INTERNATIONAL WORKSHOP ON FFAG ACCELERATORS (FFAG'11)
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FFAG SCHOOL

FFAG OPTICS

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FFAG OPTICS

SCALING FFAGs
1. Principles - scaling
2. Transverse optics
3. Longitudinal optics
4. Quasi-scaling FFAGs

NON-SCALING FFAGs
1. Linear Non-Scaling LNS-FFAGs
2. Johnstone Tune-stabilized NLNS-FFAGs
3. Rees Pumplet NLNS-FFAGs
**FFAGs – Fixed Field Alternating Gradient accelerators**

**Fixed Magnetic Field** - members of the *CYCLOTRON* family

<table>
<thead>
<tr>
<th>Magnetic field variation $B(\theta)$</th>
<th>Fixed Frequency (CW beam)</th>
<th>Frequency-modulated (Pulsed beam)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform</td>
<td>Classical</td>
<td>Synchro-</td>
</tr>
<tr>
<td>Alternating</td>
<td>Isochronous</td>
<td>FFAG</td>
</tr>
</tbody>
</table>

But FFAG enthusiasts sometimes express an alternative view: - cyclotrons are just special cases of the FFAG!

THE FFAG IDEA

- was to introduce alternating “strong” focusing to fixed-field accelerators (enabling higher rep rates and beam currents than in synchrotrons)
- either by alternating +ve and –ve bending magnets with radial edges, creating Alternating Gradient focusing (Ohkawa, Kolomensky, Symon, 1953-4)
- or by using spiral sector magnets (Kerst 1955) - as later used in cyclotrons.

BASIC CHARACTERISTICS OF FFAGs

are determined by their **FIXED MAGNETIC FIELD**

- Spiral orbits
  - needing **wider magnets, rf cavities and vacuum chambers** (compared to AG synchrotrons)
- Faster rep rates (up to kHz?) limited only by rf capabilities
  - not by magnet power supplies
- Large acceptances
- High beam current

The last 3 factors have fuelled the interest in FFAGs over 50 years!

The most intensive studies were carried out by Symon, Kerst, et al. at the **Mid-west Universities Research Association (MURA)** in the 1950s and 60s - who adopted the “**scaling**” principle
  - and built several successful electron models.
SCALING DESIGNS

Betatron resonances were a big worry in early days, because of low $\Delta E$/turn:

So “Scaling” designs were used, with:

- the same orbit shape at all energies
- the same optics
- the same tunes

$\Rightarrow$ no crossing of resonances!

To $1^{st}$ order, the tunes are given by

$$v_r^2 \approx 1 + k \quad v_z^2 \approx -k + F^2 (1 + 2 \tan^2 \varepsilon)$$

So constant high tune values require:

- constant average field index
  \[ k(r) \equiv \frac{r}{B_{av}} \frac{dB_{av}}{dr} \gg 0 \]
  where $B_{av} = \langle B(\Theta) \rangle$

  (and hence $B_{av} = B_0 \left( \frac{r}{r_0} \right)^k$ and $p = p_0 \left( \frac{r}{r_0} \right)^{(k+1)}$)

- constant magnetic flutter
  \[ F^2 = \left\langle \left( B - B_{av} \right)^2 \right\rangle / B_{av}^2 \]
  i.e. constant profile $B(\Theta)/B_{av}$
  - maximized for radial sectors by choosing $B_D = -B_F$

- constant spiral angle $\varepsilon$ (sector axis follows $R = R_0 e^{\Theta \cot \varepsilon}$)
MURA Electron FFAGs

- 400keV radial sector
- 50 MeV radial sector
- 120 keV spiral sector

In spite of the success of the electron models, none of MURA’s proposals for proton FFAGs (0.5, 10, 15, and 20 GeV) were funded. Nor were proposals for 1.5-GeV x 4-mA spallation neutron sources by Argonne and Jülich in the 1980s. The first proton FFAGs were Mori’s at KEK (1 MeV 2000, 150 MeV 2003).

Since 2000 an explosion of interest!

- 6 more now operating (for p, e, α) and 3 more (e) being built
- ~15 designs under study:
  - for protons, heavy ions, electrons and muons
  - many of novel “non-scaling” design
- with diverse applications:
  - cancer therapy
  - industrial irradiation
  - driving subcritical reactors
  - intense many-GeV proton beams
  - producing neutrinos.

