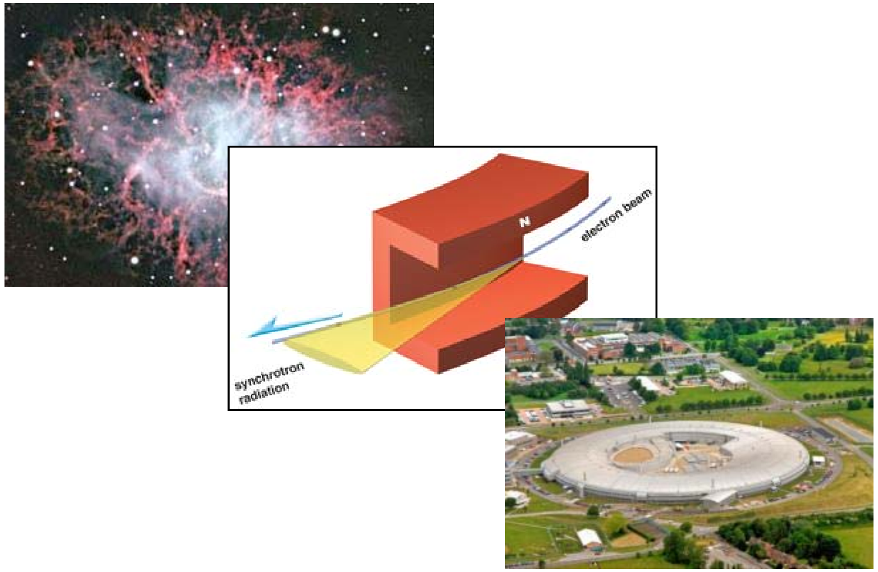
Insertion Devices
Lecture 1
Introduction to Synchrotron Radiation



Jim Clarke

ASTeC

Daresbury Laboratory

Program

4th Feb	10.30	Introduction to SR
4th Feb	11.45	Wigglers and Undulators
11th Feb	10.30	Undulator Radiation and Realisation
11th Feb	11.45	Undulator Magnet Designs
11th Feb	14.00	Tutorial

Please interrupt and ask questions during the lectures !!!

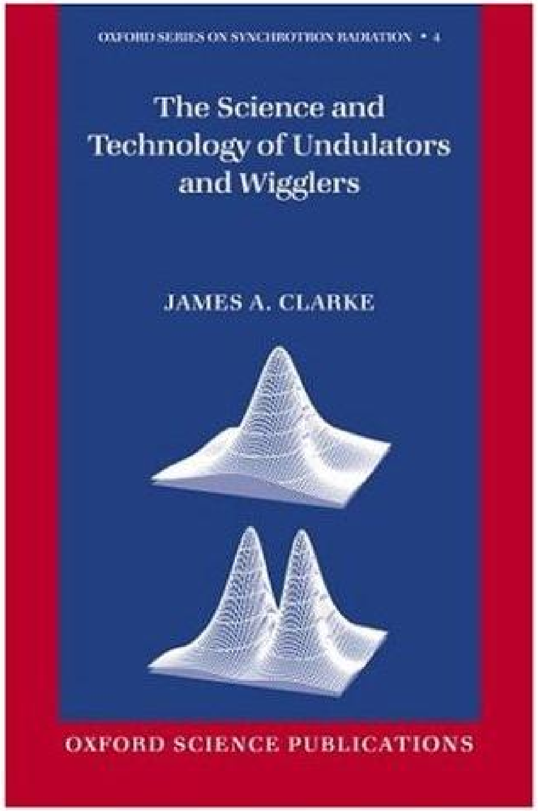
Course Book

The vast majority of the material presented is from my book, you will find much more detail in there, derivations of all the equations, and also other supplementary information.

The Science and Technology of Undulators and Wigglers,
J. A. Clarke, Oxford University Press, 2004

(Oxford series on Synchrotron Radiation - 4)

It is available in the Daresbury Library



Why is Synchrotron Radiation so Important?

All accelerator scientists and engineers need to understand SR as it impacts directly on many areas of accelerator design and performance

- RF
- Diagnostics
- Vacuum design
- Magnets
- Beam Dynamics
- It affects all charged particles
- Light sources and Free Electron Lasers are a major "customer" of advanced accelerators

All processes which change the energy of particles are important – SR is one of the most important processes

Introduction to Synchrotron Radiation

Synchrotron Radiation (SR) is a relativistic effect

Many features can be understood in terms of two basic processes:

- Lorentz contraction and
- Doppler shift

Imagine that a relativistic charged particle is travelling through a periodic magnetic field (an undulator)

In the particles rest frame it sees a magnetic field rushing towards it

If in our rest frame the magnet period is λ_u then because of Lorentz contraction the electron sees it as λ_u/γ

γ is the relativistic **Lorentz factor**

Lorentz Factor

$$\gamma = \frac{E}{E_0} \qquad \beta = \sqrt{1 - \frac{1}{\gamma^2}} \qquad \beta = v/c$$

c is the velocity of light in free space

v is the velocity of the electron

β is the relative velocity of the electron

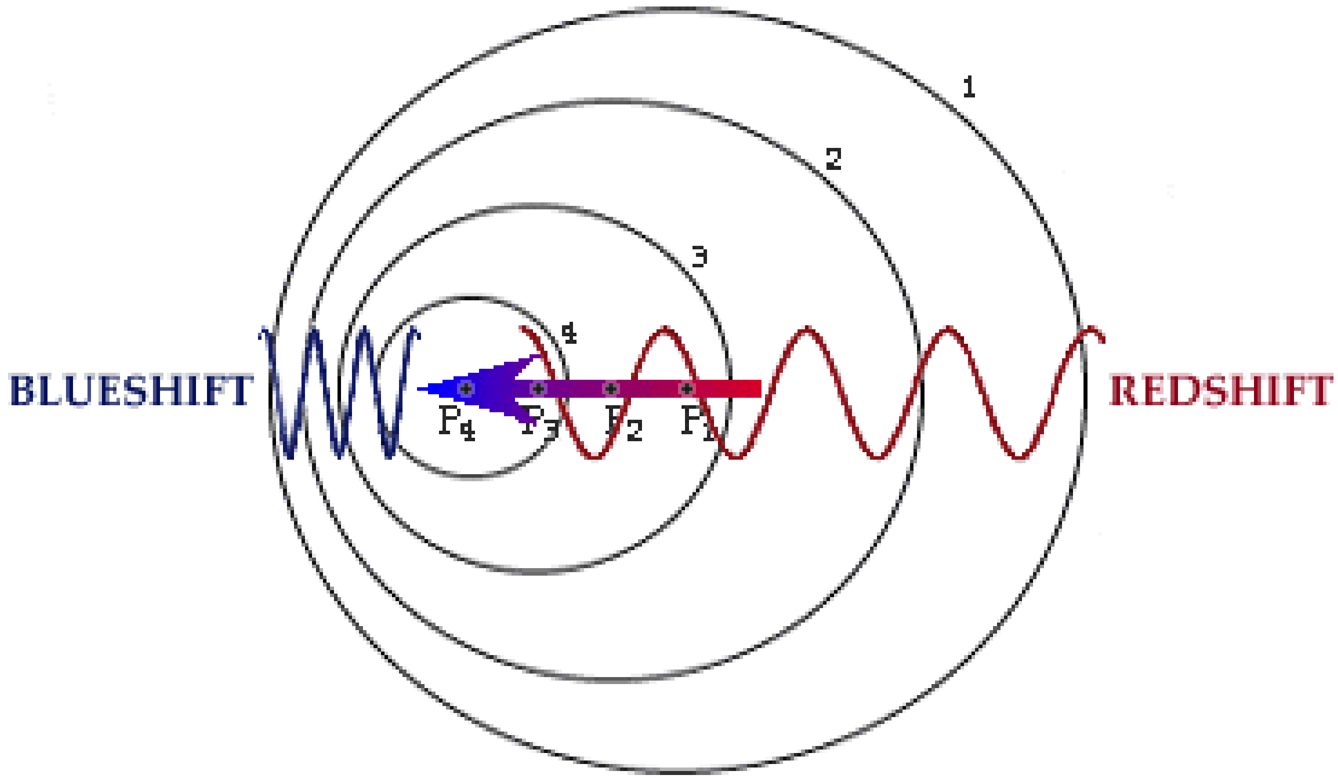
E is the Electron Energy (3000 MeV in DIAMOND)

E_o is the Electron Rest Energy (0.511 MeV)

So in DIAMOND, $\gamma \sim 6000$

This γ factor turns up again and again in SR !

Relativistic Doppler Shift



Relativistic Doppler Shift

In the **relativistic** version of the Doppler effect the frequency of light seen by an observer at rest is

$$f = \gamma f'(1 - \beta \cos\theta')$$

Source travelling away from the observer

where f' is the frequency emitted by the moving source, θ' is the angle at which the source emits the light.

