60 °C

Irreversible losses for different loadline working points as a function of temperature

(courtesy of Vacuumschmelze)
Pure Permanent Magnet Undulators

A magnet which contains no iron or current carrying coils is said to be a Pure Permanent Magnet (PPM)

Because of CSEM we can use the principle of superposition

An **ideal undulator** would have a sinusoidal magnetic field along the direction of the electron beam

To generate a sinusoidal field an ideal PPM would have **two sets (arrays)** of Permanent Magnet with their easy axis rotating smoothly through **360° per period** along the direction of the electron beam

In practice this ideal situation is approximated by splitting the system into M rectangular magnet blocks per period with the easy axis at the relevant set angle
Example PPM arrangement, $M = 4$

Side View

Top Array

Bottom Array

$\lambda u$

$e^-$

$y$

$s$

$\frac{\varepsilon \lambda u}{M}$

$h$

$g$
Lines of Magnetic Flux
The field strength between the two arrays assuming infinite width in the x – direction (2D approximation) is

\[ B_y = -2B_r \sum_{i=0}^{\infty} \cos \left( \frac{2n\pi s}{\lambda_u} \right) \cosh \left( \frac{2n\pi y}{\lambda_u} \right) \frac{\sin(n \varepsilon \pi / M)}{n\pi / M} e^{-n\pi g/\lambda_u} \left( 1 - e^{-2n\pi h/\lambda_u} \right) \]

\[ B_s = 2B_r \sum_{i=0}^{\infty} \sin \left( \frac{2n\pi s}{\lambda_u} \right) \sinh \left( \frac{2n\pi y}{\lambda_u} \right) \frac{\sin(n \varepsilon \pi / M)}{n\pi / M} e^{-n\pi g/\lambda_u} \left( 1 - e^{-2n\pi h/\lambda_u} \right), \]

Where \( n = 1 + iM \) and \( \varepsilon \) is a packing factor to allow for small air gaps between blocks.

The vertical field on axis \((y = 0)\) is a number of cosine harmonics.

As \( M \rightarrow \infty \) this reduces to a single cosine (ideal case).

The longitudinal (and horizontal) field on axis is zero.
A Practical PPM

The most popular choice is $M = 4$

This is a good compromise between on axis field strength and quality vs engineering complexity

Higher harmonics then account for $< 1\%$ of the field on axis

Away from the axis it is definitely not cosine-like

For an example PPM with 50mm period, block height of 25mm, magnet gap of 20 mm and remanent field of 1.1 T

Note that the fields increase away from the axis
Selecting $M = 4$ means that you will achieve about 90% of the theoretical limit.
Simplifying the Magnetic Field

If we assume that only the first harmonic makes a significant contribution \((n = 1)\) – a good approximation in general

Then the equation simplifies greatly on axis to

\[
B_y = -2B_r \cos \left( \frac{2\pi s}{\lambda_u} \right) \frac{\sin(\varepsilon \pi / M)}{\pi / M} e^{-\pi g / \lambda_u} (1 - e^{-2\pi h / \lambda_u})
\]

\[
B_s = 0.
\]

Important:

Note that so long as all the spatial dimensions scale together the fields on axis do not change

This is not true for electromagnets – there the current densities have to increase to maintain the same field levels
Effect of Different Block Heights

A typical block height selection is half the period length. A height of a quarter period length would make all the blocks identical, but then there is a greater field loss.

Selecting 0.5 means that you will achieve about 95% of the theoretical limit.
Peak Field Achievable

The maximum peak field achievable (in theory) is $2B_r$.

In practice with $M = 4$ and $h = \lambda_u/2$ the peak on axis field is

$$B_{y0} = 1.72 \ B_r e^{-\pi g/\lambda_u}$$

So even with an ambitious gap to period ratio of 0.1 the peak value is only $1.26B_r$.

Achieving fields of $\sim 1.5T$ requires very high $B_r$ material, small gaps and long periods!

But, higher fields are possible if we include iron in the system.

Mixing Permanent Magnets and iron poles is called a hybrid magnet.
**Tuning the Undulator**

To vary the output wavelength from the undulator – to map out the tuning curves – we need to alter the field level on the axis. We can now see that the only practical way to do this for a permanent magnet device is **to change the magnet gap**.

\[ B_{y_0} = 1.72 \, B_r e^{-\pi g/\lambda_u} \]

![Graph showing photon energy vs. flux for different harmonics](image)
Hybrid Insertion Devices

Simple hybrid example

Top Array

Bottom Array

Steel pole

\[ \lambda u \]

\[ g \]
Lines of Magnetic Flux

Including a non-linear material like iron means that simple analytical formulae can no longer be derived – linear superposition no longer works!

Accurate predictions for particular designs can only be made using special magnetostatic software in either 2D (fast) or 3D (slow)
Empirical Formulae for Hybrid Insertion Devices

A series of 2D studies were performed in the 1980s to generate an empirical formula for the peak on axis field.