KEK Proof-of-Principle 1-MeV proton FFAG
FFAG Complex at Kyoto University Research Reactor Inst.

The World’s first test of Accelerator-Driven Sub-critical Reactor (ADSR) operation was performed in March 2009.
The tune advances (denoted by $\nu_x$, $\nu_y$) and beta- and dispersion functions in a DFD cell of the KEK 150-MeV radial-sector FFAG.
These simulation results for the 150-MeV FFAG demonstrate the large acceptances (areas of stable motion) available in phase space — in both planes.

(Note the different displacement scales in the two planes.)
OBSERVED TUNES

In practice, it’s difficult to build magnets with identical field index $k$ at all radii, to give $B_z \propto r^k$, especially where $k \gg 1$.

The plots below show measured tunes for the KURRI 150-MeV FFAG, for which $k = 7.6$. (The left plot shows only the decimal part of the tune.) The tune diagram shows that 1$^\text{st}$ and 2$^\text{nd}$ order resonances are avoided, but that two 3$^\text{rd}$-order resonances are crossed.
RECAP – OFF-MOMENTUM ORBIT PARAMETERS

Momentum dispersion: \[
\eta_x(s) \equiv \frac{x(s)}{\delta p / p_0}.
\]

Momentum compaction: \[
\alpha \equiv \frac{\delta C / C}{\delta p / p} = \frac{\eta_x}{R} = \frac{1}{\gamma_t^2}
\]

Slip factor: \[
\eta = \frac{\delta \omega / \omega}{\delta p / p} = \frac{1}{\gamma^2} - \alpha = \frac{1}{\gamma^2} - \frac{1}{\gamma_t^2}
\]

Transition energy: \[
\gamma_t = \frac{1}{\sqrt{\alpha}} = \sqrt{\frac{R}{\eta_x}}.
\]

N.B. Some authors define \( \eta \) with the opposite sign.
LONGITUDINAL PARAMETERS FOR SCALING FFAGS

In a scaling FFAG, like a synchrocyclotron, the field \( B \) is constant, but instead of being uniform, it increases with radius:

\[
B \propto r^k \quad \text{and} \quad p \propto r^{k+1} \quad (k \gg 1).
\]

As the momentum \( p = qBr \), the momentum compaction equation gives:

\[
\frac{1}{\gamma_t^2} = \alpha \equiv \frac{\delta C}{C} \frac{\delta p}{p} = \frac{1}{k+1}
\]

and \( \gamma_t = \sqrt{(k+1)} \).

As in AG synchrotrons, the transition energy \( \gamma_t \) may lie within the acceleration range, and depending whether \( \gamma < \gamma_t > \gamma_t \), the slip factor \( \eta \) is given by:

\[
\eta \equiv \frac{\delta \omega}{\omega} \frac{\delta p}{p} = \frac{1}{\gamma_t^2} - \frac{1}{k+1}
\]

\[
= \frac{k}{k+1} - \beta^2 \quad > / < 0.
\]

Thus the orbital (and hence rf) frequencies at first rise with energy, reaching a peak at transition, \( \gamma_t \), but then fall away.
FREQUENCY VARIATION IN SCALING FFAGs

The explicit form of $\omega(E)$ can be obtained by integrating

$$\frac{d\omega}{\omega} = \eta \frac{\delta p}{p} = \eta \frac{dE}{\beta^2 E}$$

$$= \left[ \left( \frac{k}{k+1} \right) \frac{1}{\beta^2} - 1 \right] \frac{dE}{E},$$

giving:

$$\omega(\gamma) = \frac{\omega_B (\gamma^2 - 1)^{k/2(k+1)}}{\gamma}$$

where $\omega_B$ is an integration constant set by the field $B$.

The lower curve shows how $d\omega/dE$ changes sign at the transition energy $\gamma_t$.

(For $k < -1$ $\gamma_t$ is imaginary.)

[Figure adapted from Kolomensky & Lebedev, Theory of Cyclic Accelerators, p. 351.]
**THE SYNCHRONOUS PHASE & PHASE STABILITY**

The condition for synchronism between rf and revolution frequency in a magnetic field $B$ at energy $\gamma m_0c^2$: $\omega_{rf}(t) = \frac{hqB}{\gamma m_0(t)}$
can only be maintained during acceleration for ions at the particular “synchronous phase” $\phi_s$ for which the energy gain/turn $qV_0\sin \phi_s$

exactly matches the frequency change.