With the source travelling towards the observer $\theta' = \pi$ so

$$f = \gamma f'(1 + \beta)$$

In terms of wavelength

$$\lambda = \frac{\lambda'}{\gamma(1+\beta)} \sim \frac{\lambda'}{2\gamma}$$

Combining Lorentz and Doppler

So the particle emits light of wavelength λ_u/γ

Since it is travelling towards us this wavelength is further reduced by a factor 2γ

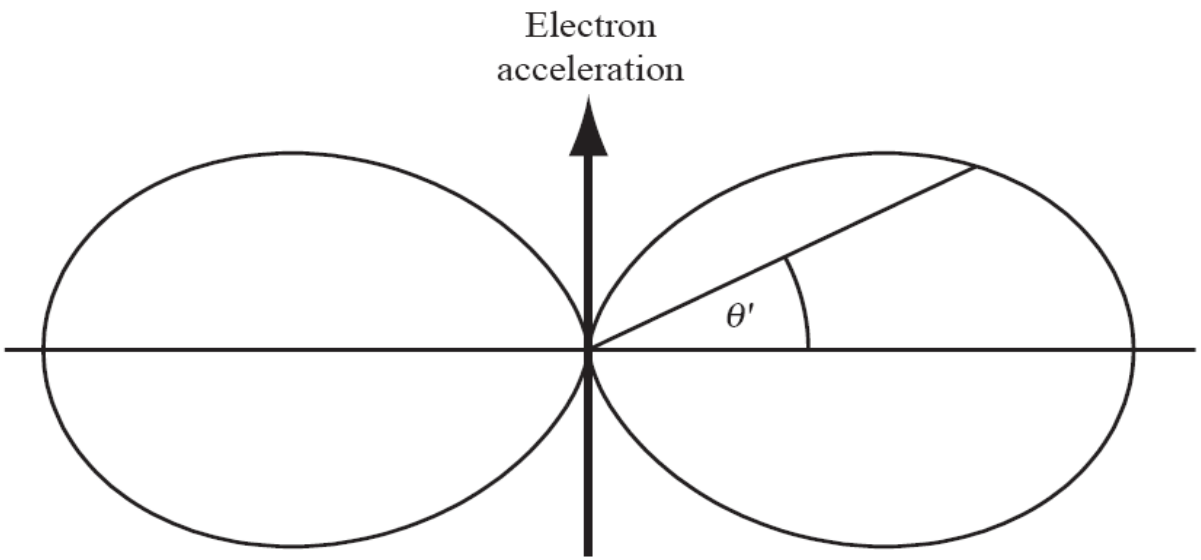
So the wavelength observed will be ~ $\lambda_u/2\gamma^2$

For GeV electron energies with γ of 1000’s, an undulator with a period of a **few cm** will provide radiation with wavelengths of **nm (X-rays)**

Angle of Emission

In the moving frame of the electron, the electron is oscillating in the periodic magnetic field with simple harmonic motion

It therefore emits in the familiar dipole pattern that has a $\sin^2 \theta'$ distribution



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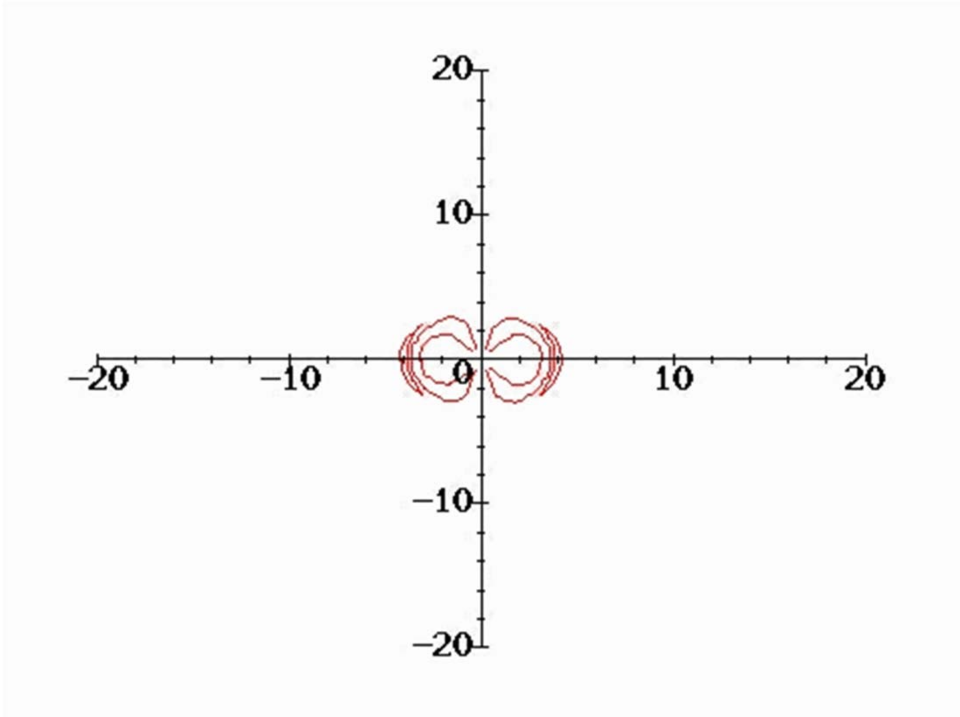


Electric field lines due to a vertically oscillating dipole

Angle of Emission

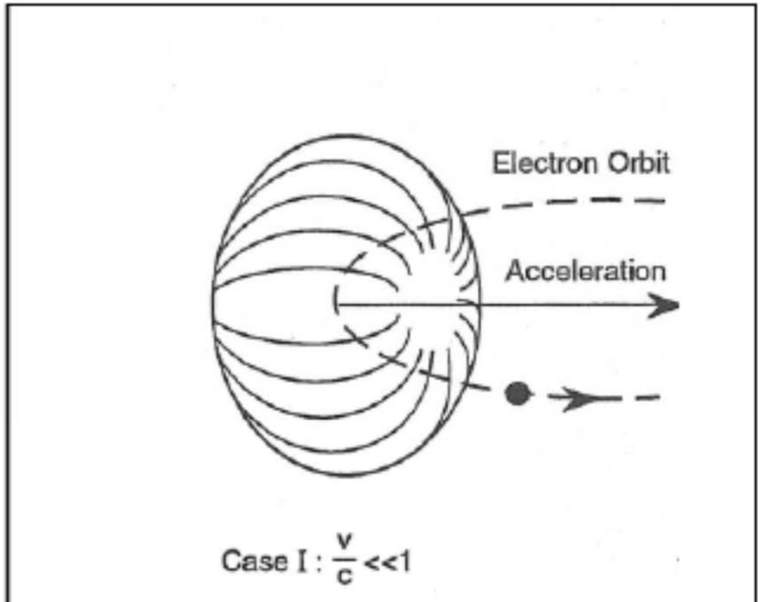
A second consequence of Doppler is that the angle with which the observer views the source will also be affected

$$\tan\theta = \frac{\sin\theta'}{\gamma(\cos\theta' - \beta)}$$

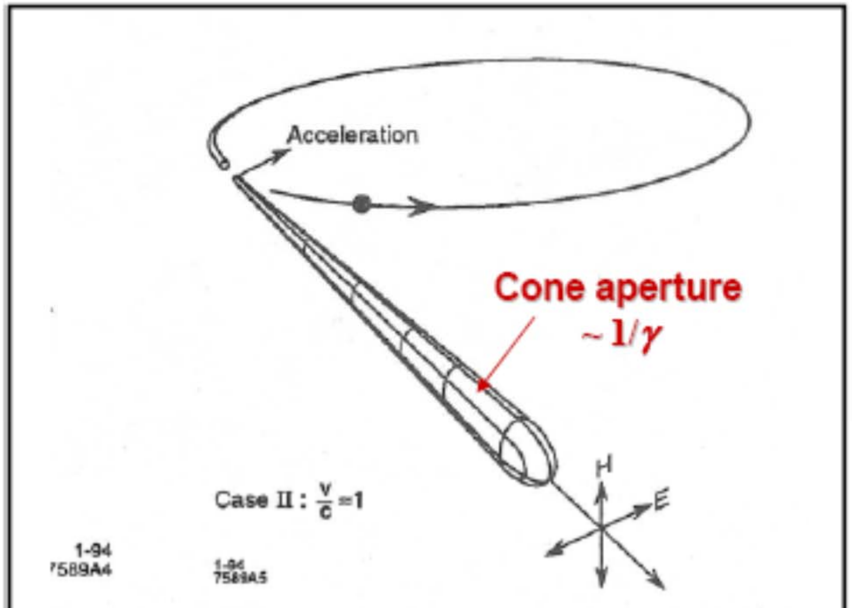
So the point at which the electric dipole has zero amplitude ($\theta' = \pm\pi/2$) appears at the angle $\theta \sim \pm 1/\gamma$

The peak of the emission is orthogonal to the direction of the electrons acceleration so for an electron on a circular path the radiation is emitted in a forward cone at a tangent to the circle

Effect of Relativity



At low electron velocity (non-relativistic case) the radiation is emitted in a non-directional pattern



When the electron velocity approaches the velocity of light, the emission pattern is folded sharply forward.