For $Br = 0.9T$

$$B_{y_0} = 3.33 \exp \left( -5.47 \frac{g}{\lambda_u} + 1.8 \frac{g^2}{\lambda_u^2} \right)$$

For $Br = 1.1T$

$$B_{y_0} = 3.44 \exp \left( -5.08 \frac{g}{\lambda_u} + 1.54 \frac{g^2}{\lambda_u^2} \right)$$

These are valid over the range $0.07 < g/\lambda_u < 0.7$

Recently these have been updated for $Br = 1.3T$

$$B_{y_0} = 4.3 \exp \left( -6.45 \frac{g}{\lambda_u} + \frac{g^2}{\lambda_u^2} \right)$$

Valid over the range $0.04 < g/\lambda_u < 0.2$
Field Levels for Hybrid and PPM Insertion Devices

Assuming $B_r = 1.1\text{T}$ and gap of 20 mm

When $g/\lambda_u$ is small the impact of the iron is very significant
Insertion Device End Termination Design

We want undulators and wigglers to have zero net effect on the electron trajectory

Otherwise, operating one undulator would affect all the other users

Remember (from Lecture 2)

\[ \ddot{x} = \frac{eB_y}{\gamma m_0 c} \]

So, the electron exit angle is found by integrating the field over the full length of the device

\[ I_y = \int_{-\infty}^{\infty} B_y(s) \, ds \]

Such that the exit angle, \( \alpha \), is

\[ \alpha = \frac{e}{\gamma m_0 c} I_y \]
Insertion Device End Termination Design

The electron position after the undulator is found by integrating a second time

\[ II_y = \int_{-\infty}^{\infty} \int_{-\infty}^{s} B_y(s') ds' ds \]

Such that

\[ x = \frac{e}{\gamma m_0 c} II_y \]

The requirement is that the first and second field integrals should both equal zero at all operating points. This is achieved (in theory!) by the selection of suitable end terminations (entrance and exit) for the magnets.
An alternative expression for the final beam position at the exit is to project it back to the centre of the device.

Then

$$\delta = -\frac{e}{\gamma m_0 c} \int_{-\infty}^{\infty} s B_y(s) ds$$

Realistic electron trajectory in a non-ideal undulator
Symmetric and Antisymmetric Insertion Devices

The parameter $\delta$ will be zero when the field is symmetric about $s = 0$ (centre of the undulator is the peak of a pole).

The end designs then need to be set so that $\alpha$ is also zero.

In the antisymmetric case, the centre of the magnet is a zero crossing.

In this case, $\alpha$ will automatically be zero.

The ends then need to be chosen so that $\delta$ is also zero.

For PPMs the end design is relatively simple because of the superposition principle.

For hybrids the non-linear effects mean that the integrals are harder to control so active compensation is generally used – these might be electromagnetic coils or moving permanent magnets.
PPM End Termination Design

Symmetric field end design – the simplest solution

The end terminations are just half length blocks of the same permanent magnet material

Q. Why does this give zero first field integral?

\[ \delta = -\frac{e}{\gamma m_0 c} \int_{-\infty}^{\infty} s B_y(s) ds \]

\[ \alpha = \frac{e}{\gamma m_0 c} I_y \]

\[ I_y = \int_{-\infty}^{\infty} B_y(s) ds \]
Remember the principle of superposition

The field on-axis due to this block ...

... will be exactly cancelled by the field due to this block
Blocks of the same colour cancel each other
PPM End Termination Design

The same is true for the blocks with vertical easy axis – an ‘up’ block cancels a ‘down’ block

Finally, we are left with two ‘up’ blocks and these are cancelled by four half length ‘down’ blocks

They have to be half length otherwise the magnet would not be symmetric and both integrals would not be zero
Half Block End Termination Design

10 period model
50 mm period

3GeV trajectory
\( \alpha \) and \( \delta \) are zero

Note that there is a small position offset through the undulator but the light is emitted parallel to the axis
Antisymmetric Solution

The first integral is zero automatically

\[ I_y = \int_{-\infty}^{\infty} B_y(s) ds \]

The second integral is set to zero by choosing an appropriate length of end block – this depends on the number of periods in the undulator
Antisymmetric Solution

10 period model
50 mm period

3GeV trajectory
$\alpha$ and $\delta$ are zero

The final position and angle of the electron are both zero but the light will be emitted at an angle to the axis.
Other Popular Solutions

A field strength series of $+\frac{1}{4}$, $-\frac{3}{4}$, $+1$, $-1$, … works for both symmetric and antisymmetric cases.
The +¼, -¾, +1 Solution

10 period model
50 mm period

3GeV trajectory
α and δ are zero

The electron now oscillates about the axis so the light will be emitted along the axis
Summary

A Permanent Magnet can independently support a flux in an air gap – no coils are needed.

Permanent Magnets are operated in the 2\textsuperscript{nd} quadrant of the BH curve – ideally they have a linear behaviour.

Permanent Magnets can be modelled as current sheets so we can add the field contributions from each block linearly.

To generate a sinusoidal field we use two arrays of magnet blocks – one above and one below the electron beam.

The field limit is \( \sim 1.5 \text{T} \) but if we include iron (hybrid magnet) then \( >2 \text{T} \) is easily achievable.

The end terminations of undulators are important.

The end design can depend on whether the magnet is symmetric or asymmetric but the design principles are the same.