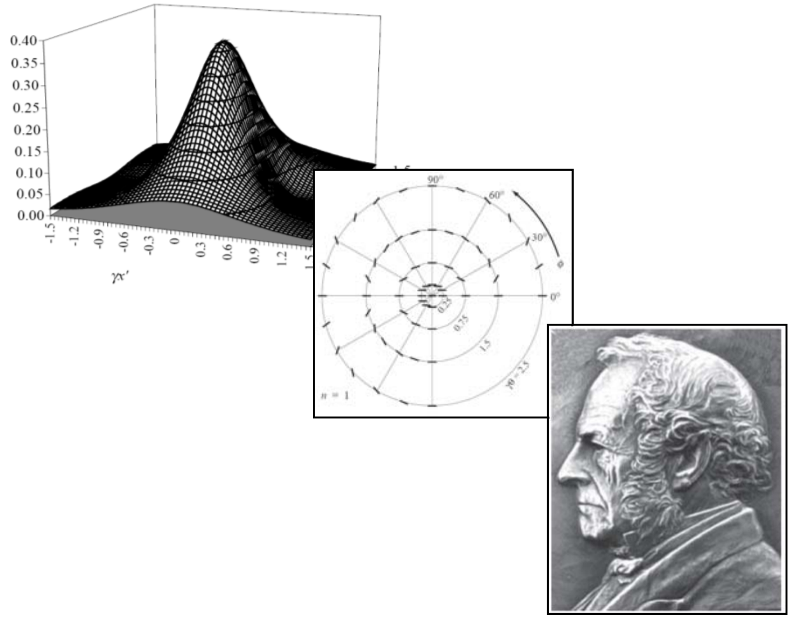
Accelerator Science and Technology Centre

Insertion Devices
Lecture 3
Undulator Radiation and Realisation



Jim Clarke

ASTeC

Daresbury Laboratory

The Cockcroft Institute of Accelerator Science and Technology

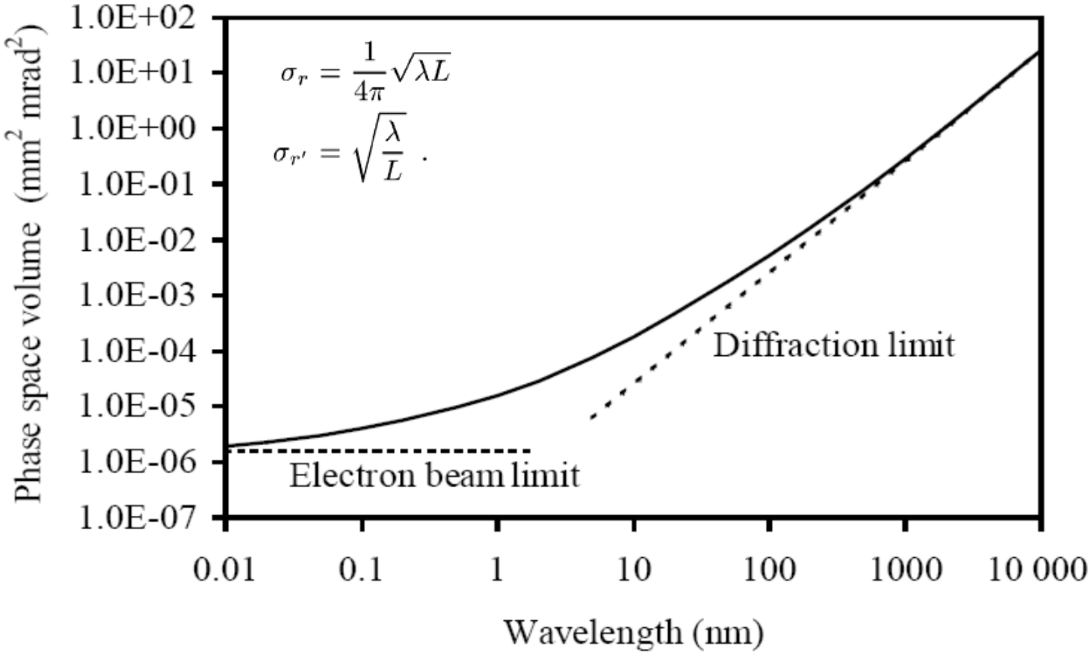
Science & Technology Facilities Council

Diffraction Limited Sources

Light source designers strive to reduce the electron beam size and divergences to maximise the brightness (minimise emittance & coupling)

But when $\sigma_r \gg \sigma_{x,y}$ and $\sigma_{r'} \gg \sigma_{x',y'}$ then there is nothing more to be gained

In this case, the source is said to be **diffraction limited**



Example **phase space volume** of a light source

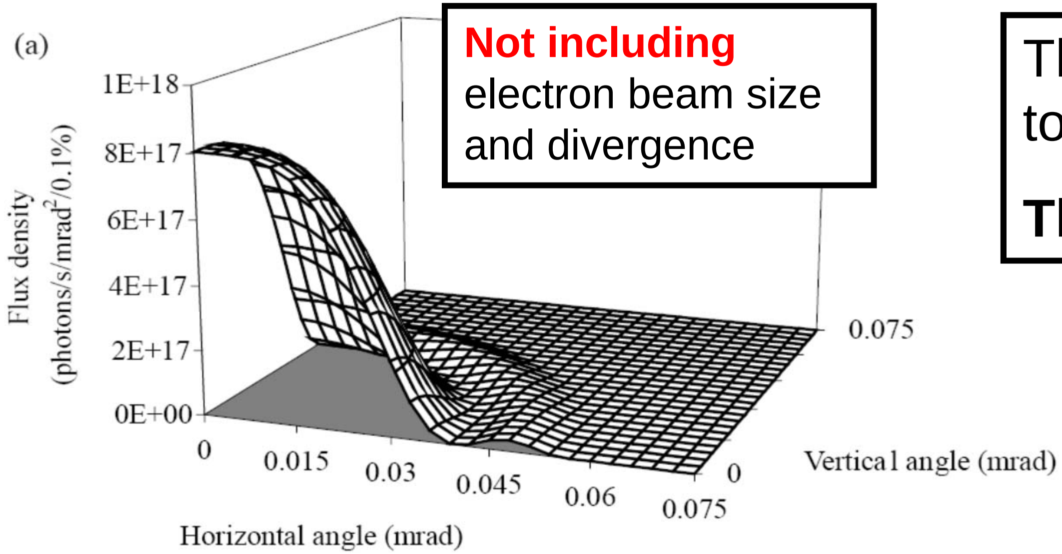
$$4\pi^2 \Sigma_x \Sigma_y \Sigma_{x'} \Sigma_{y'}$$

Using electron parameters

$\sigma_x = 100\ \mu\text{m},\ \sigma_y = 10\ \mu\text{m},\ \sigma_{x'} = 20\ \mu\text{rad, and}\ \sigma_{y'} = 2\ \mu\text{rad}$

In this example, for wavelengths of >100nm, the electron beam has virtually no impact on the undulator brightness

Undulator Output Including Electron Beam Dimensions



Not including electron beam size and divergence

The effect of the electron beam is to smear out the flux density

The total flux is unchanged

50mm period

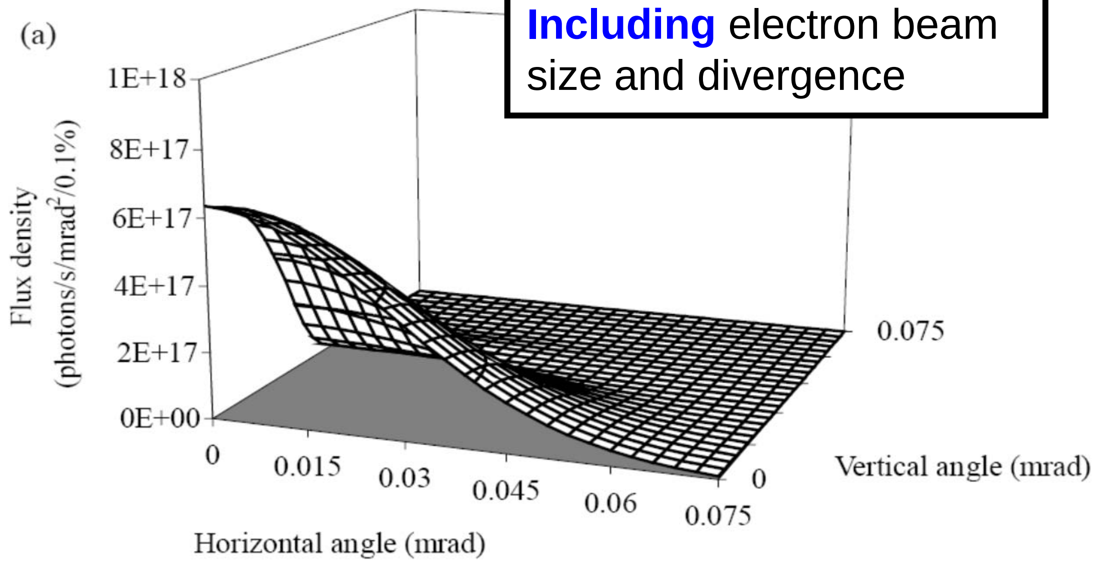
100 periods

3GeV

300mA

K = 3

λ = 4 nm, 1st harmonic



Including electron beam size and divergence

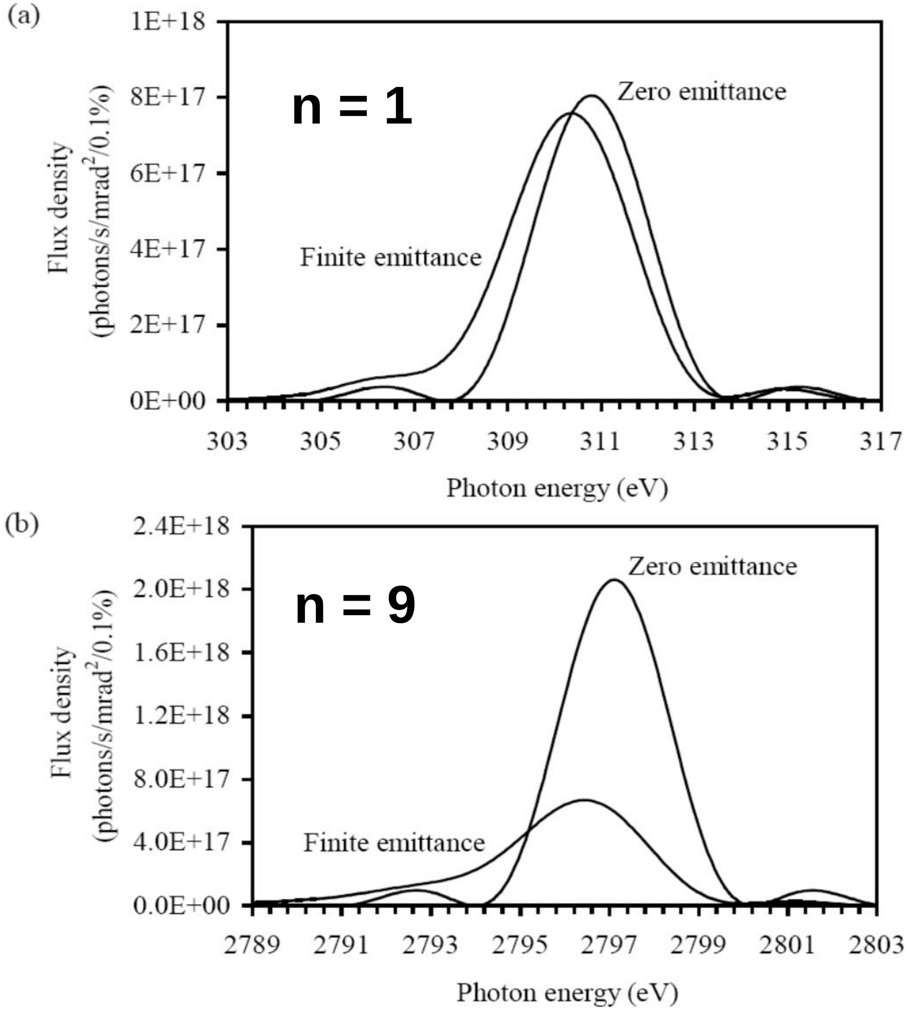
$\sigma_x = 100\ \mu\text{m},\ \sigma_y = 10\ \mu\text{m},\ \sigma_{x'} = 20\ \mu\text{rad, and}\ \sigma_{y'} = 2\ \mu\text{rad}$