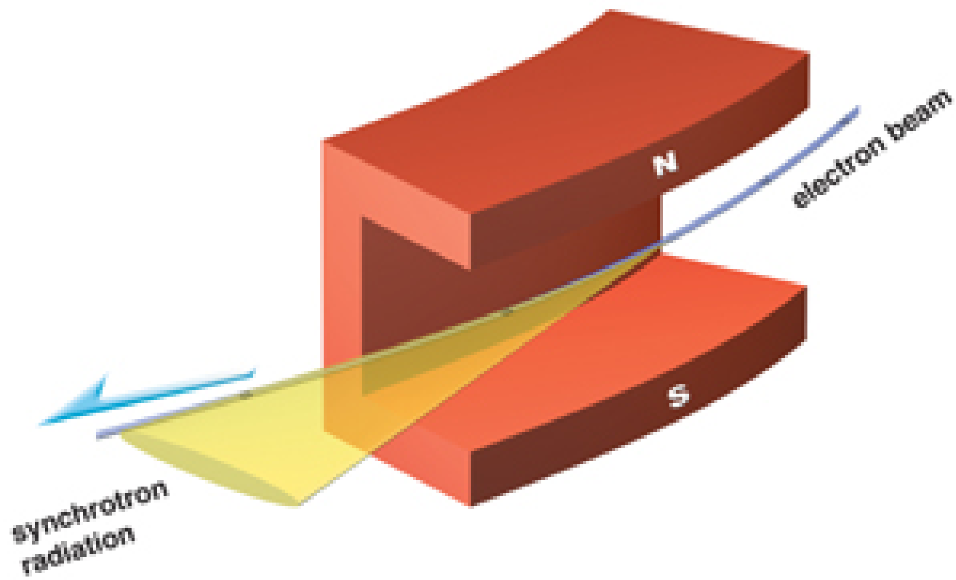
SR from Bending Magnets

A bending magnet or dipole has a uniform magnetic field

The electron travels on the arc of a circle of radius set by the magnetic field strength

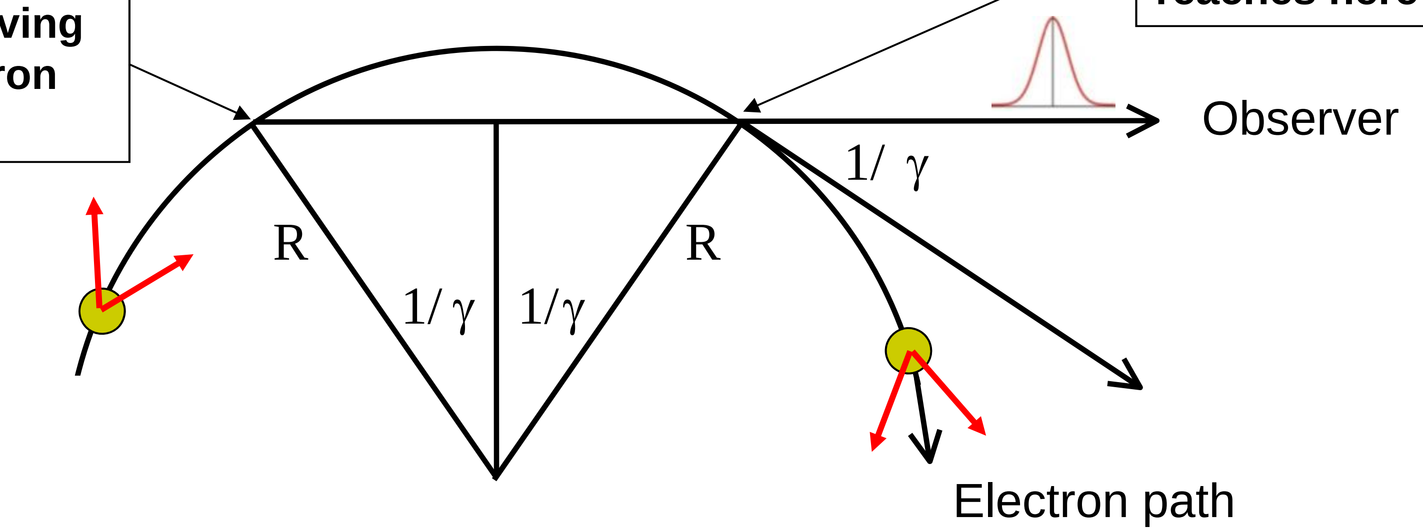
Horizontally the light beam sweeps out like a lighthouse - the intensity is flat with horizontal angle

Vertically it is in a narrow cone of typically $\pm 1/\gamma$ radians



SR from Bending Magnets

Viewed from above



The electrons in a bending magnet are accelerated as they are forced to bend along a circular path in a strong magnetic field.

Typical Wavelength

Pulse Duration = Time for electron - Time for photon

$$\approx \frac{4R}{3c\gamma^3}$$

So, "Typical Wavelength"