On average, the energy gain per unit path length is:

$$\frac{qV_0 \sin \phi_s}{2\pi r} = \frac{dE}{ds} = \frac{dE}{\nu dt} = \frac{E \beta^2}{\omega \eta \nu dt}.$$

[Here $V_0$ denotes the total voltage/turn, and $\phi = 0$ where $V = 0$.]

The synchronous phase is therefore defined by:

$$qV_0 \sin \phi_s = \frac{2\pi E \beta^2}{\omega^2 \eta} \frac{d\omega}{dt} = \frac{2\pi E}{\omega^2 \eta} \left( \frac{\gamma^2 - 1}{1 - \gamma^2 / (k+1)} \right) \frac{d\omega}{dt}.$$

(Note that $\eta$ and $d\omega/dt$ always have the same sign, so $qV_0\sin \phi_s > 0$.)

But do ions at neighbouring phases oscillate stably about $\phi_s$ - or not? Veksler and McMillan showed that only certain ranges of $\phi_s$ are stable.
LONGITUDINAL EQUATIONS OF MOTION

For particles differing from the synchronous one by $\Delta T$, $\Delta p$, $\Delta \omega$, etc., the rate of change of the phase:

$$\frac{d\phi}{dt} = \frac{\Delta \phi}{\tau} = -\frac{h\Delta \omega \tau}{\tau} = -h \eta \omega_0 \frac{\Delta p}{p_0} = h \eta \omega_0 \frac{\Delta T}{mv^2} = -h \eta R^2 \gamma \left( \frac{\Delta T}{\omega_0} \right).$$

Comparing the energy gain/turn

$$\frac{2\pi}{\omega} \frac{dT}{dt} = qV_0 \sin \phi$$

with that for the synchronous particle, we also find:

$$\frac{d}{dt} \left( \frac{\Delta T}{\omega_0} \right) = \frac{qV_0}{2\pi} (\sin \phi - \sin \phi_s).$$

Noting that the “angular momentum” coordinate canonically conjugate to the phase angle is

$$W \equiv \frac{\Delta T}{\omega_0} = R \Delta p$$

we see that the equations above have the form of Hamilton’s equations:

$$\frac{d\phi}{dt} = \frac{\partial H}{\partial W}$$
and

$$\frac{dW}{dt} = -\frac{\partial H}{\partial \phi}.$$

with the Hamiltonian:

$$H(\phi, W) = -\frac{h \eta}{2m_0 R^2 \gamma} W^2 + \frac{qV_0}{2\pi} \left[ \cos \phi - \cos \phi_s + (\phi - \phi_s) \sin \phi_s \right].$$
Curves of constant $H$ in the $\phi-W$ plane represent particle trajectories. Note the parabolic variation of $H$ around $W = 0$ for fixed $\phi$. 

$$H(\phi, W) = -\frac{h \eta}{2m_0 R_s^2 \gamma} W^2 + \frac{qV_0}{2\pi} \left[ \cos \phi - \cos \phi_s + (\phi - \phi_s) \sin \phi_s \right]$$

The synchronous points are surrounded by regions of stable motion - "buckets" - bounded by pear-shaped separatrices - outside which the motion is unstable.
The area is greatest for \( \phi_s = 0 \) or \( \pi \), and drops to zero for \( \phi_s = \pi/2 \).

\[ \therefore \text{choosing } \phi_s \text{ is a design compromise between } \phi_s \approx \pi/2 \text{ with maximum acceleration but zero acceptance, and } \phi_s \approx 0 \text{ or } \pi \text{ with the largest buckets but zero acceleration.} \]

Popular choices, giving reasonable acceleration and bucket area, are:

\[ \phi_s \approx 30^\circ \text{ or } 150^\circ. \]

Note \( W = \Delta T/\omega_0 \propto \Delta T, \text{ not } T \) - the buckets are centred on the synchronous particle - imagine them rising with \( T_s \) as it rises.

Also \( W-\phi \text{ area is conserved by Liouville's theorem - but not } \Delta T-\phi \).
In very-high-energy FFAGs - such as those proposed as muon accelerators for a Neutrino Factory at J-PARC (shown above) - a particle's speed, radius - and therefore orbit time - change very little, so it's possible to consider operating at fixed rf frequency - a great simplification. Two methods have been proposed............
FIXED-FREQUENCY I: STATIONARY BUCKET

Here a large rf bucket is created, spanning the whole energy range. 
(At fixed frequency in fixed magnetic field, it’s fixed in energy.)