Effect of the Electron Beam on the Spectrum

Including the finite electron beam emittance reduces the wavelengths observed (lower photon energies)

$$\lambda = \frac{\lambda_u}{2n\gamma^2}\left(1 + \frac{K^2}{2} + \theta^2\gamma^2\right)$$

At the higher harmonics the effects are more dramatic since they are more sensitive by factor n

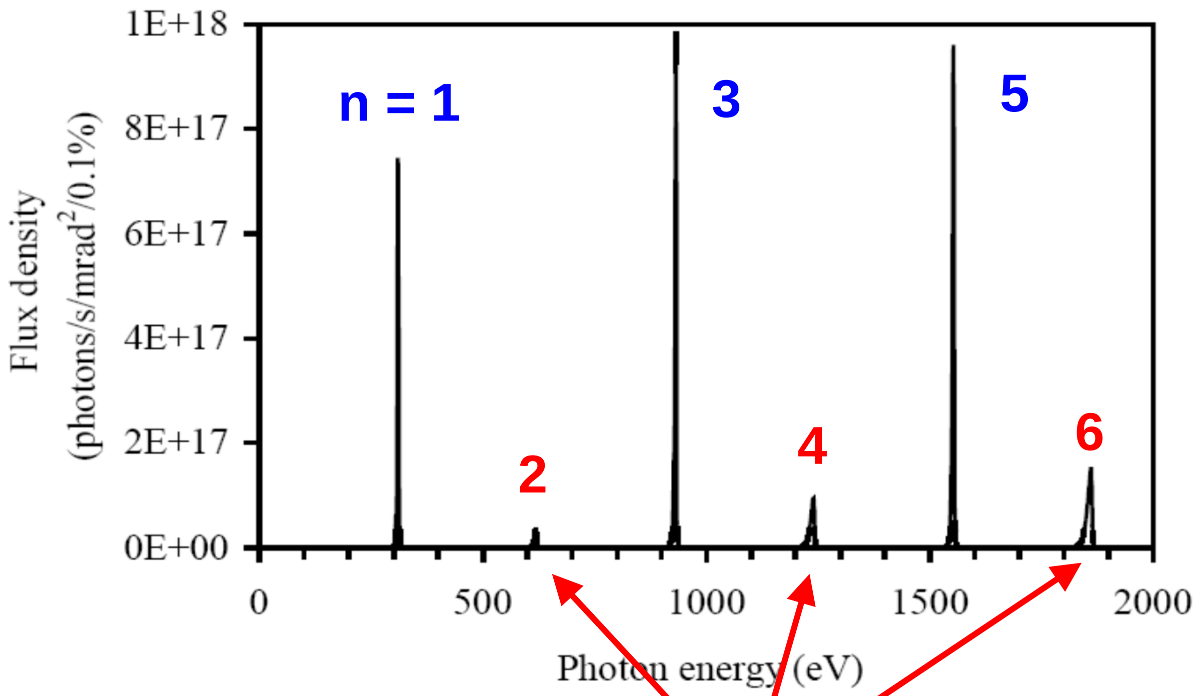


$\sigma_x = 100\ \mu\text{m},\ \sigma_y = 10\ \mu\text{m},\ \sigma_{x'} = 20\ \mu\text{rad, and}\ \sigma_{y'} = 2\ \mu\text{rad}$

Effect of the Electron Beam on the Spectrum

The same view of the on-axis flux density but over a wider photon energy range

The odd harmonics stand out but **even harmonics** are now present because of the electron beam divergence



Even harmonics observed on axis because of electron divergence (θ^2 term in undulator equation)

Flux Through an Aperture

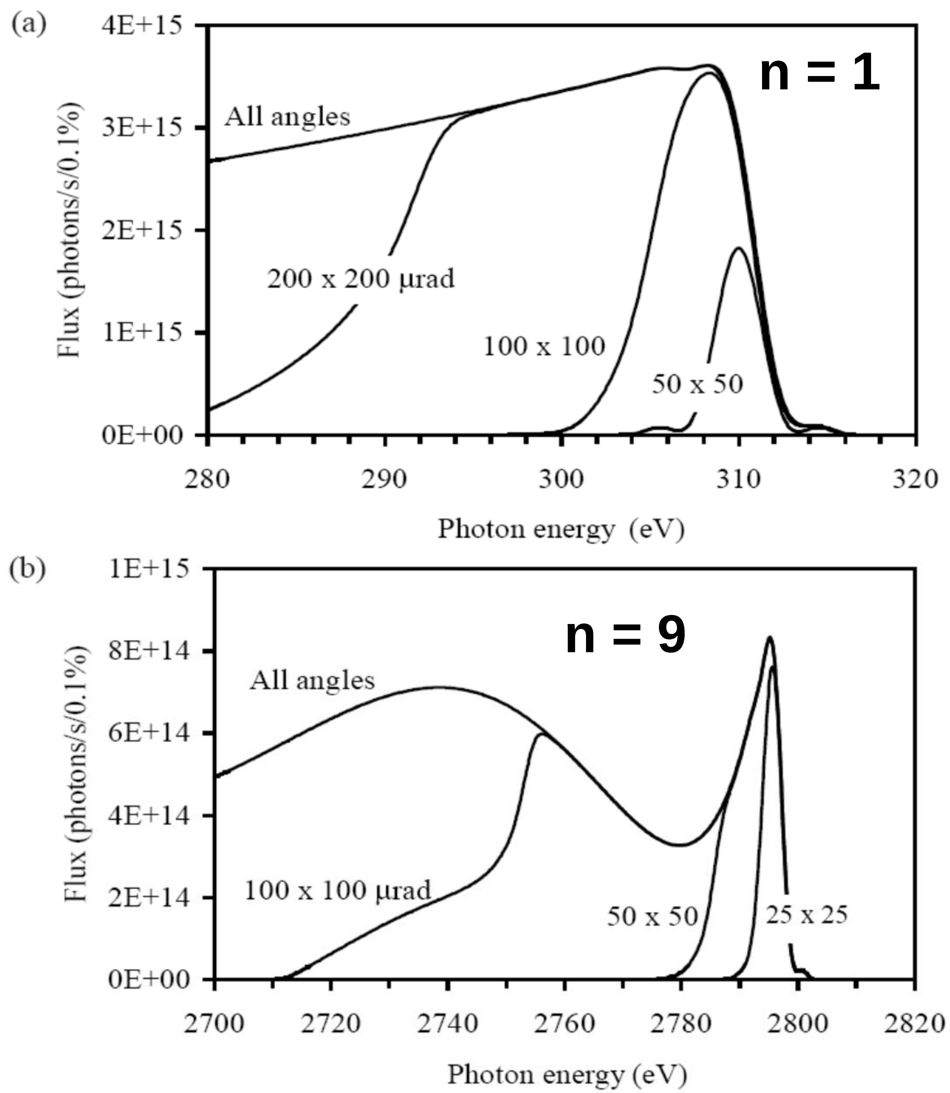
The actual total flux observed depends upon the aperture of the beamline

As the aperture increases, the flux increases and the spectrum shifts to lower energy (θ^2 again)

The narrower divergence of the higher harmonic is clear

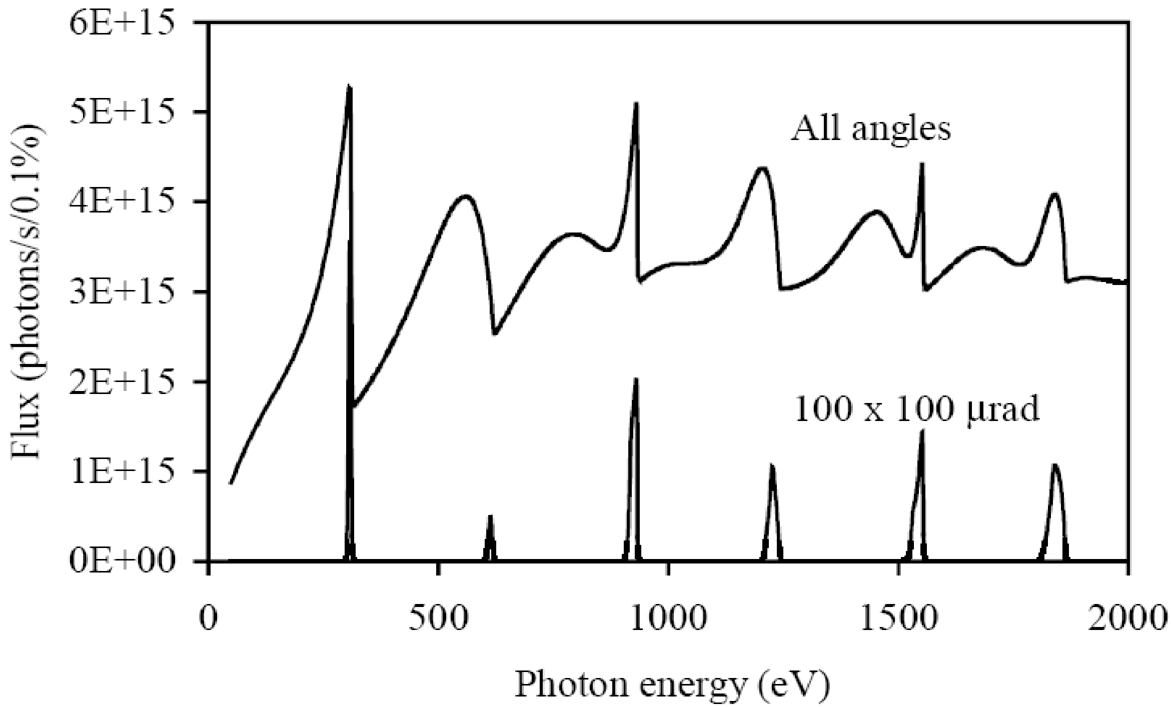
$$\sigma_{r'} = \sqrt{\frac{\lambda}{N\lambda_u}} = \sqrt{\frac{\lambda}{L}}$$

Zero electron emittance assumed for plots



Flux Through an Aperture

As previous slide but over a much wider photon energy range and **all harmonics that can contribute are included** – hence greater flux values



Many harmonics will contribute at any particular energy so long as **large enough angles** are included (θ^2 again!)