$$\approx \frac{4R}{3\gamma^3}$$

For DIAMOND,

R ~ 7.1 m, γ ~ 6000

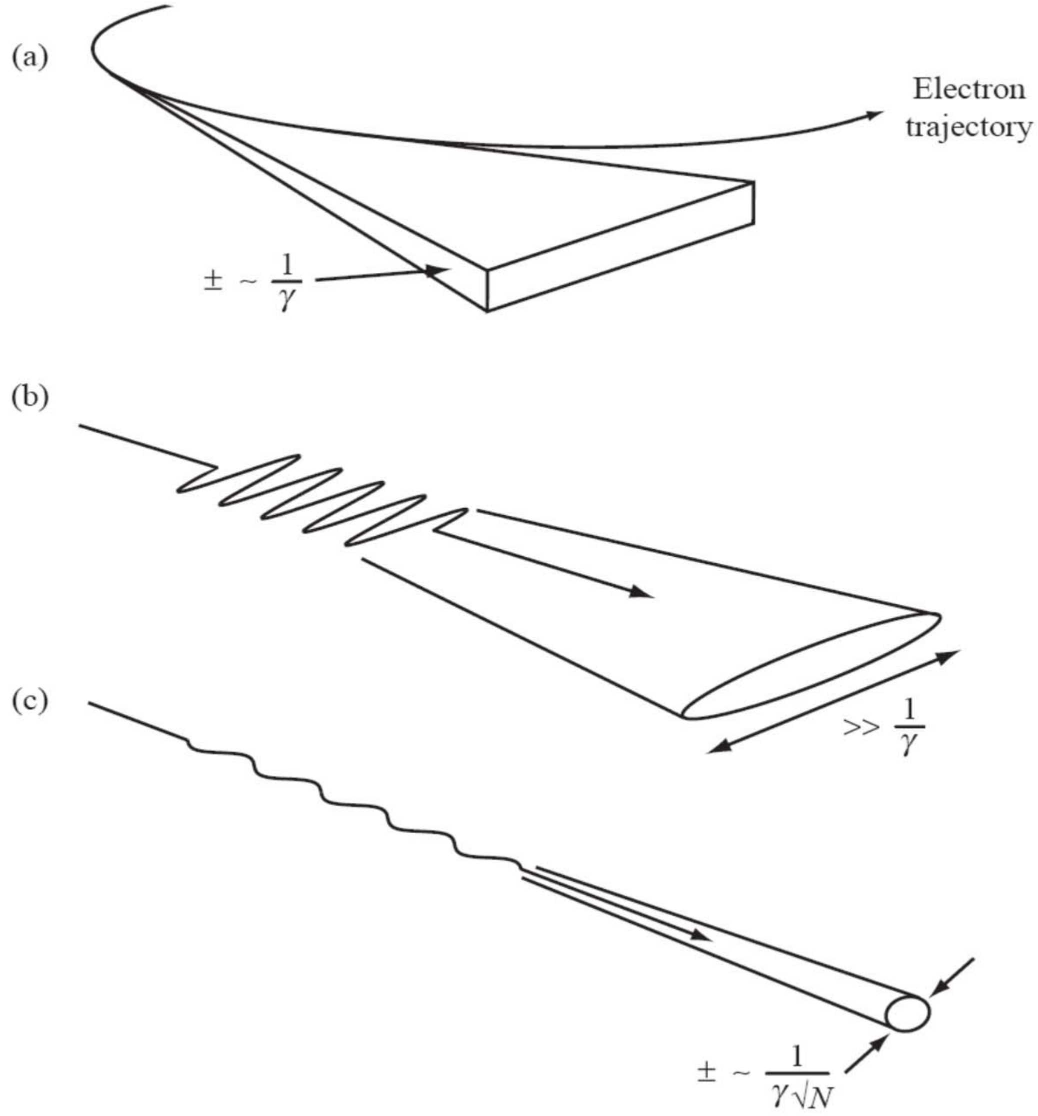
Wavelength ~ 0.04 nm

Summary of the Three Basic Sources

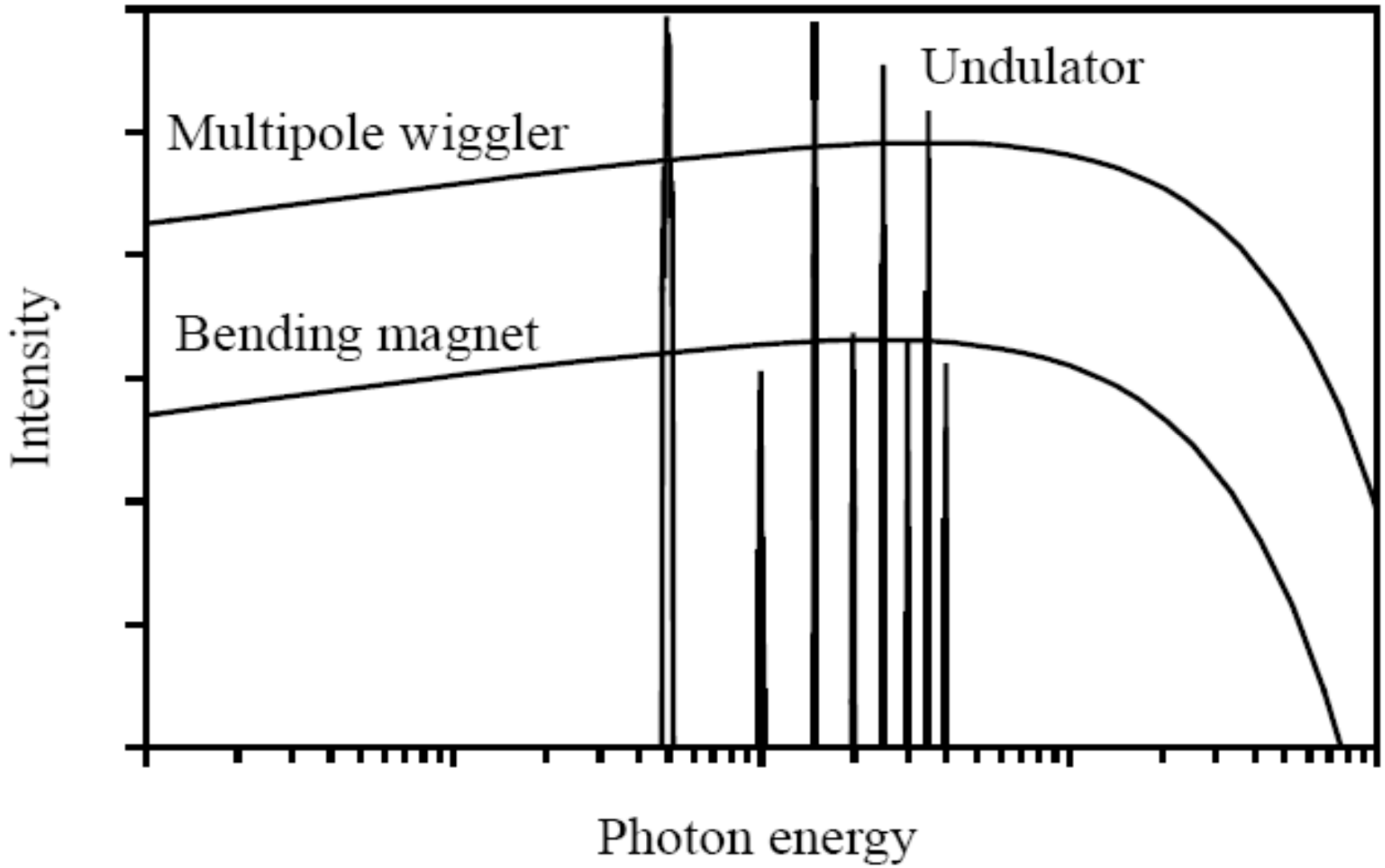
Bending magnet or Dipole

(Multipole) Wiggler

Undulator



A Typical Spectrum



Definition of SR

Synchrotron Radiation is electromagnetic radiation that is emitted by relativistic charged particles due to their acceleration.

The First Ever Recorded Observation

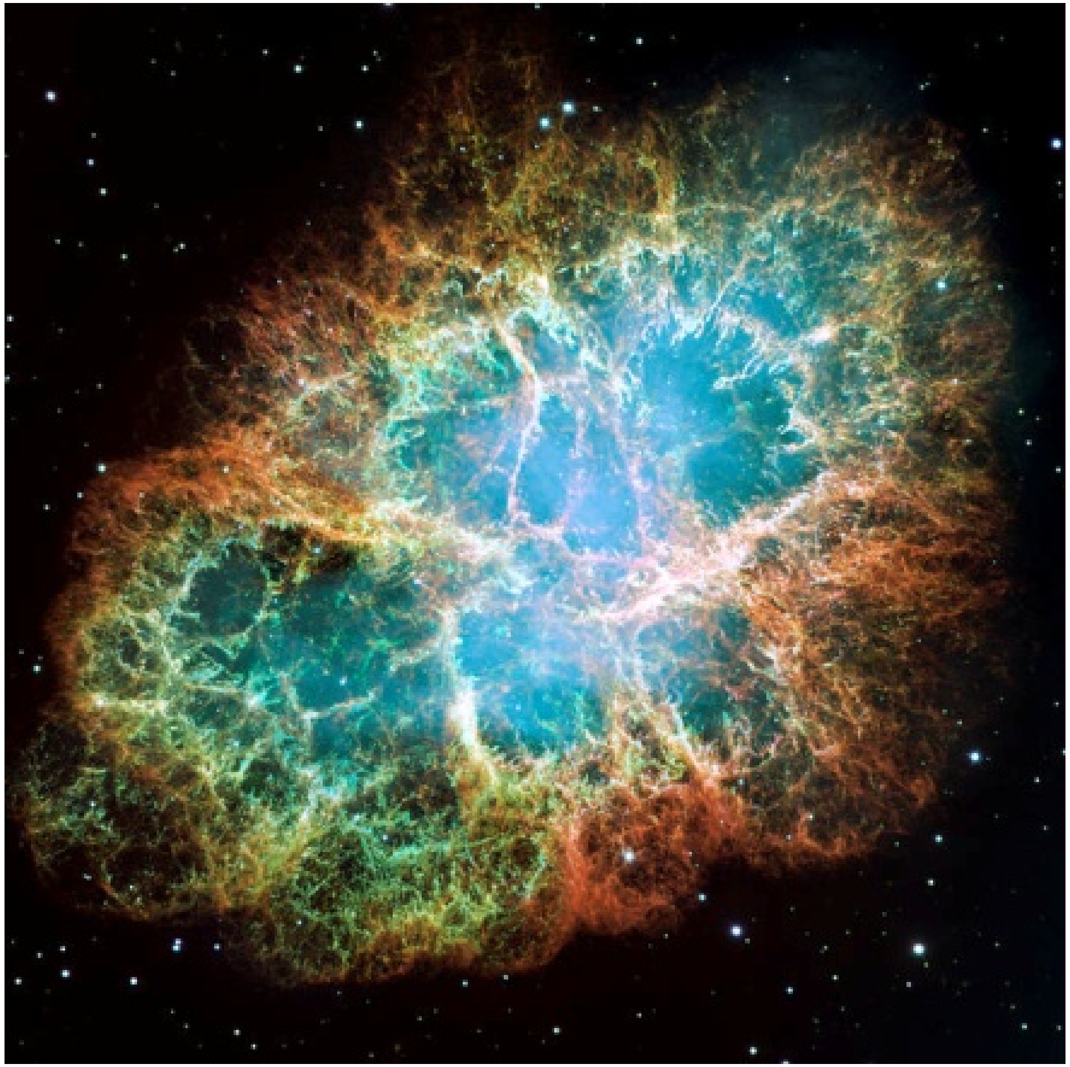


Image taken by the Hubble Space Telescope

The Crab nebula is the expanding remains of a star's supernova explosion that was observed by Chinese & Japanese astronomers in the year 1054 AD.

At the heart of the nebula is a rapidly-spinning neutron star, a pulsar, and it powers the strongly polarised bluish 'synchrotron' nebula.