As bucket height $\propto \sqrt{\text{voltage gain/turn}}$, large rf voltages are needed - but for muons ($\tau = 2.2\ \mu s$) these are essential anyway for rapid acceleration and survival.

The diagrams (Planche, Mori et al. IPAC'10) refer to a 3.6-12.6 GeV muon FFAG with 1.8 GV/turn at 200 MHz (h=675), giving 6-turn acceleration. 
(Left - Right): Longitudinal, horizontal and vertical phase space.

Initial and final transverse emittances
**FIXED-FREQUENCY II: HARMONIC NUMBER JUMP**

HNJ is a technique where the harmonic number $h (\equiv \tau_{\text{orbit}}/\tau_{\text{rf}})$ is increased by an integer on each turn (often $\Delta h = 1$) - originally devised by Veksler (1944) for electrons in a fixed-frequency microtron, so that as the orbit radius increased, $\tau_{\text{orbit}}$ remained a harmonic of the fixed $\tau_{\text{rf}}$.

Planche, Mori et al. have also simulated this method for a 3.6-12.6 GeV muon FFAG with 2.1 GV/turn at 400 MHz, giving 8.5-turn acceleration. The practical challenge is to provide the correct energy gain each turn.

*Initial and final transverse emittances*
QUASI-SCALING FFAG: PAMELA (Adams Inst., Oxford)

31–250 MeV protons
- for cancer therapy
- 12-cell FDF
- radius $\approx 6.25$ m
- 4-T magnets

Machida "semi-scaling" lattice
- High field index $k$ (i.e. $B \sim r^k$) for small orbit excursions
- approximate $r^k$ locally by $\Sigma b_n x^n$
  with $n = 0, 1, 2, 3$ only
- flat tunes, good dynamic aperture

400-MeV/u $C^+$ ions require a similar 2nd stage (radius 9.2 m).
The small ring delivers 31-250 MeV protons
The large ring delivers 68-400 MeV/u $C^{6+}$ ions
Superconducting 2-, 4-, 6- & 8-pole magnets keep the tunes constant.
Lattice modeling

\[ B = B_0 \left( \frac{r}{r_0} \right)^k = B_0 \left( 1 + \frac{k}{r_0} x + \frac{k(k-1)}{2!r_0^2} x^2 + \cdots \right) \]

⇒ \( r_0 \) should be the center of magnet

⇒ Now, accelerating speed is determined by clinical requirements, not by beam dynamics

\[ \Delta \nu_H < 0.1, \Delta \nu_H < 0.05 \]

Tracked by ZGOUBI

Before correction (FFAG08) after correction
Magnet

Challenges: Large aperture, short length, strong field

- Applicable to superconducting magnet
- Each multipole can be varied independently
  ⇒ Operational flexibility
- Present lattice parameters are within engineering limit

Superposition of helical field can form multipole field

Dipole
Quadrupole
Sextupole
Octupole

By H. Witte
LINEAR NON-SCALING (LNS) FFAGs

FFAGs look attractive for accelerating muons in μ Colliders or ν Factories

- Large acceptance (in r & p) eliminates cooling & phase rotation stages
- Rapid acceleration (<20 turns) makes resonance crossing ignorable (Mills '97)
- Less expensive than recirculating linacs.

NON-SCALING approach first tried by Carol Johnstone (arc 1997, ring 1999)

- strong positive-bending Ds + negative Fs - i.e. negative field gradients!
- "LINEAR" constant-gradient magnets (i.e. quadrupoles).

- Greater momentum compaction (& hence narrower radial apertures);
- Less orbit-time variation \(\rightarrow\) fixed rf frequency & cw operation;
- No multipole field components to drive betatron resonances >1\(^{st}\) order;
- Simpler construction (\(B' = \) constant, rather than \(B' \propto r^{k-1}\)).
LINEAR NON-SCALING FFAGs (cont’d)

Note that for LNS-FFAGs, orbit circumference $C$ varies quadratically with energy rather than rising monotonically:

$$C(p) = C(p_m) + \frac{12\pi^2}{e^2 S^2 NL_{f}\text{d}} (p - p_m)^2$$

So, compared to a scaling FFAG, there’s less variation in $C$ and orbit period - enabling fixed rf frequency operation when $v \approx c$.