If a beamline can accept very wide apertures the discrete harmonics are replaced by a continuous spectrum

Polarised Light

A key feature of SR is its polarised nature

By manipulating the magnetic fields correctly any polarisation can be generated: linear, elliptical, or circular

This is a big advantage over other potential light sources

Polarisation can be described by several different formalisms

Most of the SR literature uses the **“Stokes parameters”**

Sir George Stokes

1819 - 1903

Stokes Parameters

Polarisation is described by the relationship between two orthogonal components of the Electric E-field

$$E_x = E_{x_0} \cos(\omega t)$$
$$E_y = E_{y_0} \cos(\omega t + \delta)$$

There are **3 independent parameters**, the field amplitudes and the phase difference

When the phase difference is zero the light is **linearly polarised**, with angle given by the relative field amplitudes

If the phase difference is $\pi/2$ and $E_{x_0} = E_{y_0}$ then the light will be **circularly polarised**

However, these quantities cannot be measured directly so Stokes created a formalism based upon observables

Stokes Parameters

The **intensity** can be measured for different polarisation directions:

Linear erect I_x and I_y

Linear skew I_{45° and I_{135°

Circular I_R and I_L

The Stokes Parameters are:

$$
\begin{aligned}
S_0 &= I_x + I_y = I_{45^\circ} + I_{135^\circ} = I_R + I_L \\
S_1 &= I_x - I_y \\
S_2 &= I_{45^\circ} - I_{135^\circ} \\
S_3 &= I_R - I_L \quad .
\end{aligned}
$$

Polarisation Rates

The polarisation rates, dimensionless between -1 and 1, are

$$P_1 = S_1/S_0$$
$$P_2 = S_2/S_0$$
$$P_3 = S_3/S_0$$

Total polarisation rate is $(P_1^2 + P_2^2 + P_3^2)^{1/2}$

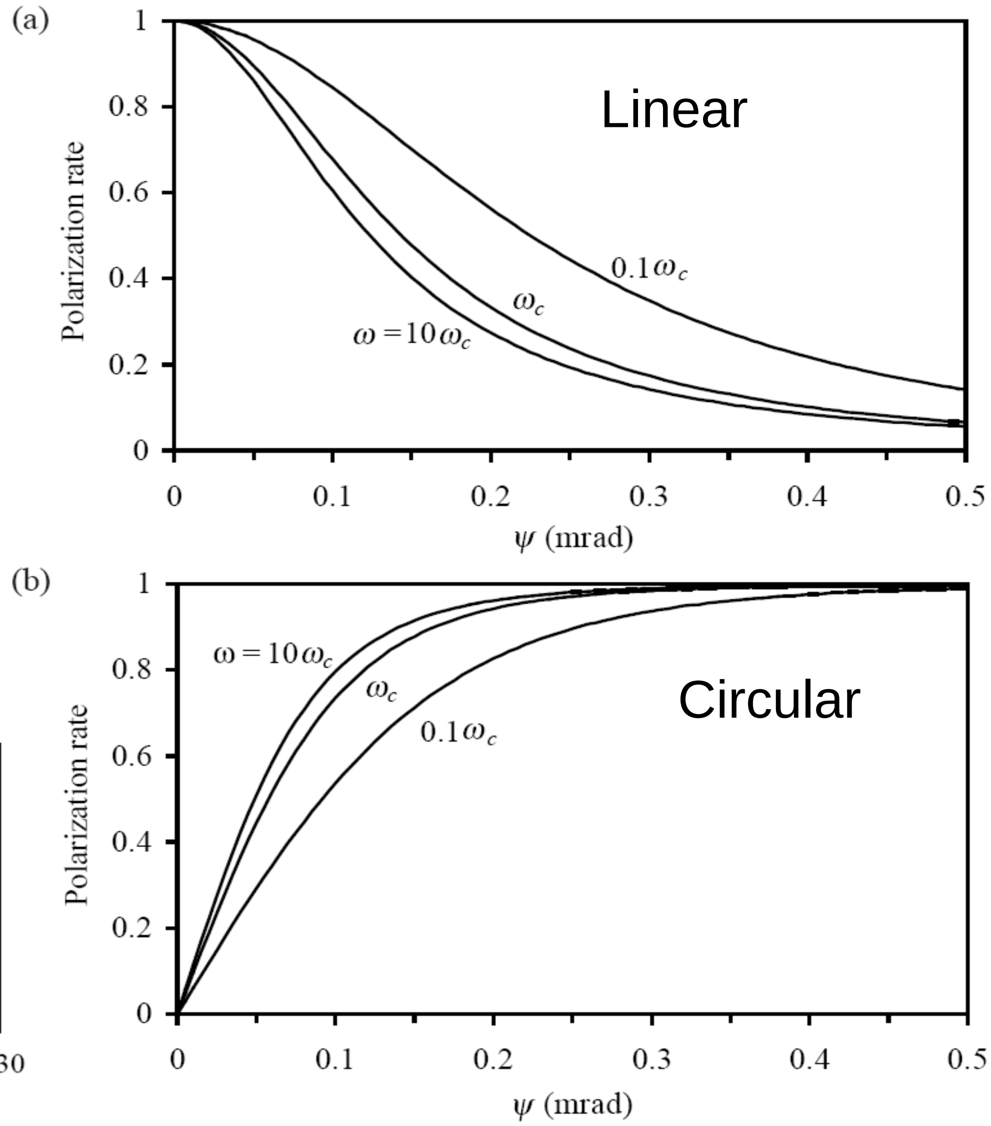
The natural or **unpolarised** rate is $P_4 = 1 - (P_1^2 + P_2^2 + P_3^2)^{1/2}$

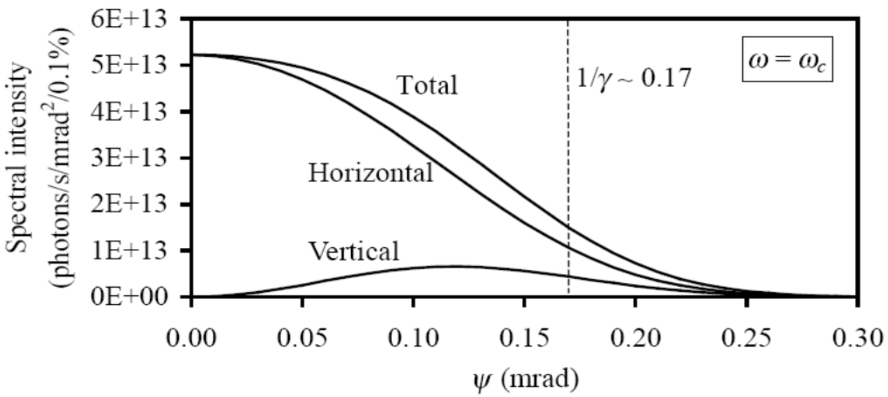
Bending Magnet Polarisation

Circular polarisation is zero on axis and increases to 1 at large angles (ie **pure circular**)

But, remember that **flux falls to zero** at large angles!

So have to make **intensity and polarisation compromise** if want to use circular polarisation from a bending magnet





Qualitative Explanation

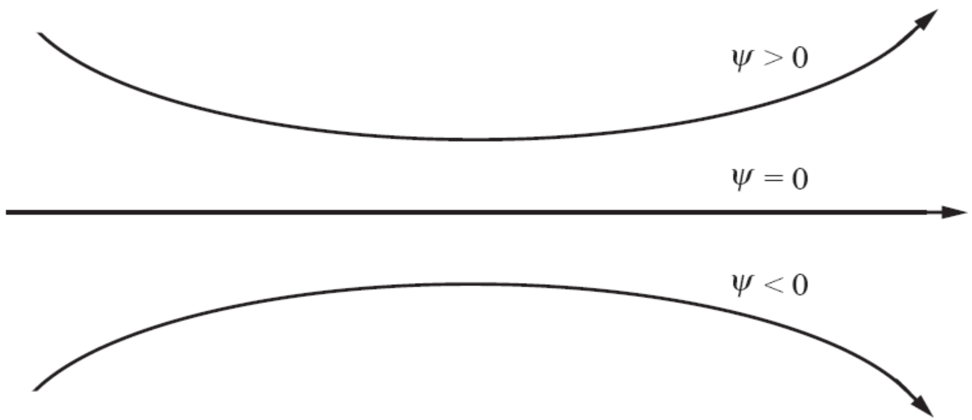
Consider the trajectory seen by an observer:

In the horizontal plane he sees the electron travel along a line (giving **linear polarisation**)