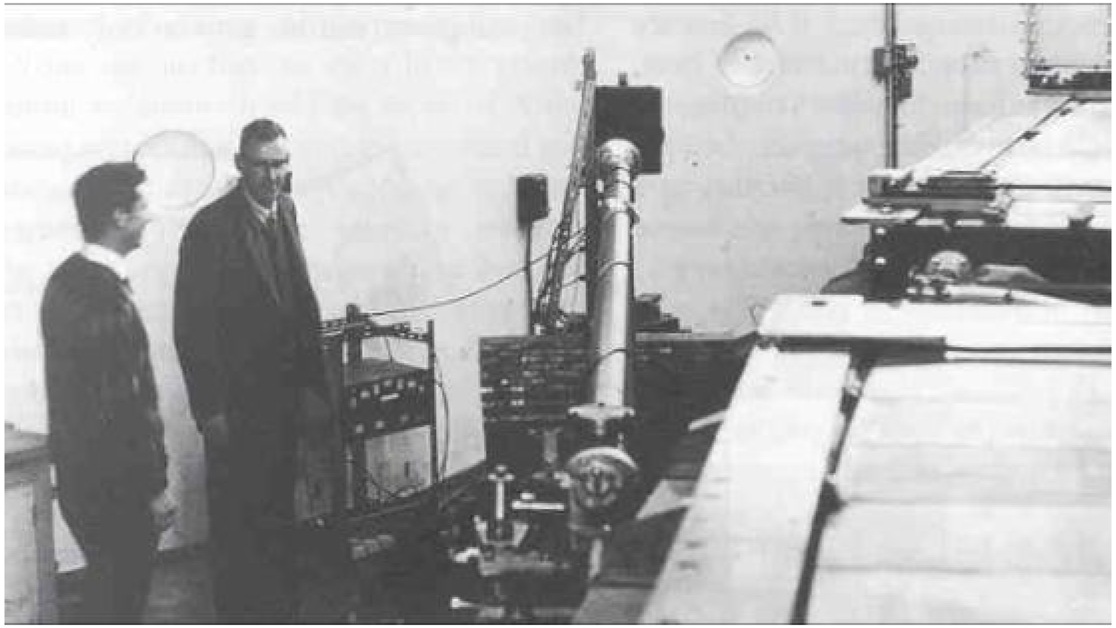
A Brief History of SR – First Use

1st Generation SR sources

Electron synchrotrons start to be built for high energy physics use (rapidly cycling accelerators not Storage Rings!)

There is interest from other physicists in using the “waste” SR

The first users are **parasitic**



The first beamline on NINA at Daresbury constructed in 1966/67 by Manchester University

NINA was a 5 GeV electron synchrotron devoted to particle physics

A Brief History of SR – Dedicated Facilities

2^{nd} Generation SR sources

Purpose built accelerators start to be built – late 70's

First users ~1980 (at SRS, Daresbury)

Based primarily upon **bending magnet radiation**

The VUV ring at Brookhaven in 1980 before the beamlines are fitted

Not much room for undulators!

A Brief History of SR – Enhanced Facilities

3rd Generation SR sources

Primary light source is now the undulator

First built in the late 80's/early 90's

First users ~1994

A Brief History of SR – The Next Generation

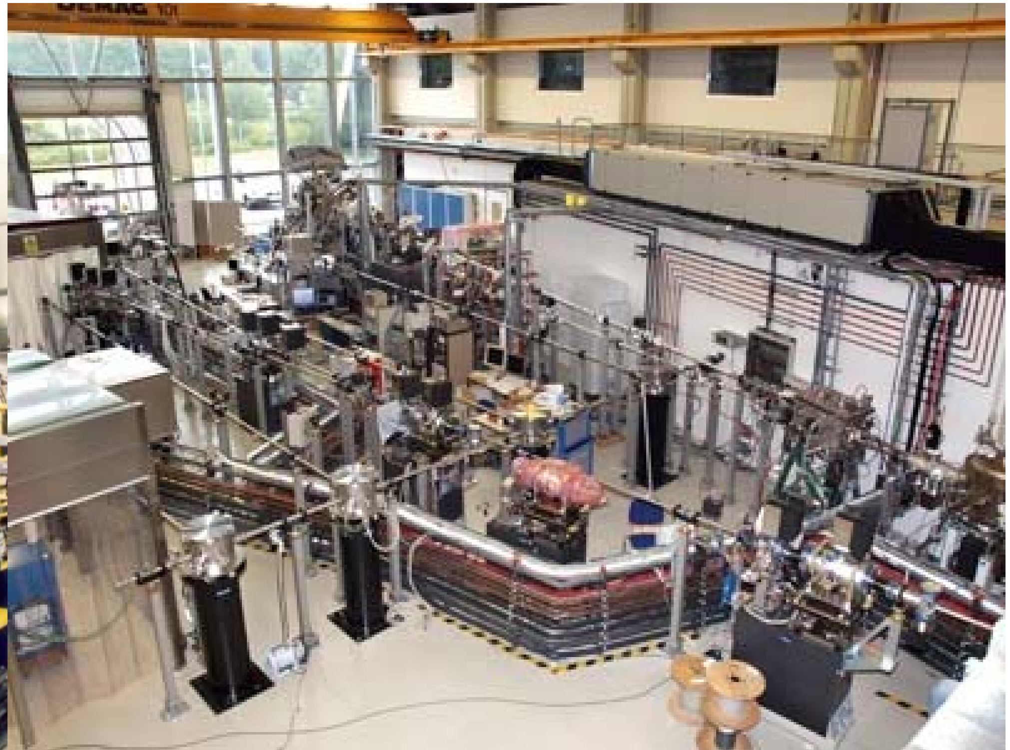
4th Generation SR sources

FELs are totally reliant upon undulators

Now the most advanced light source is the single pass **Free Electron Laser**

First built ~2000

First users ~2006



FLASH FEL facility at DESY, Germany

The Impact of X-Rays on Science

21 Nobel Prizes so far …

1901 Rontgen (Physics)

1914 von Laue (Physics)

1915 Bragg and Bragg (Physics)

1917 Barkla (Physics)

1924 Siegbahn (Physics)

1927 Compton (Physics)

1936 Debye (Chemistry)

1946 Muller (Medicine)

1962 Crick, Watson & Wilkins (Medicine)

1962 Perutz and Kendrew (Chemistry)

1964 Hodgkin (Chemistry)

1976 Lipscomb (Chemistry)

1979 Cormack Hounsfield (Medicine)

1981 Siegbahn (Physics)

1985 Hauptman and Karle (Chemistry)

1988 Deisenhofer, Huber & Michel (Chemistry)

1997 Boyer and **Walker** (Chemistry)

2003 Agre and Mackinnon (Chemistry)

2006 Kornberg (Chemistry)

2009 Yonath, Steitz & **Ramakrishnan** (Chemistry)

2012 Lefkowitz and Kobilka (Chemistry)

These last 5 all relied on SR, 2 of them used the SRS at Daresbury

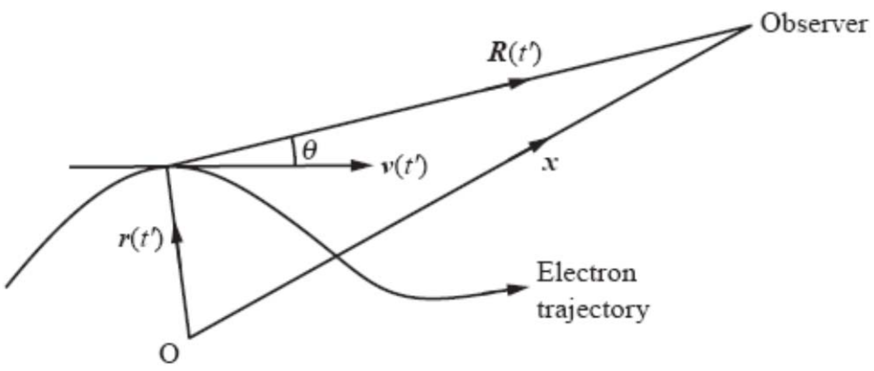
How to formally derive the properties of SR …

1. Consider the geometry of the system for a relativistic electron on an arbitrary trajectory
2. Relate the time of **emission** to the time of **observation**
3. We know the accelerating charge emits EM radiation
4. Use Maxwell's equations to derive the electric and magnetic fields of the EM radiation that the observer sees as a function of time
5. Convert this from time to frequency to predict the spectrum of EM radiation that is observed
6. Apply these general results to specific cases – bending magnets, wigglers and undulators

Electric Field at the Observer

The electron emits at time t' (retarded or emission time)

The photon (travelling at speed c) arrives at the observer at time t (observation time)



$$\boldsymbol{E}(t) = \frac{e}{4\pi c\epsilon_0}\left(\frac{c(1-\beta^2)(\boldsymbol{n}-\boldsymbol{\beta})}{R^2(1-\boldsymbol{n}\cdot\boldsymbol{\beta})^3} + \frac{\boldsymbol{n}\times((\boldsymbol{n}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}})}{R(1-\boldsymbol{n}\cdot\boldsymbol{\beta})^3}\right)_{t'}$$

Where **n** is the unit vector pointing along **R**(t')

When R is large we can ignore the first term

The Far Field Case

Ignoring the first term and assuming that **R** does not vary with time (d**n**/dt=0):

$$\boldsymbol{E}(t) = \frac{e}{4\pi c\epsilon_0}\left(\frac{\boldsymbol{n}\times((\boldsymbol{n}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}})}{R(1-\boldsymbol{n}\cdot\boldsymbol{\beta})^3}\right)_{t'}$$

Often see this in text books but remember this only holds in the *far field*. How far away is the *far field*?

Most SR calculations can (and do!) ignore the near field.