Thus in a high-energy muon accelerator:
- the muons oscillate in phase across the rf voltage peak (3 crossings)
  - just as in a real, imperfectly isochronous, cyclotron (see below)!

The International Design Study for a Neutrino Factory chose LNS-FFAGs of 12.6-25 GeV and 25-50 GeV for the final stages of muon acceleration - with designs developed by a consortium led by Johnstone (FNAL), Berg (BNL), and Koscielniak (TRIUMF).
LNS-FFAG ORBITS I

We assume FDF triplet cells with **thin** quadrupoles (lengths $L_{f/d} \ll L_0$, the cell length), and define a **central orbit** - an N-sided polygon - which is on axis in the F quad for momentum $p_c$, and for which all bending occurs at D:

$$p_c = qB\rho_c = qB_c \frac{L_d/2}{\pi/N}.$$ 

Assuming equal quad strengths:

$$S \equiv B_f' L_f = -B_d' L_d$$

and writing  $\sigma \equiv qS/2$, then for other momenta $p$:

$$B_f = B_f' x_f \quad \psi_f = \left(\frac{qB_f'}{p}\right) \frac{L_f}{2} x_f = \frac{\sigma}{p} x_f$$

$$B_d = B_c + B_d' x_d \quad \psi_d = \frac{\pi}{N} \frac{p_c}{p} - \frac{\sigma}{p} x_d$$

Note that as $L_{fd} \to L_0/2$
the FDF cell approaches FODO form.
As the total bend \( \psi_f + \psi_d = \frac{\pi}{N} \),

then:

\[
x_f - x_d = \frac{\pi}{N\sigma} (p - p_c).
\]

But for large \( N \), \( \pi/N \ll 1 \), \( \psi_f \ll 1 \), so we also have:

\[
x_f - x_d \approx L_{fd} \psi_f = \frac{\sigma L_{fd}}{p} x_f
\]

Thus:

\[
x_f = \frac{\pi}{\sigma^2 N L_{fd}} p (p - p_c)
\]

\[
x_d = \frac{\pi}{\sigma^2 N L_{fd}} (p - p_c) (p - \sigma L_{fd}).
\]

i.e. the orbit offsets vary quadratically with momentum.
We have \( L \cos \psi_f \approx L_{fd} + x_d \sin(\pi/N) \).

\[
\therefore L \approx \left( L_{fd} + \frac{\pi}{N} x_d \right) \left[ 1 + \frac{1}{2} \left( \frac{\sigma x_f}{p} \right)^2 \right]
\]

and \( \Delta L \equiv L - L_{fd} \approx \frac{\pi}{N} x_d + \frac{1}{2} \sigma^2 L_{fd} \left( \frac{x_f}{p} \right)^2 \)

Substituting for \( x_d \) and \( x_f \):

\[
\Delta L(p) = \frac{3\pi^2}{2\sigma^2 N^2 L_{fd}} (p - p_c) \left[ p - \frac{1}{3} (p_c + 2\sigma L_{fd}) \right]
\]

Around the whole circumference:

\[
\Delta C(p) = \frac{12 \pi^2}{q^2 S^2 NL_{fd}} (p - p_m)^2
\]

where \( \Delta C = 0 \) for \( p_m = \frac{1}{3} (2p_c + \sigma L_{fd}) \).
If \( v \approx c \), the parabolic momentum dependence of path lengths \( L(p) \) leads to a parabolic form for time-of-flight \( \tau(p) \) also. If the FFAG consists of \( N \) identical rf cells (peak voltage \( V_0 \)), then for each cell:

\[
\tau(p) = \tau_m + \Delta \tau \left( \frac{p - p_m}{\Delta p / 2} \right)^2.
\]

If the particles are synchronous with the rf for some ToF \( \tau_m + \tau_0 \) at harmonic \( h \), then the phase advance/cell

\[
\Delta \phi = \frac{2\pi}{\tau_{rf}} \left[ \tau(p) - (h / N) \tau_{rf} \right].
\]

But \( h \tau_{rf} = N(\tau_m + \tau_0) \), so the average rate of advance \( d\phi/d\theta \) is

\[
\frac{\partial H}{\partial p} = \frac{d\phi}{d\theta} = \frac{N}{\tau_{rf}} \left[ \frac{4\Delta \tau}{\Delta p^2} (p - p_m)^2 - \tau_0 \right],
\]

giving us the first Hamilton equation.
LONGITUDINAL MOTION... (continued)