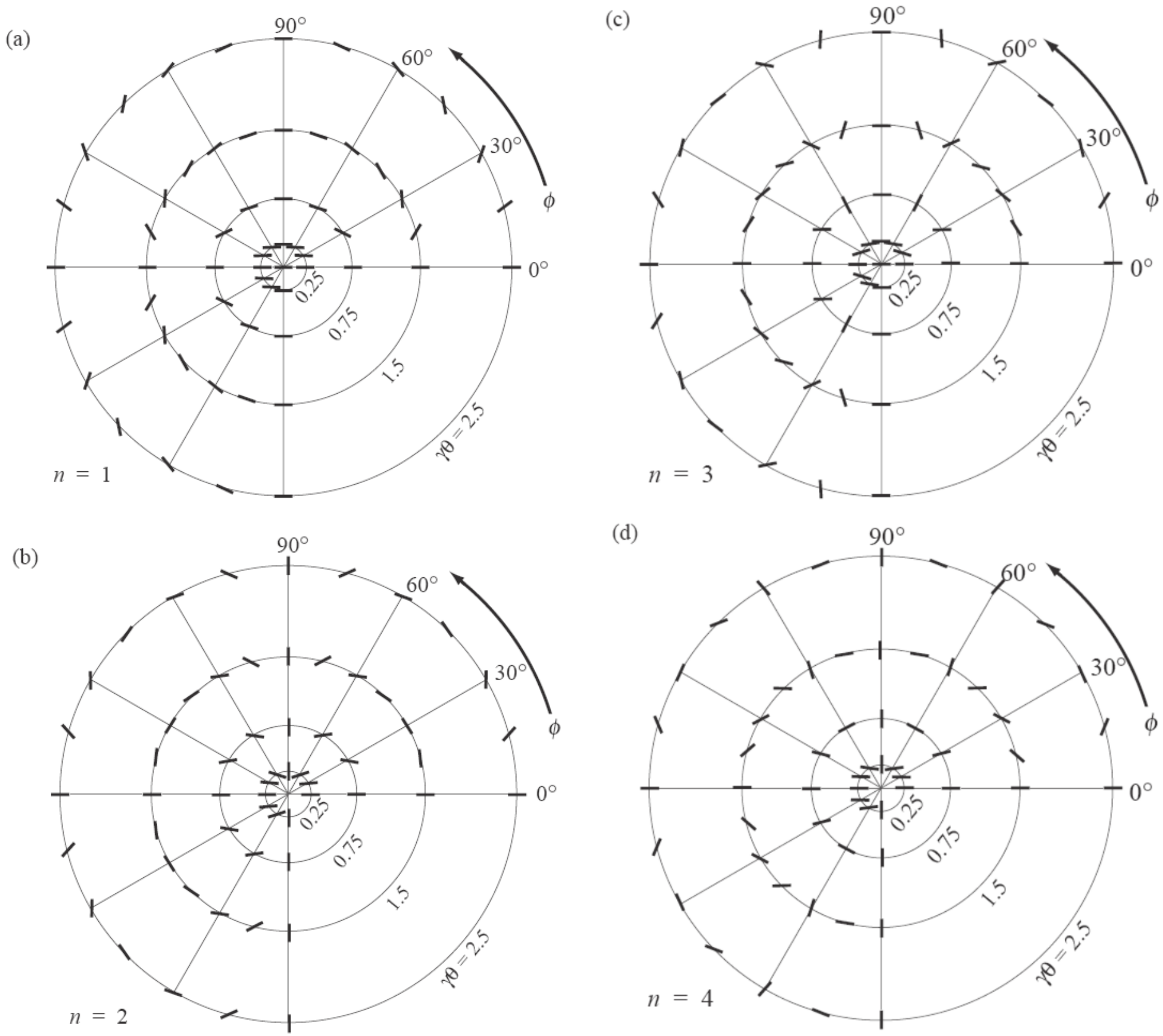
Above the axis the electron has a curved trajectory (containing an element of **circular polarisation**)

Below the axis the trajectory is again curved but of opposite handedness (containing an element of **circular polarisation but of opposite handedness**)

As the vertical angle of observation increases the curvature observed increases and so the **circular polarisation rate increases**



Undulator Polarisation



Standard undulator ($K_x = 0$) emits **linear polarisation** but the angle changes with observation position and harmonic order

Odd harmonics have **horizontal** polarisation close to the central on-axis region

Helical (or Elliptical) Undulators

Include a finite horizontal field of the same period so the electron takes an elliptical path when viewed head on

Consider two orthogonal fields of equal period but of different amplitude and phase

$$B_x = B_{x0} \sin\left(\frac{2\pi s}{\lambda_u} - \phi\right)$$
$$B_y = B_{y0} \sin\left(\frac{2\pi s}{\lambda_u}\right) \quad .$$

3 independent variables

Low K regime so interference effects dominate

Q. What happens to the undulator wavelength?

Undulator Equation

Same derivation as before:

Find the electron velocities in both planes from the B fields

Total electron energy is fixed so the total velocity is fixed β

Find the longitudinal velocity β_s

Find the average longitudinal velocity $\hat{\beta}_s$

Insert this into the interference condition to get

$$\lambda = \frac{\lambda_u}{2n\gamma^2}\left(1 + \frac{K_x^2}{2} + \frac{K_y^2}{2} + \theta^2\gamma^2\right)$$

Extra term now

Very similar to the standard result

Note. Wavelength is independent of phase

Polarisation Rates

For the helical undulator

$$P_1 = \frac{B_{y_0}^2 - B_{x_0}^2}{B_{y_0}^2 + B_{x_0}^2}$$
$$P_2 = \frac{2B_{x_0}B_{y_0}\cos\phi}{B_{y_0}^2 + B_{x_0}^2}$$
$$P_3 = \frac{2B_{x_0}B_{y_0}\sin\phi}{B_{y_0}^2 + B_{x_0}^2}$$

Only 3 independent variables are needed to specify the polarisation so a helical undulator can **generate any polarisation state**

Pure circular polarisation (P_3 = 1) when $B_{x0} = B_{y0}$ and $\phi = \pi/2$

Q. What will the trajectory look like?

Flux Levels

For the helical case, when $\phi = \pi/2$ results are very similar to planar case:

Angular flux density on axis (photons/s/mrad²/0.1% bandwidth)

$$\left.\frac{d\dot{N}}{d\Omega}\right|_{\theta=0} = 1.74 \times 10^{14}\, N^2 E^2 I_b F_n(K_x, K_y)$$

where

$$F_n(K_x, K_y) = \frac{n^2}{(1 + K_x^2/2 + K_y^2/2)^2}\Big(K_x^2\left(J_{(n+1)/2}(Y) + J_{(n-1)/2}(Y)\right)^2 + K_y^2\left(J_{(n+1)/2}(Y) - J_{(n-1)/2}(Y)\right)^2\Big)$$

$$Y = \frac{n(K_y^2 - K_x^2)}{4(1 + K_x^2/2 + K_y^2/2)}$$

Flux in the Central Cone

$$\dot{N} = 1.43 \times 10^{14}\, N I_b Q_n(K_x, K_y)$$

(photons/s/0.1% bandwidth)

where

$$Q_n(K_x, K_y) = \frac{1 + K_x^2/2 + K_y^2/2}{n} F_n(K_x, K_y)$$

In this pure helical mode get circular polarisation only ($P_3 = 1$)

The Bessel function term in $Q_n(K)$ is always zero except when $n = 1$, this means that only the 1st harmonic will be observed on axis in helical mode

Flux Levels

These equations can be further simplified since $K = K_x = K_y$ and so Y = 0.

Angular flux density on axis (photons/s/mrad²/0.1% bandwidth)

$$\left.\frac{d\dot{N}}{d\Omega}\right|_{\theta=0} = 3.49 \times 10^{14}\, N^2 E^2 I_b \frac{K^2}{(1+K^2)^2}$$

And flux in the central cone (photons/s/0.1% bandwidth)

$$\dot{N} = 2.86 \times 10^{14}\, N I_b \frac{K^2}{1+K^2}$$

Power from Helical Undulators

Use the same approach as previously

$$P_{\text{total}} = 1265.5\, E^2 I_b \int_0^L B(s)^2 ds$$

For the case $K_x = K_y \text{ and } \phi = \pi/2$ B is constant (though rotating) and so

$$P_{\text{total}} = 1265.5\, E^2 B_0^2 L I_b$$

Exactly **double** that produced by a planar undulator

The power density is found by integrating the flux density over all photon energies

On axis

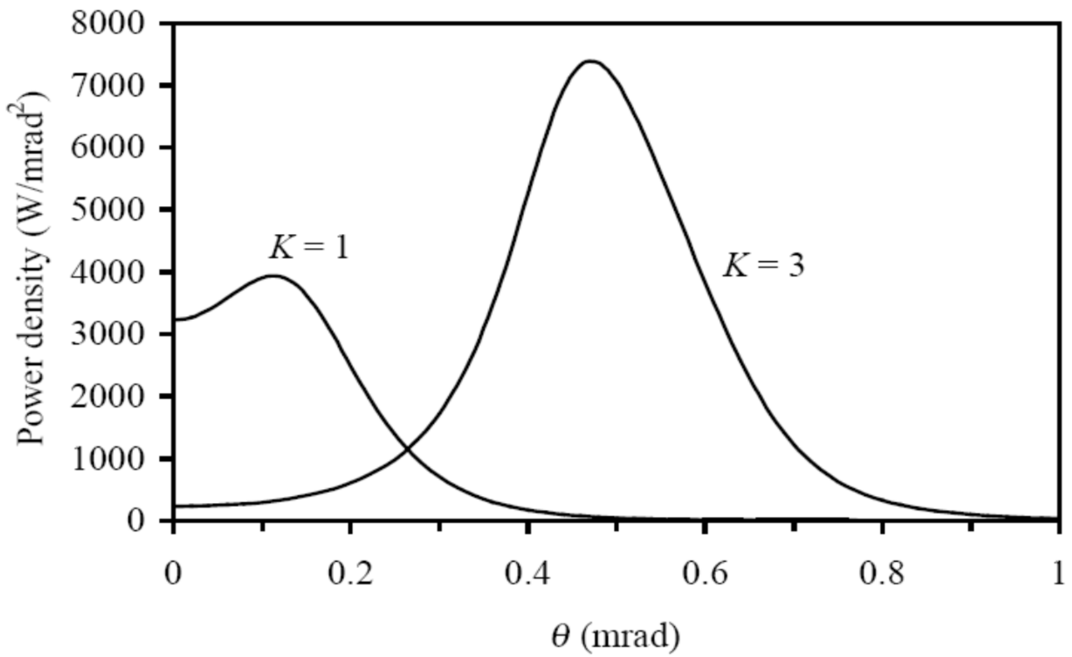
$$\left.\frac{dP}{d\Omega}\right|_{\theta=0} = 4625.7\, \frac{B_0^2 E^4 L I_b}{(1+K^2)^3} \qquad K = K_x = K_y$$

Helical Undulator Power Density

Example for 50 mm period, length of 5 m and energy of 3 GeV

Bigger K means lower on-axis power density but greater total power

Get lower power density on axis as only the 1st harmonic is on axis and as K increases the photon energy decreases



Summary So Far

The effect of the finite electron beam emittance is to smear out the radiation (in both wavelength and in angle)

The θ^2 term has an important impact on the observed radiation

- width of the harmonic
- the presence of even harmonics
- the effect of the beamline aperture

Undulators are excellent sources for experiments that require **variable, selectable polarisation**

Generating Periodic Magnetic Fields

So far we have discussed at length what the properties of SR are, when it is generated, and how it can be tailored to suit our needs (wavelength, polarisation, flux, etc)

But, how do we know what magnetic fields are actually achievable?

In this part we will look at **how periodic fields are generated** and what the limitations are

Later we will look at the present state of the art and some future possibilities

What are the possibilities?

To generate magnetic field we can use:

Electromagnets

- Normal conducting or
- Superconducting

Permanent Magnets

Both types can also include iron if required

Permanent Magnet Basics

I will give a brief introduction only

The magnetic properties of materials is a big topic!

Further reading:

P Campbell, Permanent magnet materials and their applications, CUP 1994

also

H R Kirchmayr, Permanent magnets and hard magnetic materials, J Phys D:Appl Phys 29:2763, 1996

What is a Permanent Magnet ?