Fourier Transform of the Electric Field (Far Field)

Far field case of electron moving on arbitrary path:

$$\boldsymbol{E}(\omega) = \frac{ie\omega}{4\pi\sqrt{2\pi}\,c\epsilon_0 R}\int_{-\infty}^{\infty}(\boldsymbol{n}\times(\boldsymbol{n}\times\boldsymbol{\beta}))e^{i\omega(t'+\frac{R(t')}{c})}\,dt'$$

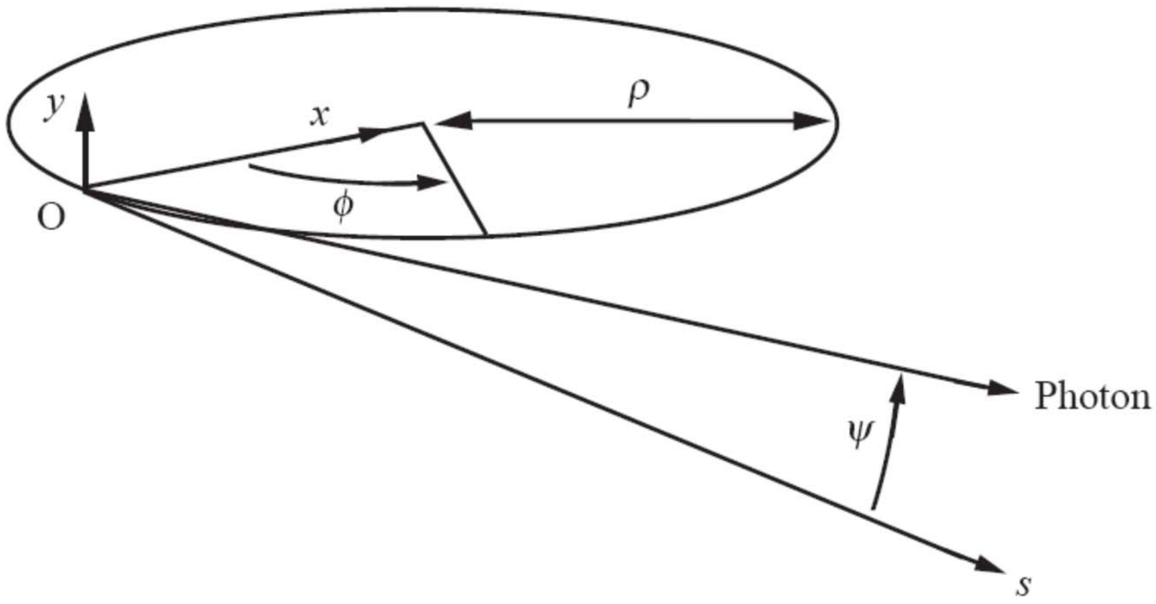
SR from a Bending Magnet

A bending magnet is a uniform dipole

The electron moves on purely circular path

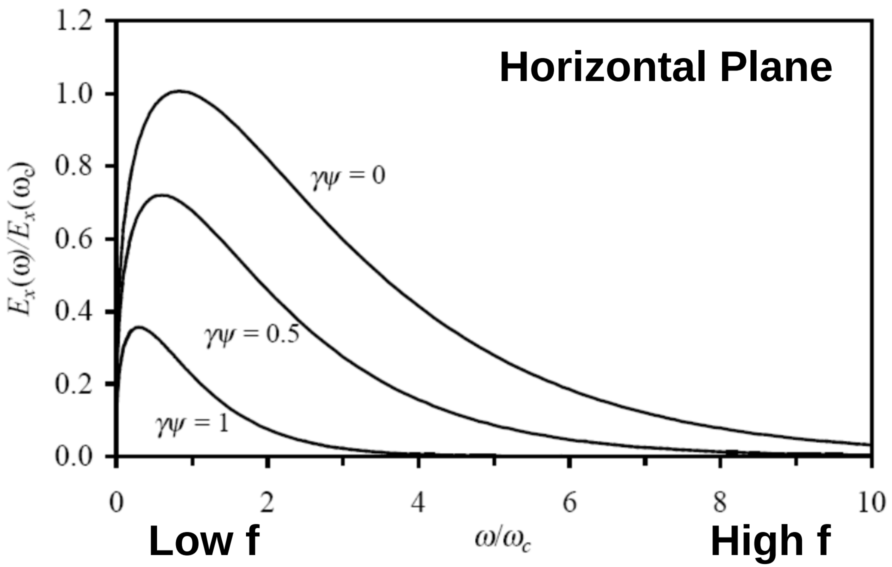
Angular velocity:

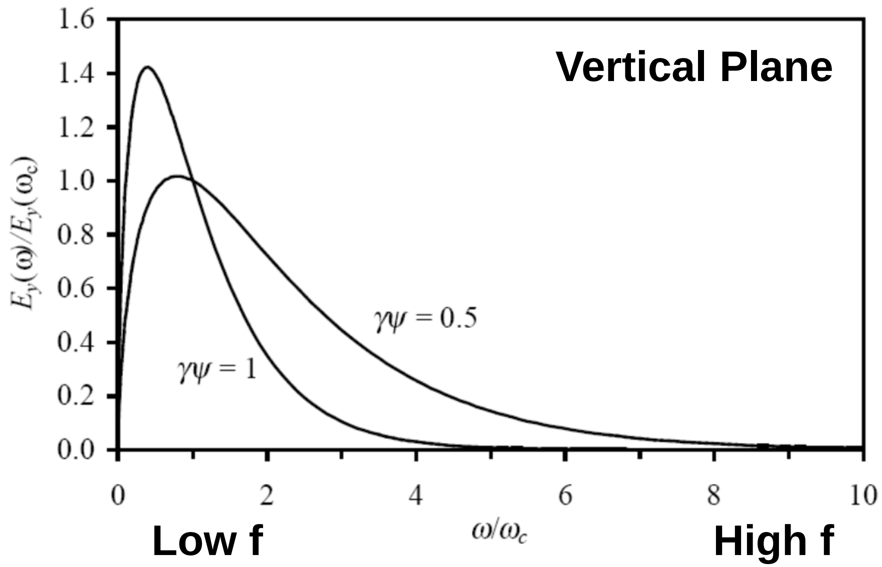
$$\omega_0 = \frac{\beta c}{\rho}$$



ρ is the bending radius

Electric Fields in the Horizontal (x) and Vertical (y)





$\gamma\psi$ is effectively the vertical angle of the radiation

On axis ($\gamma\psi = 0$) there is **zero** vertical Electric field

At larger angles, low frequencies (longer wavelengths) become more dominant, short wavelengths are no longer observed

Only E_x is observed on axis – *the light is polarised completely in the horizontal plane*

Critical Frequency in a Bending Magnet ω_c

If we integrate the power emitted from 0 to ω_c then we find that it contains **half the total power** emitted

$$\boxed{\omega_c = 3c\gamma^3/2\rho}$$

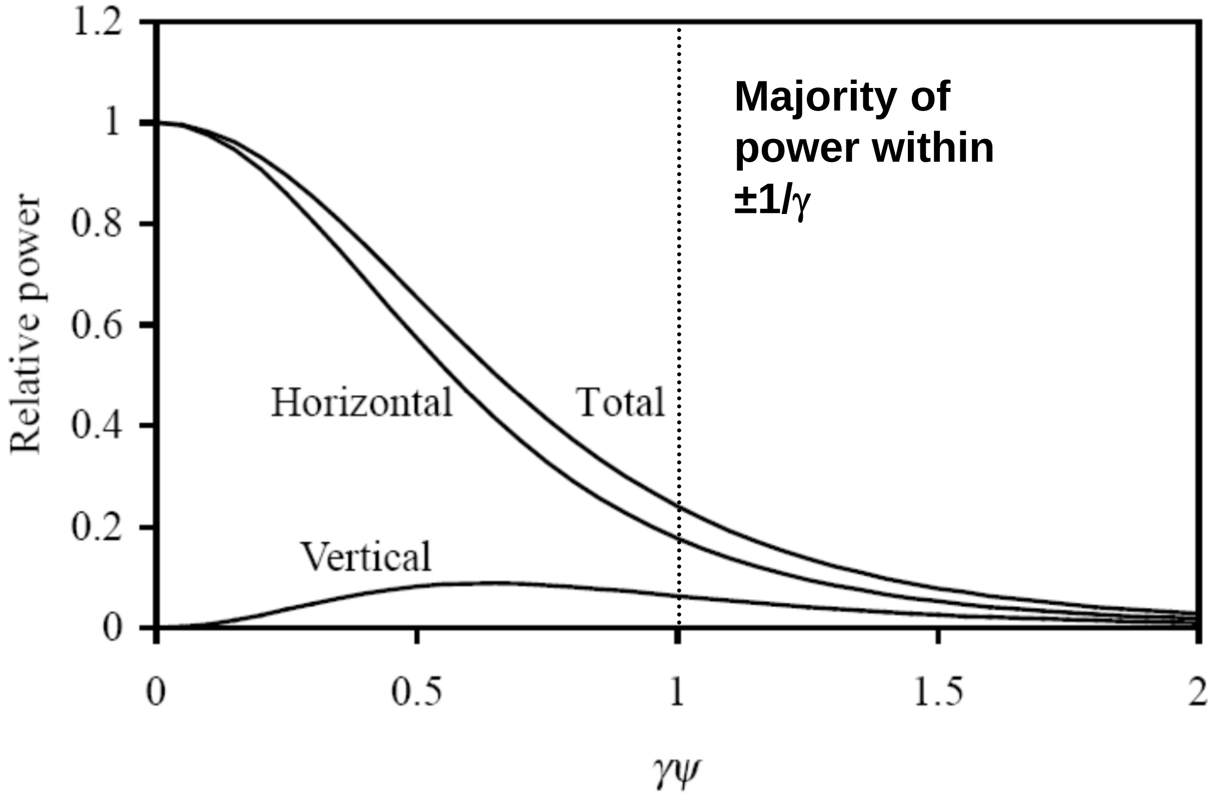
In other words, ω_c splits the power spectrum for a bending magnet into two equal halves