Definition:

A magnet is said to be **Permanent** (or **Hard**) if it will independently support a useful flux in the air gap of a device

A material is magnetically **Soft** if it can only support such a flux with the help of an external circuit (eg iron is soft)

A PM can be considered as a passive device analogous to a spring (which stores mechanical energy)

An electron in a microscopic orbit has a magnetic dipole moment – can be modelled as a current flowing in a loop

In a Permanent Magnet these ‘molecular’ currents can be identified with atoms with unfilled inner shells

eg 3d metals (Fe, Co, Ni) or 4f rare earths (Ce to Yb)

Permanent Magnets

Permanent Magnet materials are manufactured so that their magnetic properties are enhanced along a preferred axis

To do this, advantage is taken of crystal lattices

The direction of alignment is called the **easy axis**

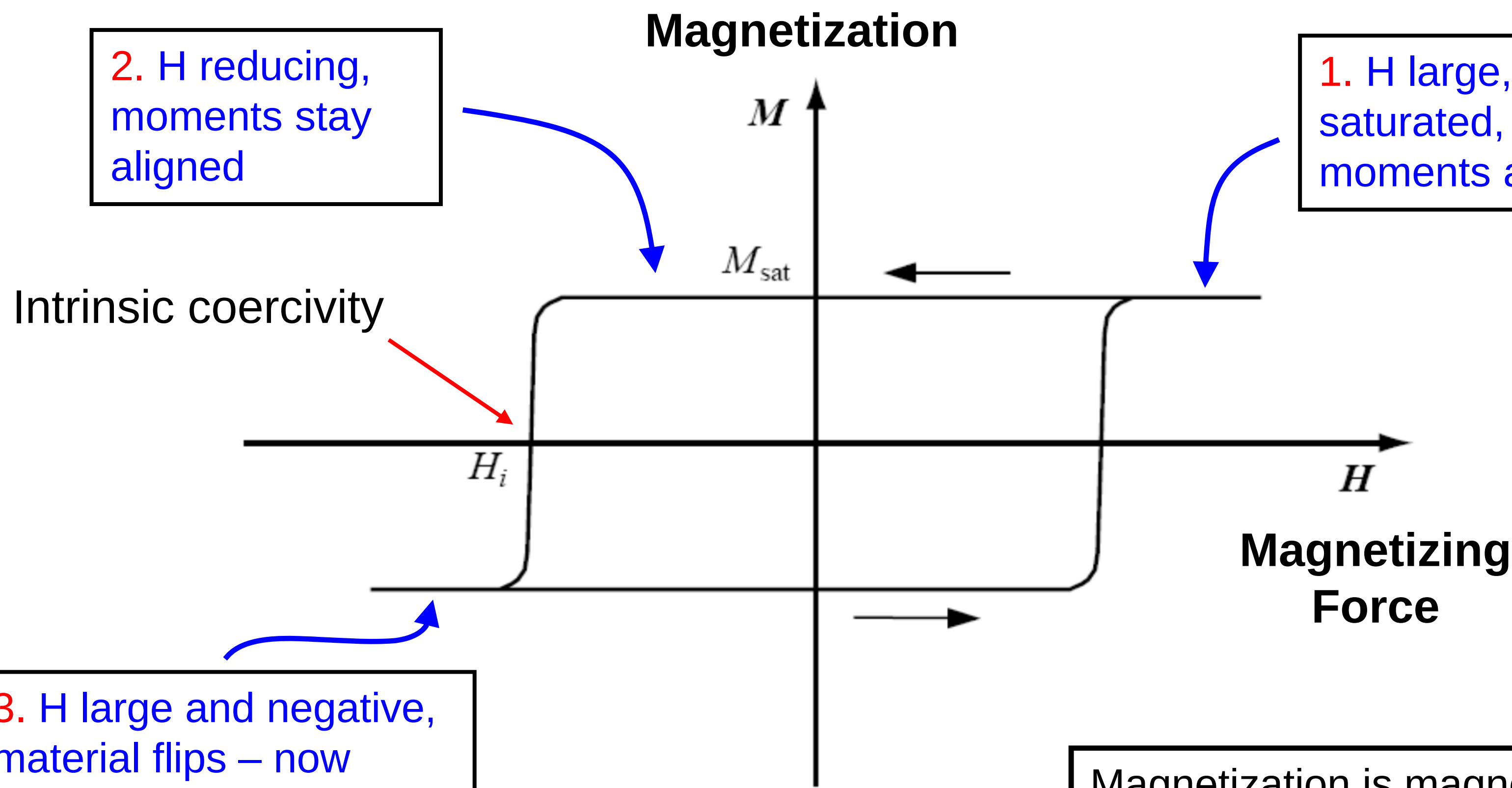
When a magnetic field, B, is applied to a magnetic material each dipole moment tries to align itself with the field direction

When B is strong enough (at saturation) all of the moments are aligned, overcoming other atomic forces which resist this

A Permanent Magnet must be able to maintain this alignment after B is removed

An Ideal Permanent Magnet

The characteristics of a Permanent Magnet are determined by its behaviour under an external magnetization force H



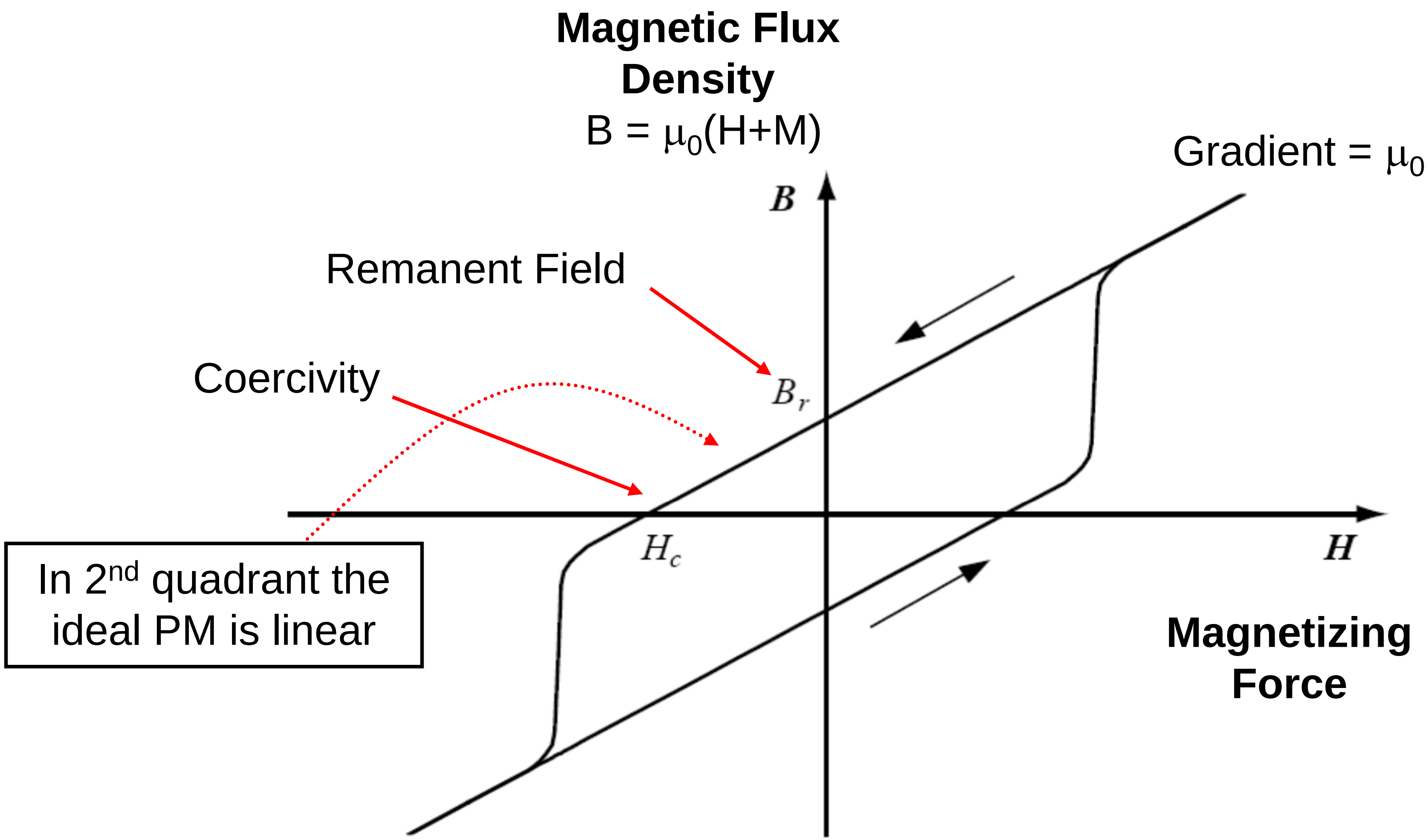
2. H reducing, moments stay aligned

1. H large, material saturated, all moments aligned

3. H large and negative, material flips – now aligned with opposite direction

Magnetization is magnetic dipole moment per unit volume, $B = \mu_0 M$

The Ideal BH Curve

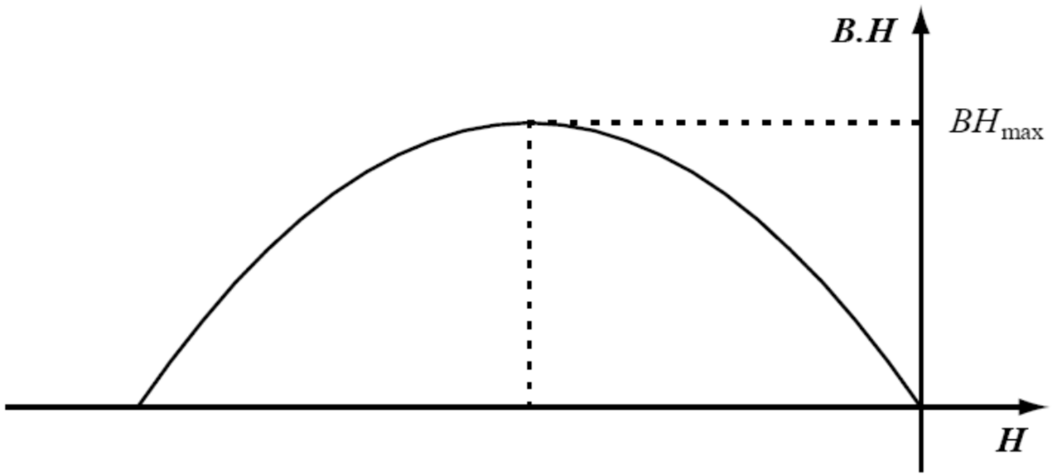


The BH Product

Permanent Magnets are operated in the 2nd quadrant – no external fields are present, moments are aligned along the easy axis

The product BH represents the **energy density** of the material

Examining the peak BH value in the 2nd quadrant is a good way of comparing the strength of different materials



Current Sheet Equivalent Materials (CSEM)

An ideal Permanent Magnet is uniformly magnetized (homogeneous)

The equivalent current model is a sheet of current flowing on the surface with no internal (volume) currents

The relative permeability is ~1 so we can consider the bulk material to be vacuum