It is a useful parameter, and it can be used to ***compare*** bending magnet sources

Expressed as a wavelength or a photon energy:

$$\boxed{\lambda_c = 2\pi c/\omega_c} \qquad \boxed{\epsilon_c = \hbar\omega_c = \frac{3hc\gamma^3}{4\pi\rho}}$$

Vertical Angular Power Distribution



The maximum power is emitted on axis

The power is symmetrical with vertical angle

There is no vertically polarised power on axis

The On Axis Spectral Angular Flux Density

$$\left.\frac{d\dot{N}}{d\Omega}\right|_{\psi=0} = 1.33 \times 10^{13}\, E^2 I_b \left(\frac{\omega}{\omega_c}\right)^2 K_{2/3}^2\left(\frac{\omega}{2\omega_c}\right)$$

In units of photons/s/mrad^2/0.1% bandwidth

Monochromator bandwidth

E is the electron energy in GeV, I_b is the beam current in A

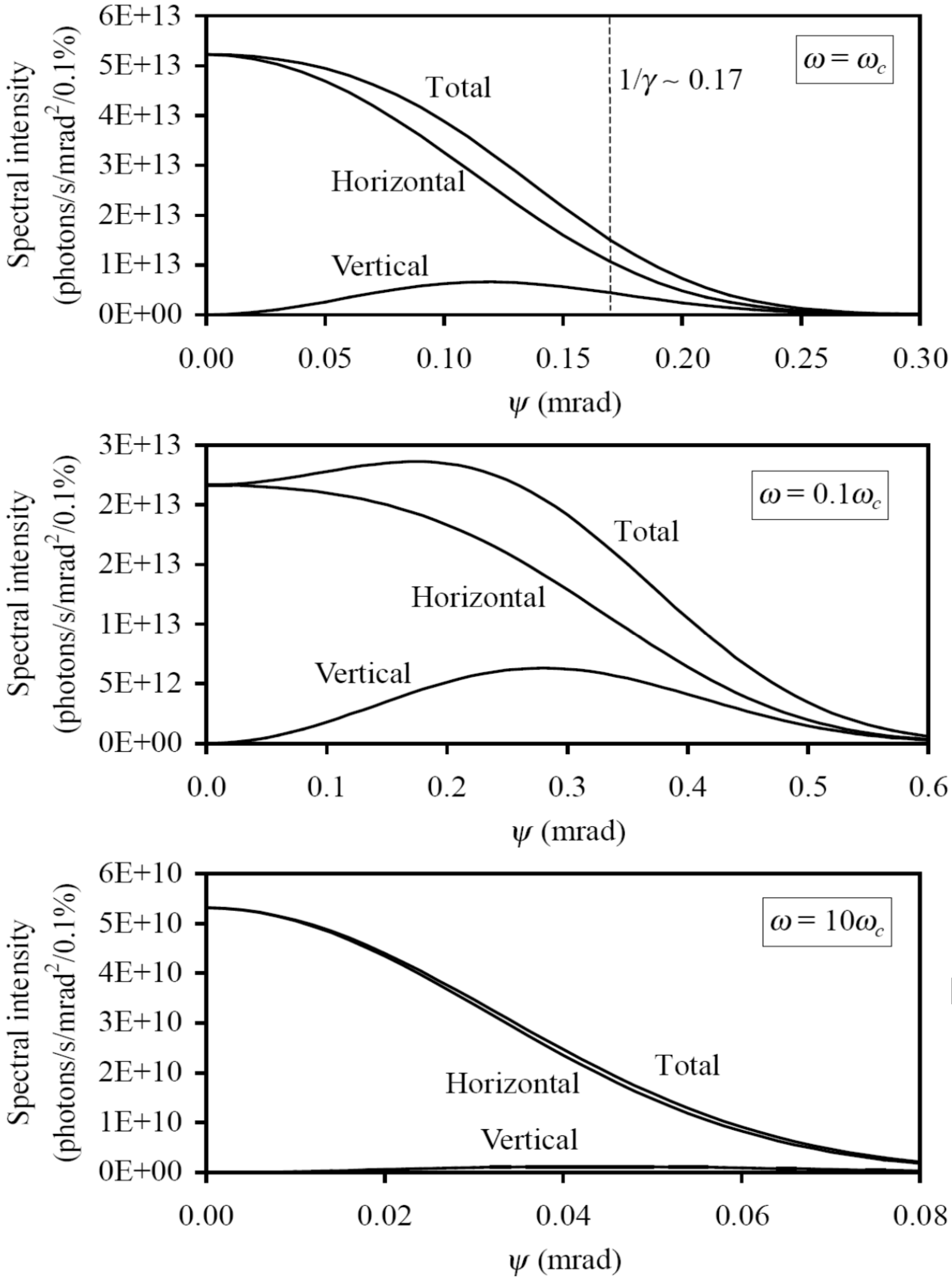
$K_{2/3}$ is a “modified” Bessel function

Example

Angular Flux Density

3 GeV
300 mA
1.4 T Dipole

Note the change of scales!



Low f

High f

Photon Flux

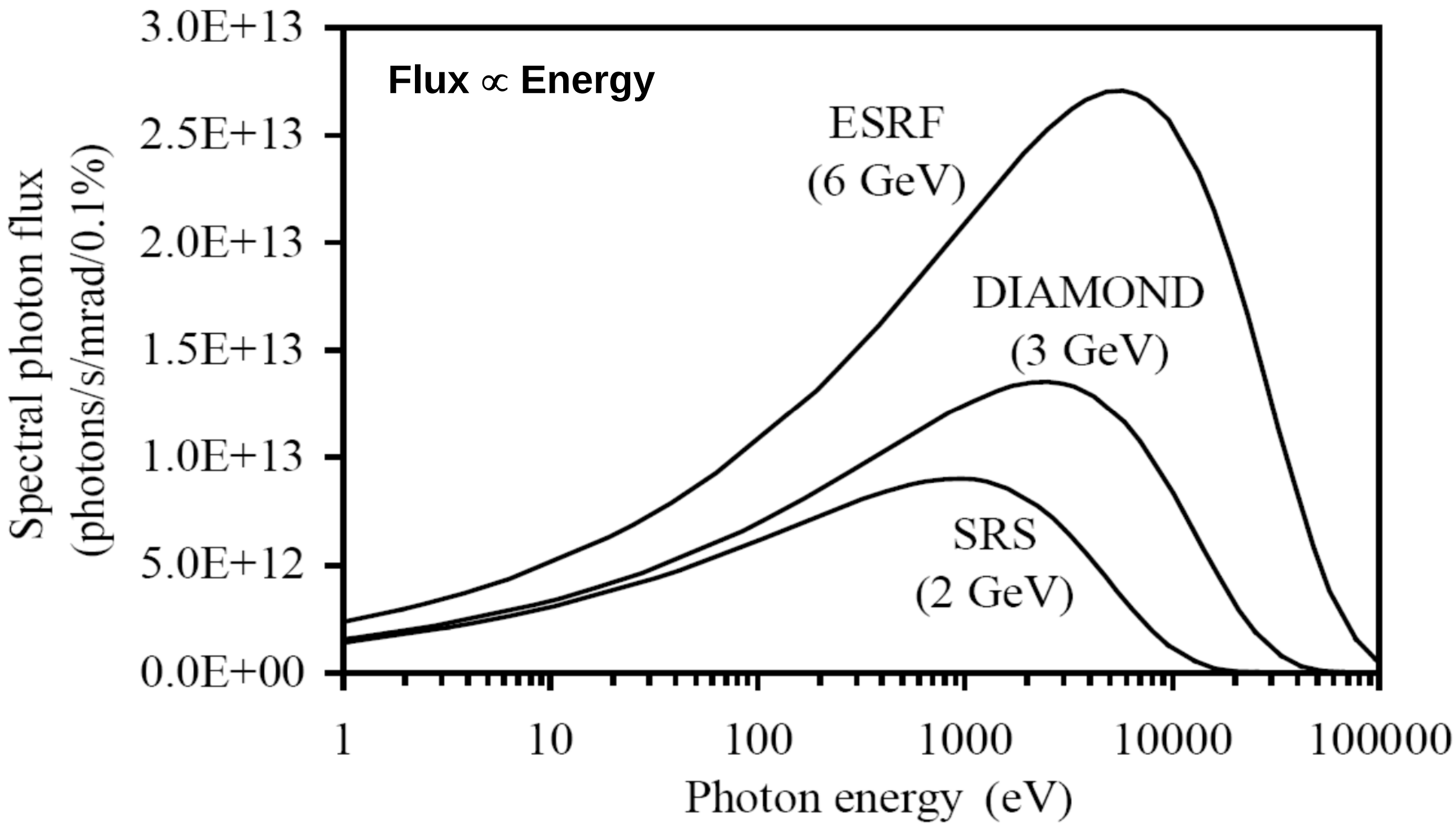
The **spectral photon flux** or the **vertically integrated spectral flux** is given by

$$\dot{N} = 2.46 \times 10^{13} \, E I_b \left(\frac{\epsilon}{\epsilon_c}\right) \int_{\epsilon/\epsilon_c}^{\infty} K_{5/3}(u) \, du$$

in units of photons/s/mrad horizontally/0.1% bandwidth

Examples for Photon Flux

log-linear scale, 200mA beam current assumed for all sources



Power

Virtually all SR facilities have melted vacuum chambers or other components due to the SR hitting an uncooled surface

The average power is high but the **power density is very high** – the power is concentrated in a tight beam. The total power emitted by an electron beam in 360° of bending magnets is

$$P_{\text{total}} = 88.46\,\frac{E^4 I_b}{\rho_0}$$

where the power is in kW, E is in GeV, I_b is in A, ρ_0 is in m.