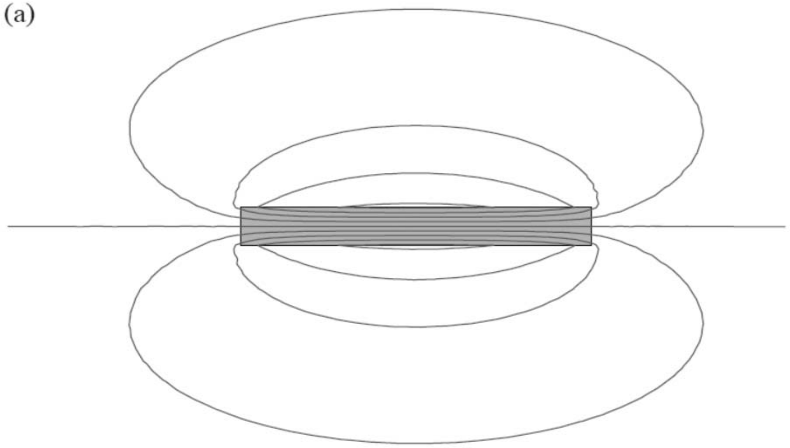
This CSEM model implies that the contributions from different magnets **can be added linearly** (just like adding contributions from currents)

Analytical calculations then become fairly simple because we can calculate the field at a point from every block independently and just add all the individual contributions up

Current Sheet Equivalent Materials (CSEM)

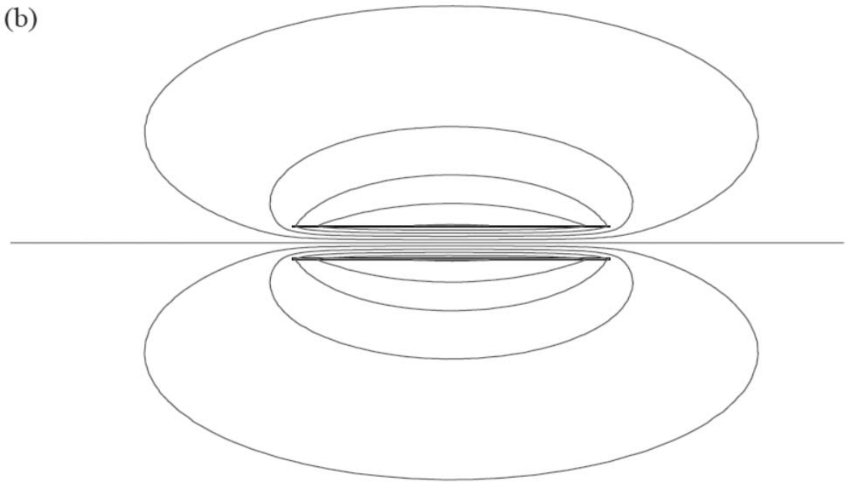
Lines of flux for an **ideal** Permanent Magnet

(a)



Lines of flux for a CSEM model

(b)



Temperature Effects

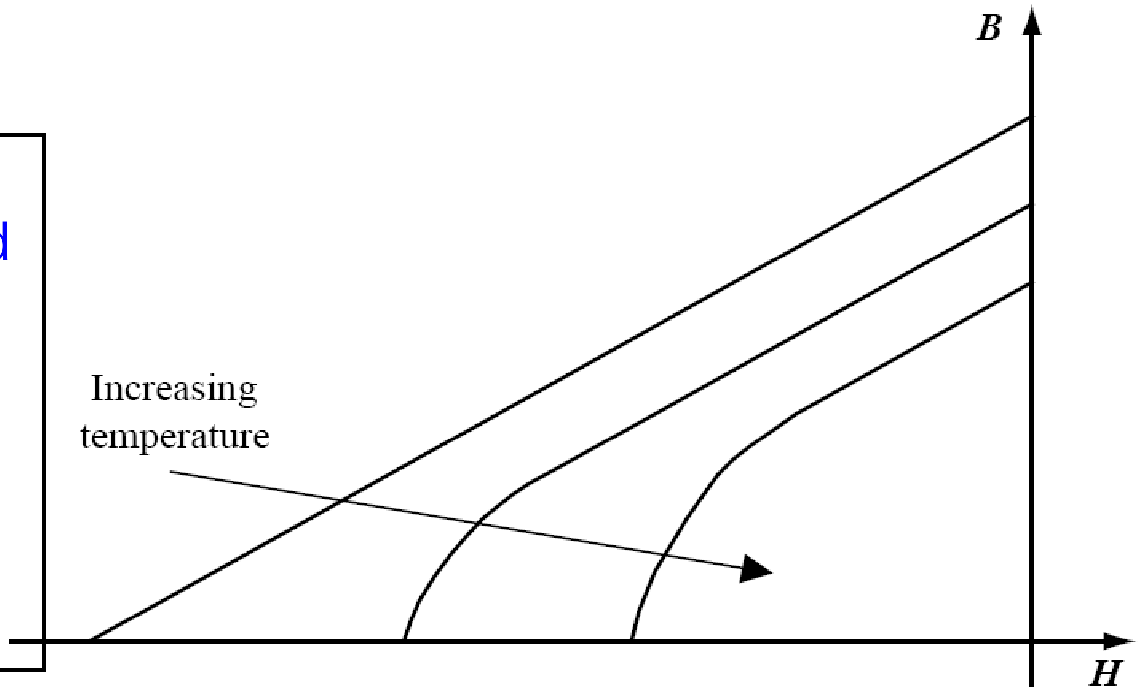
At higher temperatures the materials become more non-linear

So long as the working point stays in the linear region this is a reversible effect

Note that the remanent field drops with increasing temperature – the reverse is also true, cold magnets have a higher remanent field

Day to day temperature variations are important and must be controlled (minimised)

Many undulators are operated in air conditioned environments to keep their output more stable



Available Materials

Two types of permanent magnet are generally used – Samarium Cobalt (SmCo) and Neodymium Iron Boron (NdFeB)

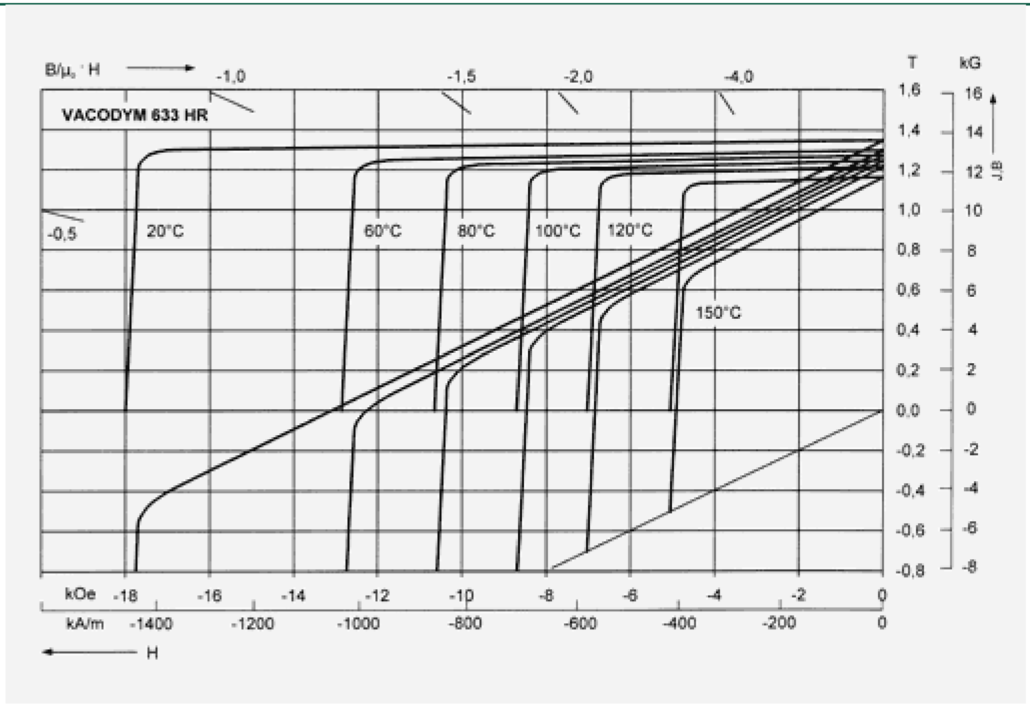
	SmCo	NdFeB
Remanent Field	0.85 to 1.05 T	1.1 to 1.4 T
Coercivity	600 to 800 kA/m	750 to 1000 kA/m
Relative Permeability	1.01 parallel, 1.04 perp	1.05, 1.15
Temperature Coefficient	-0.04 %/°C	-0.11 %/°C
Max Energy Density	150 to 200 kJ/m^3	200 to 350 kJ/m^3
Max operating temperature	~300°C	~100°C
Comment	Brittle, easily damaged, better intrinsic radiation resistance, expensive	Less brittle but still liable to chip, easier to machine, expensive

An Example Material

Vacodym 633 HR

M-H and B-H curves are shown (2nd quadrant)

The material is linear at 20°C but non-linear above about 60 °C



(courtesy of Vacuumschmelze)

Pure Permanent Magnet Undulators

A magnet which contains no iron or current carrying coils is said to be a Pure Permanent Magnet (PPM)