Other useful values are the power per horizontal angle (in W/mrad) and power density on axis (in W/mrad^2)

$$\frac{dP}{d\theta} = 14.08\,\frac{E^4 I_b}{\rho_0} \qquad \left.\frac{dP}{d\Omega}\right|_{\psi=0} = 18.08\,\frac{E^5 I_b}{\rho_0}$$

Examples

Ring	Energy (GeV)	ρ (m)	I_b (mA)	P_{total} (kW)	$dP/d\theta$ (W/mrad)	$dP/d\Omega$ (W/mrad^2)
SRS	2	5.56	200	50.9	8.1	20.8
DIAMOND	3	7.15	300	300.7	47.9	184.4
ESRF	6	25.0	200	916.5	145.9	1124.0

All this power has to come from the RF system – very expensive!

Bending Magnet Spectrum

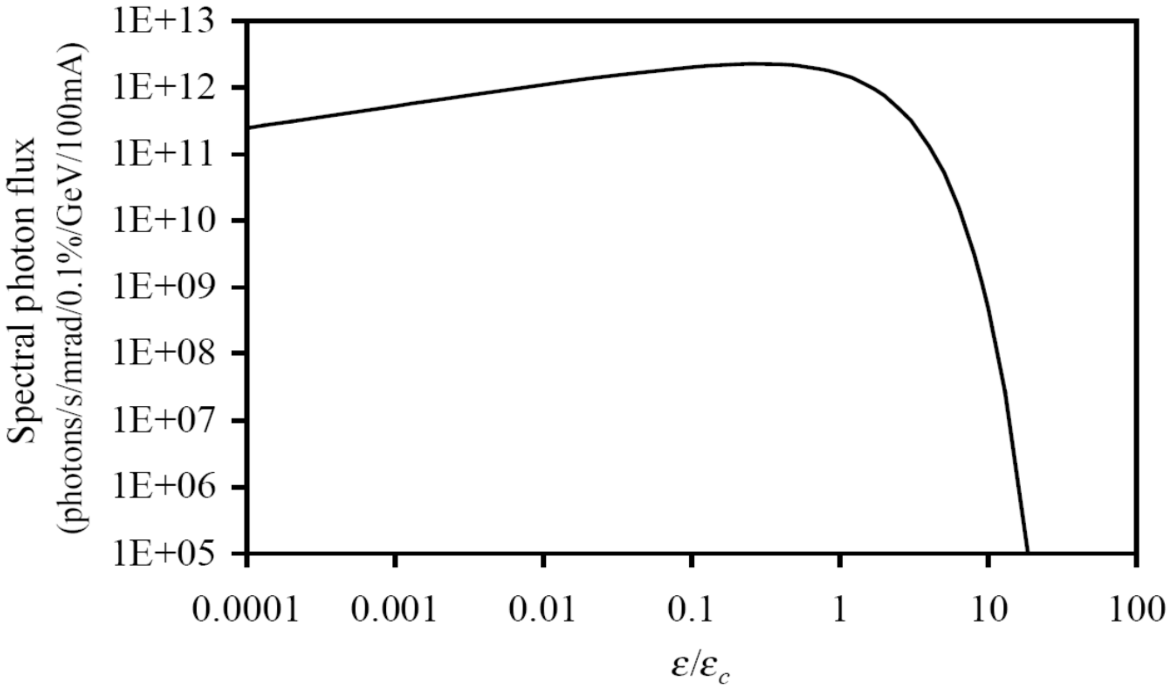
A plot of $\dot{N}$ versus ϵ/ϵ_c gives the **universal curve**

All flux plots from bending magnets have the same characteristic shape

Once the **critical energy is known** it is easy to find (scale off) the photon flux

The amplitude changes with E and I_b so higher energies and higher beam currents give more flux

Note the **log-log** scale



Bending Magnet Spectrum

In a storage ring of fixed energy, the spectrum can be shifted sideways along the photon energy axis if a different critical energy can be generated.

$$\epsilon_c = \hbar\omega_c = \frac{3hc\gamma^3}{4\pi\rho}$$

Need to change ρ (B Field)

Used especially to shift the rapidly falling edge (high energy photons, short wavelengths)

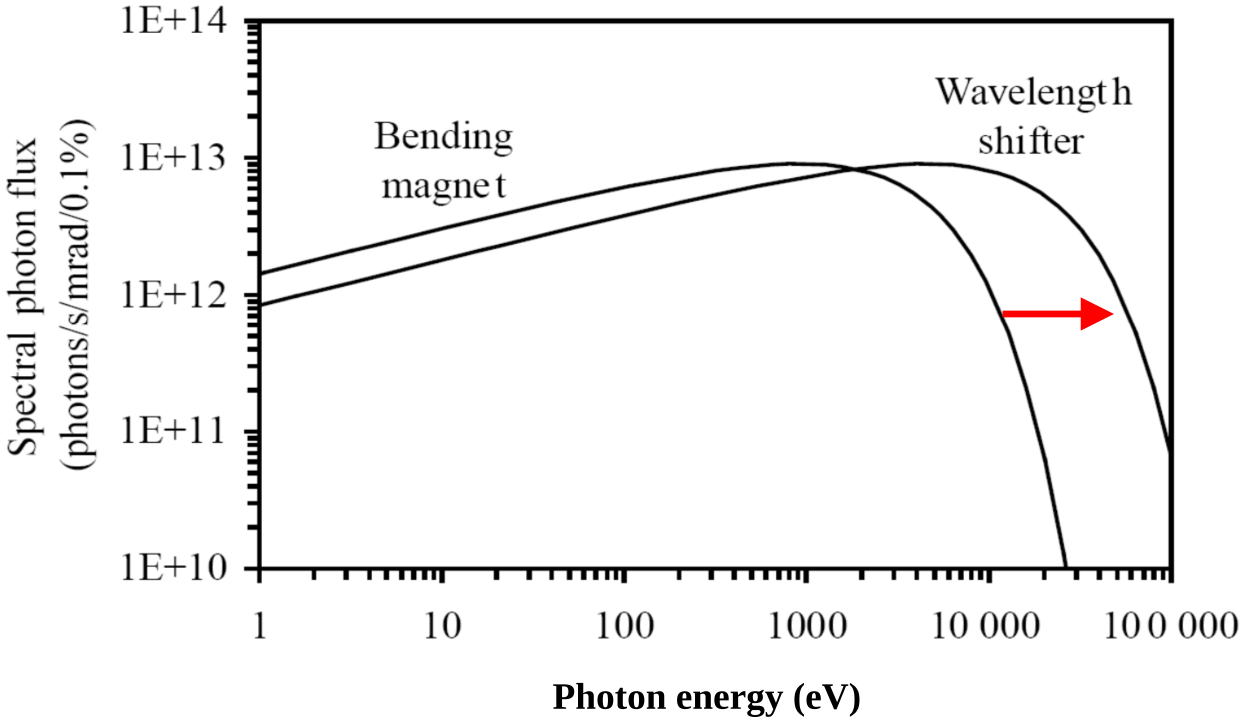
Special magnets that do this are called **wavelength shifters**

An alternative is to replace individual bending magnets with higher strength ones (superbends) – this is not so popular but it has been done

Wavelength Shifters

Shift the critical energy by locally changing the bending magnet field

The shape of the curve is unchanged but the spectrum is shifted



SRS Example

1.2 T BM & 6 T WS.

2 GeV, 200 mA.

Flux at 30 keV increased by x100

Summary

Synchrotron Radiation is emitted by accelerated charged particles

The combination of Lorentz contraction and the Doppler shift turns the **cm length scale into nm wavelengths** (making SR the best possible source of X-rays)

Apply Maxwell's equations to the particle, taking care to relate the emitted time to the observed time

Bending magnet radiation is characterised by a critical frequency

The power levels emitted can be quite extreme in terms of the total power and also the power density

Wavelength shifters are used to 'shift' the spectrum so that shorter wavelengths are generated