Because of CSEM we can use the principle of superposition

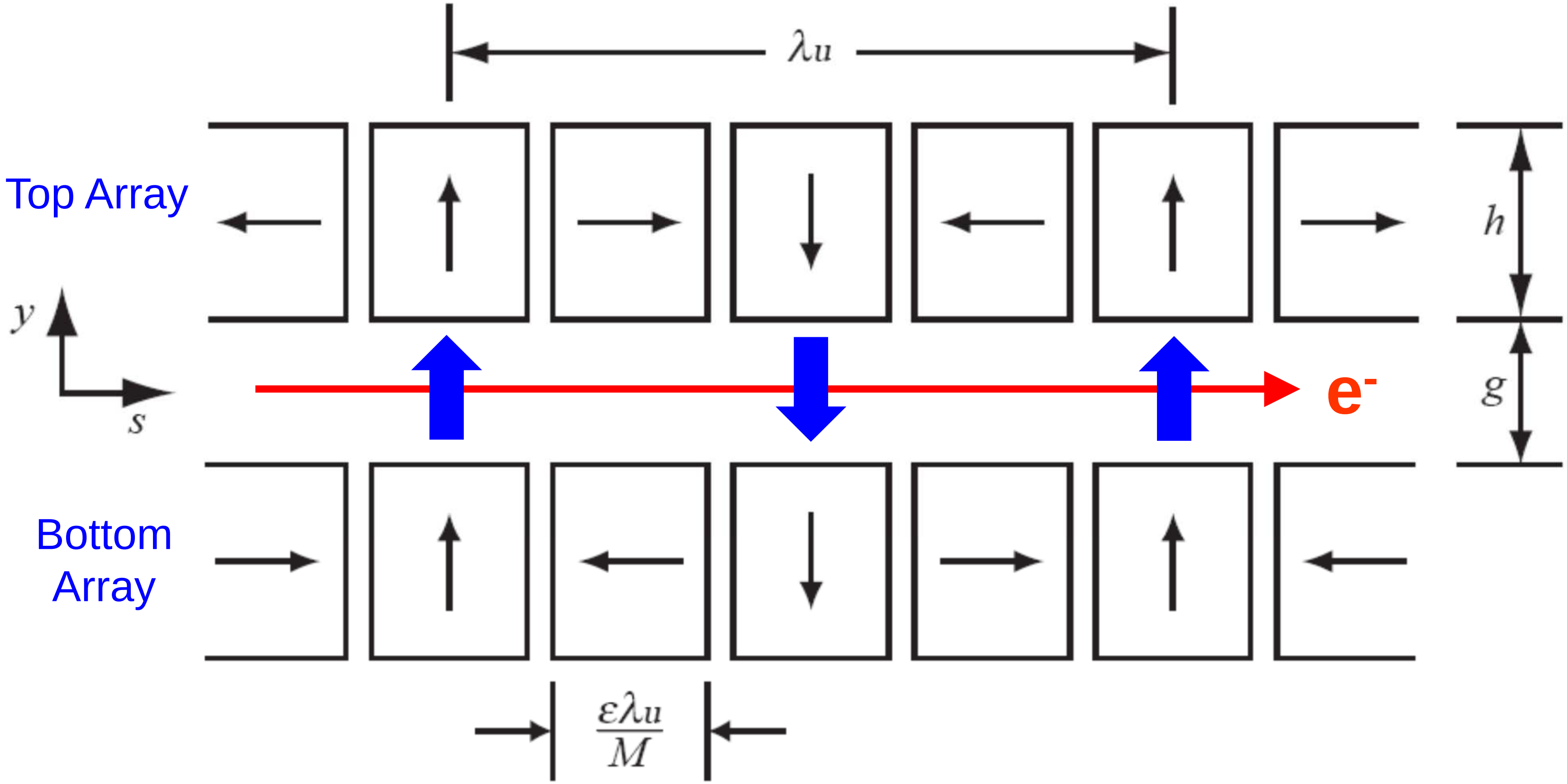
An **ideal undulator** would have a sinusoidal magnetic field along the direction of the electron beam

To generate a sinusoidal field an ideal PPM would have **two sets (arrays)** of Permanent Magnet with their easy axis rotating smoothly through **360° per period** along the direction of the electron beam

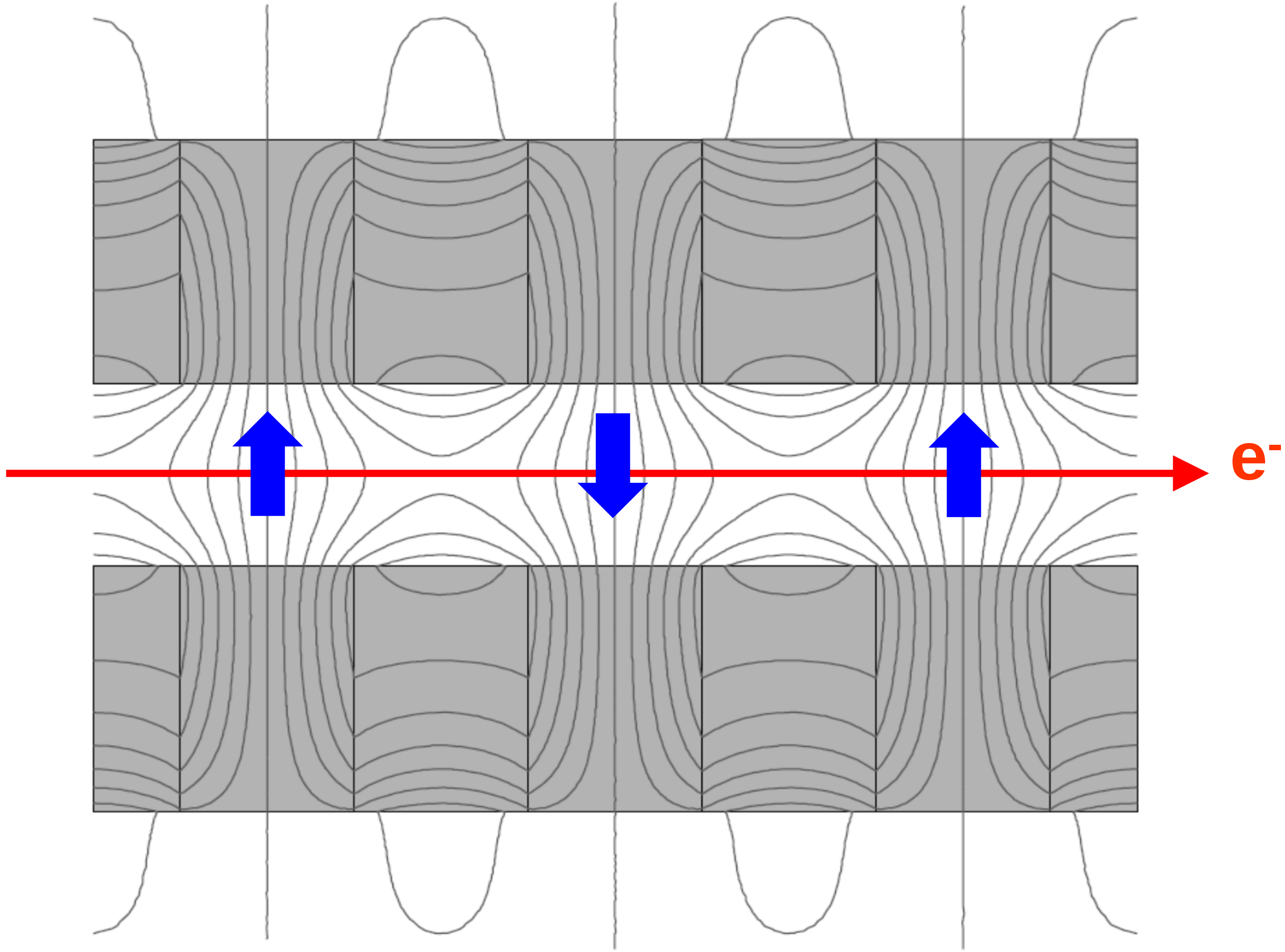
In practice this ideal situation is approximated by splitting the system into M rectangular magnet blocks per period with the easy axis at the relevant set angle

Example PPM arrangement, M = 4

Side View



Lines of Magnetic Flux



Magnetic Field

The field strength between the two arrays assuming infinite width in the x – direction (2D approximation) is

$$B_y = -2B_r \sum_{i=0}^{\infty} \cos\left(\frac{2n\pi s}{\lambda_u}\right) \cosh\left(\frac{2n\pi y}{\lambda_u}\right) \frac{\sin(n\varepsilon\pi/M)}{n\pi/M} e^{-n\pi g/\lambda_u} (1 - e^{-2n\pi h/\lambda_u})$$

$$B_s = 2B_r \sum_{i=0}^{\infty} \sin\left(\frac{2n\pi s}{\lambda_u}\right) \sinh\left(\frac{2n\pi y}{\lambda_u}\right) \frac{\sin(n\varepsilon\pi/M)}{n\pi/M} e^{-n\pi g/\lambda_u} (1 - e^{-2n\pi h/\lambda_u}) \ ,$$

Where $n = 1 + iM$ and ε is a packing factor to allow for small air gaps between blocks

The vertical field on axis (y = 0) is a number of cosine harmonics

As $M \longrightarrow \infty$ this reduces to a single cosine (ideal case)

The longitudinal (and horizontal) field on axis is zero

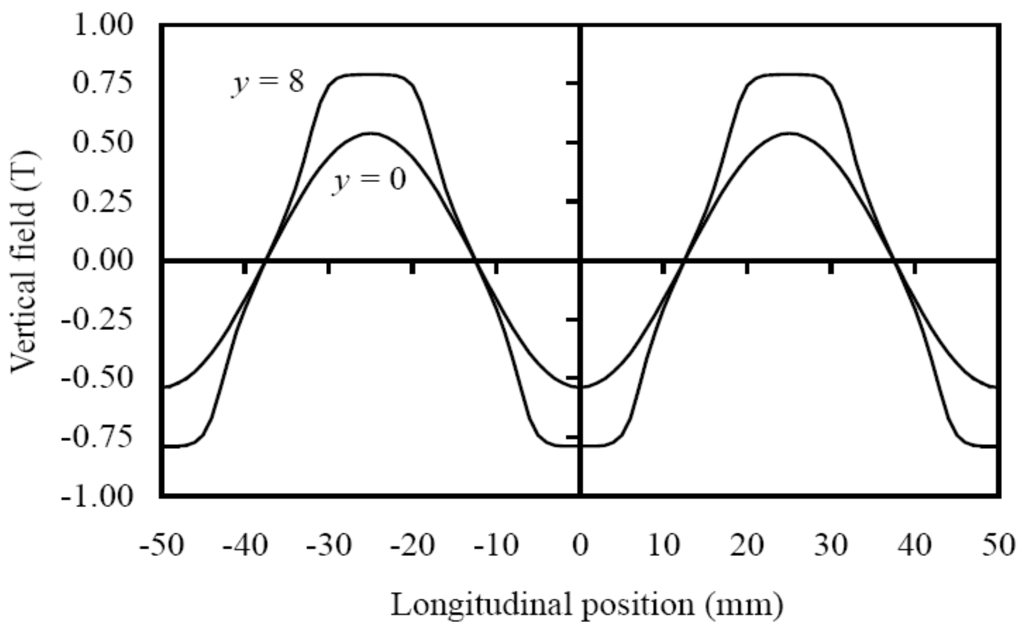
A Practical PPM

The most popular choice is M = 4

This is a good compromise between on axis field strength and quality vs engineering complexity

Higher harmonics then account for < 1% of the field on axis

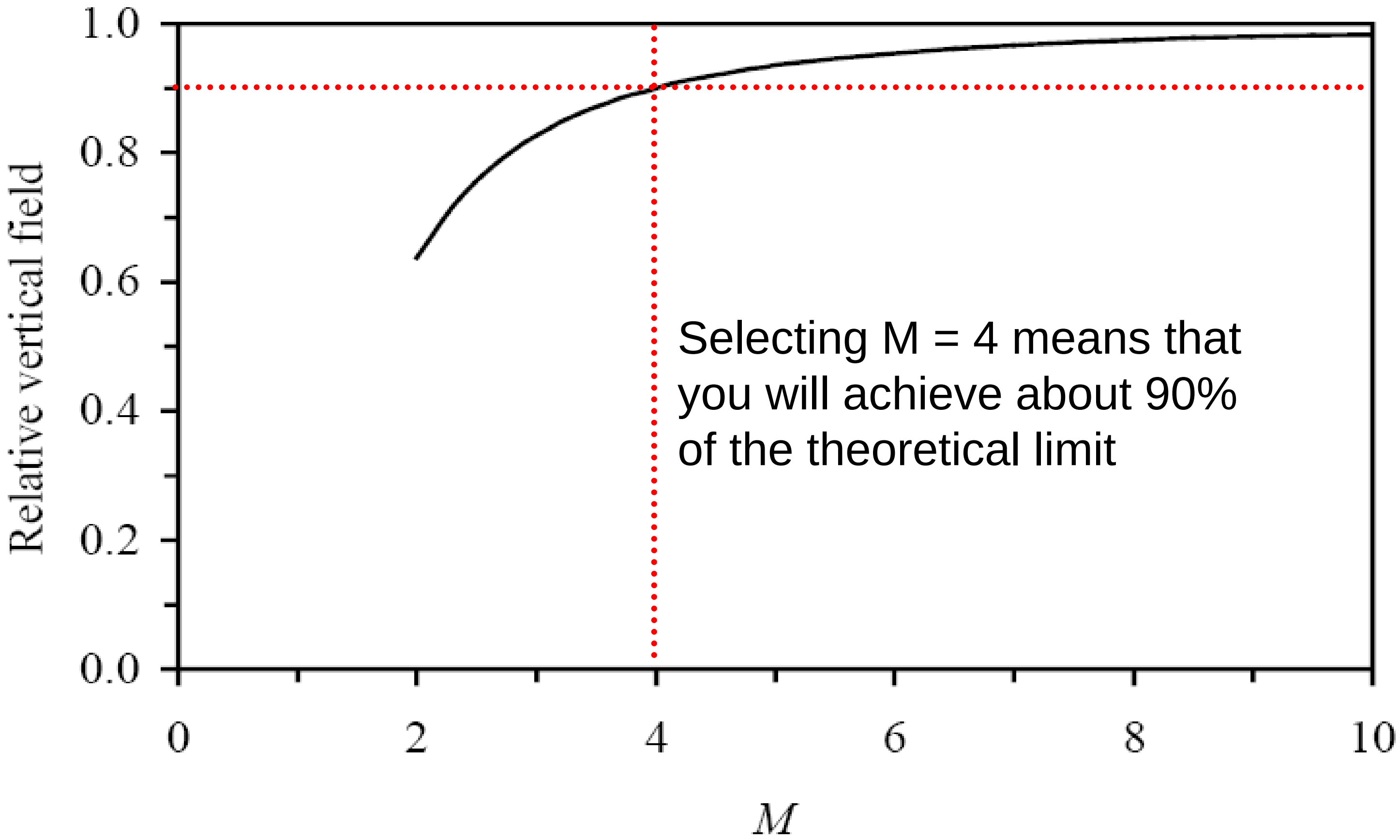
Away from the axis it is definitely **not cosine-like**



For an example PPM with 50mm period, block height of 25mm, magnet gap of 20 mm and remanent field of 1.1 T

Note that the fields increase away from the axis

Peak Vertical Field vs M



Simplifying the Magnetic Field

If we assume that only the first harmonic makes a significant contribution (n = 1) – a good approximation in general

Then the equation simplifies greatly on axis to

$$B_y = -2B_r \cos\left(\frac{2\pi s}{\lambda_u}\right)\frac{\sin(\epsilon\pi/M)}{\pi/M}e^{-\pi g/\lambda_u}(1 - e^{-2\pi h/\lambda_u})$$

$$B_s = 0 \ .$$

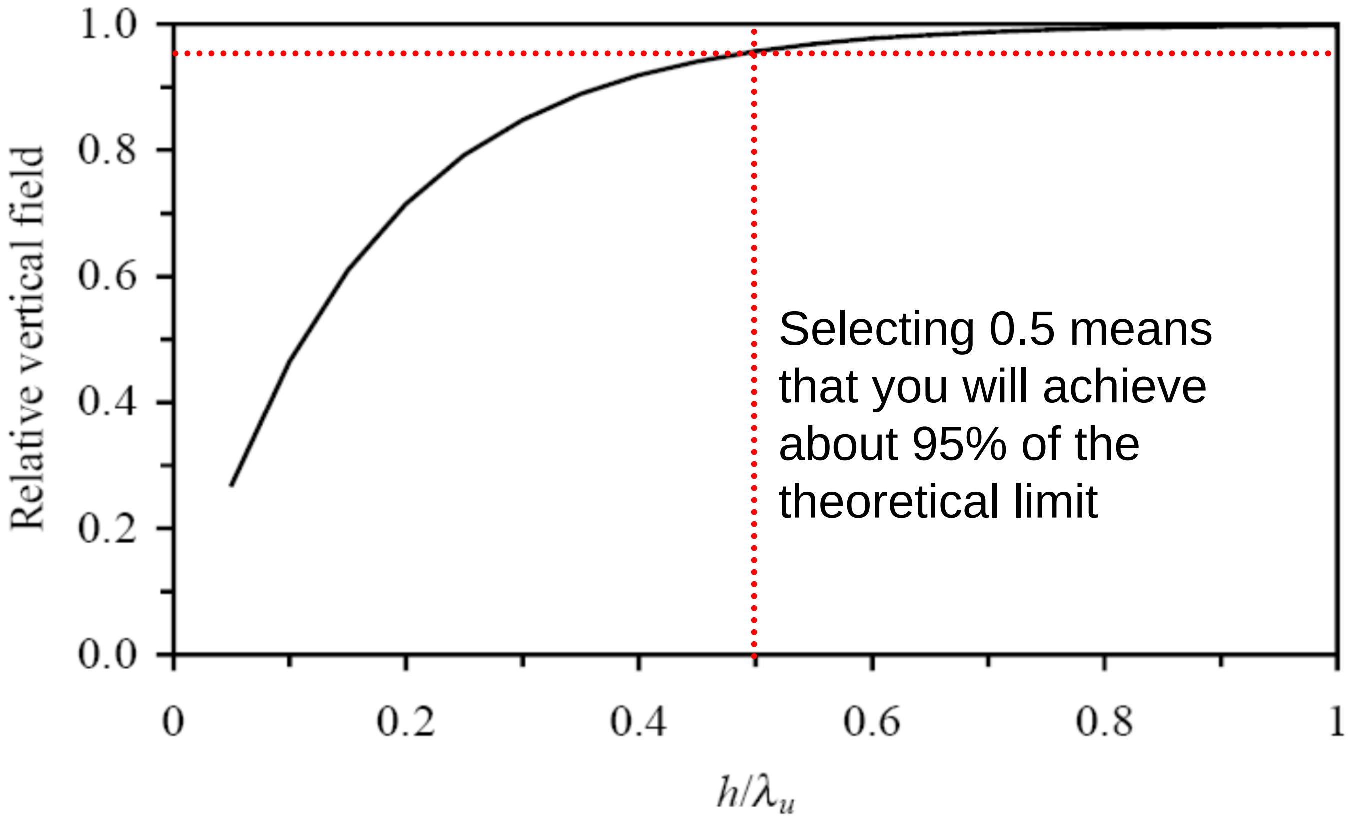
Important:

Note that so long as all the spatial dimensions scale together the fields on axis **do not change**

This is not true for electromagnets – there the current densities have to increase to maintain the same field levels

Effect of Different Block Heights

A typical block height selection is half the period length



Peak Field Achievable

The maximum peak field achievable (in theory) is **2B$_r$**

In practice with M = 4 and h = $\lambda_u/2$ the peak on axis field is

$$B_{y_0} = 1.72\, B_r e^{-\pi g/\lambda_u}$$

So even with an ambitious gap to period ratio of 0.1 the peak value is only **1.26B$_r$**

Achieving fields of ~1.5T requires very high B_r material, small gaps and long periods!

But, higher fields are possible if we **include iron in the system**

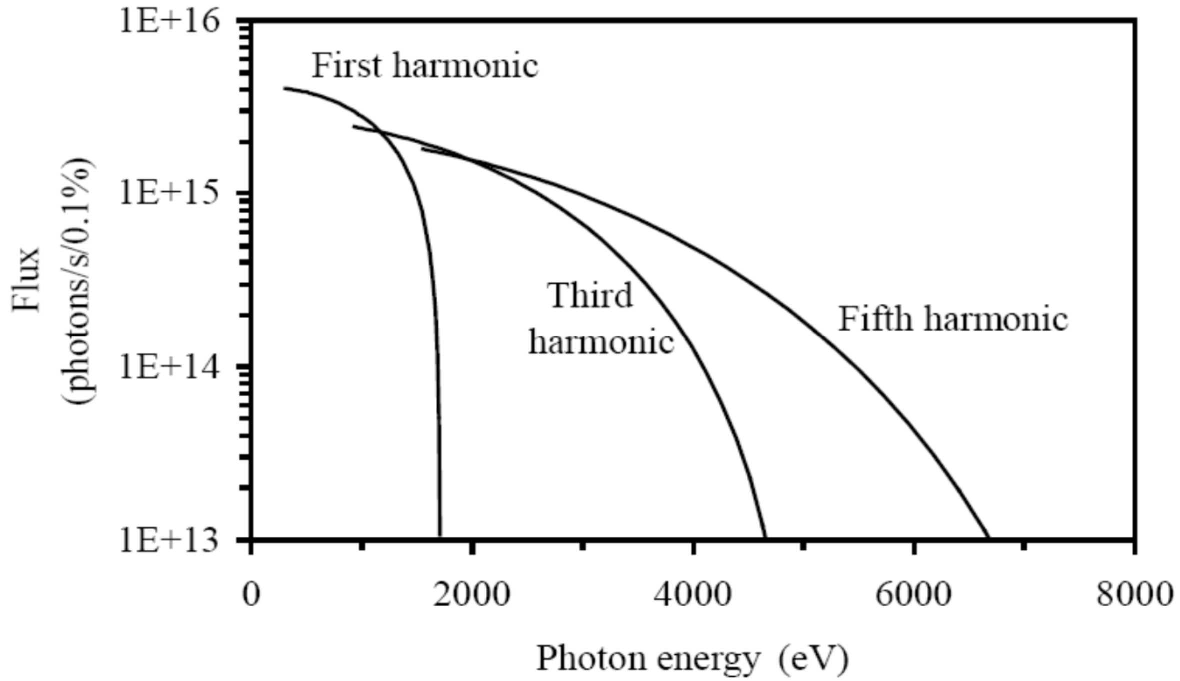
Mixing Permanent Magnets and iron poles is called a **hybrid** magnet

Tuning the Undulator

To vary the output wavelength from the undulator – to map out the tuning curves – we need to alter the field level on the axis

We can now see that the only practical way to do this for a permanent magnet device is **to change the magnet gap, g**

$$B_{y_0} = 1.72\, B_r e^{-\pi g/\lambda_u}$$



Summary

A Permanent Magnet can independently support a flux in an air gap – no coils are needed

Permanent Magnets are operated in the 2nd quadrant of the BH curve – ideally they have a linear behaviour

Permanent Magnets can be modelled as current sheets so we can add the field contributions from each block linearly

To generate a sinusoidal field we use two arrays of magnet blocks – one above and one below the electron beam