The first equation is supplemented by a second from the energy gain:

\[
\frac{\partial H}{\partial p} = \frac{d \phi}{d \theta} = \frac{N}{\tau_{rf}} \left[ \frac{4 \Delta \tau}{\Delta p^2} (p - p_m)^2 - \tau_0 \right], \quad -\frac{\partial H}{\partial \phi} = \frac{dp}{d \theta} = \frac{N q V_0}{2\pi} \cos \phi,
\]

(assuming \( E \approx c p \), and setting \( \phi = 0 \) at peak voltage). These lead to a Hamiltonian with a cubic dependence on momentum and energy:

\[
H = \frac{4 N \Delta \tau}{3 \tau_{rf} \Delta p^2} (p - p_m)^3 - \frac{N \tau_0}{\tau_{rf}} (p - p_m) - \frac{N q V_0}{2\pi} \sin \phi,
\]

or, introducing the dimensionless parameters

\[
P \equiv \frac{1}{2} + \frac{p - p_m}{\Delta p}, \quad b \equiv \frac{\tau_0}{\Delta \tau}, \quad a \equiv \frac{q V_0}{\omega_{rf} \Delta \tau \Delta p}
\]

we get:

\[
H = \frac{N \Delta \tau \Delta p}{\tau_{rf}} \left[ \frac{4}{3} \left( P - \frac{1}{2} \right)^3 - b \left( P - \frac{1}{2} \right) - a \sin \phi. \right]
\]
SERPENTINE ACCELERATION IN LNS-FFAGs

The cubic dependence of $H$ on energy creates stable fixed buckets at two different energies. With sufficient rf voltage $V_0$ a continuous ‘serpentine’ channel opens between them.

- Accelerate between the buckets - not within them

- Follow the golden trail!
Energy-phase plots for $b = \frac{1}{4}$ and increasing values of $a \propto V_0$. A complete serpentine channel only appears for $a \geq 1/24$ (S. Koscielniak, 2003).
SEPARATRICES IN THE PHASE-ENERGY PLANE

From J.S. Berg, FFAG 2003 (n.b. his $p$ = our $P$; his $x$ = our $\phi$).
The curves show lower limits of $a \propto V_0$ for a continuous channel from $P = 0$ to 1.
SERPENTINE ACCELERATION IN CYCLOTRONS

- Real cyclotrons are only imperfectly isochronous
- Acceleration occurs along a serpentine path
If the orbits cross the magnet ends perpendicularly:
- the tunes fall sharply with energy, crossing betatron resonances
- possibly leading to loss of beam quality/quantity
- danger lessened by rapid energy gain, but very expensive
- for muons ($\tau = 2 \mu s$): expensive but essential anyhow
- for ions: just expensive
Unfortunately, for large-emittance beams, the radial longitudinal coupling in LNS-FFAGs makes transfer matching difficult. Mitigation techniques exist, but the ν Factory ISS concluded that >2 LNS-FFAGs would not be practical - and opted for the more costly recirculating linacs below 12.6 GeV.
ELECTRON MODEL LNS-FFAG “EMMA”

A Proof of Principle machine for linear non-scaling FFAGs to demonstrate their two novel features:

- safe passage through many low-order structural resonances
- acceleration outside buckets.

EMMA has relativistic parameters similar to those of a 10-20 GeV muon FFAG, with a doublet lattice based on offset quadrupoles:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy</td>
<td>10-20 MeV</td>
</tr>
<tr>
<td>Circumference</td>
<td>16.57 m</td>
</tr>
<tr>
<td>Cells</td>
<td>42</td>
</tr>
<tr>
<td>N.T. Acceptance</td>
<td>3 mm</td>
</tr>
<tr>
<td>F quad length</td>
<td>5.88 cm</td>
</tr>
<tr>
<td>D quad length</td>
<td>7.57 cm</td>
</tr>
<tr>
<td>RF frequency</td>
<td>1.3 GHz</td>
</tr>
<tr>
<td>Cavities</td>
<td>19 x 120 kV</td>
</tr>
<tr>
<td>Injector</td>
<td>ALICE (7-35 MeV)</td>
</tr>
</tbody>
</table>

UK funding ($16M) started April 2007.
Commissioning under way at Daresbury Lab.
• Demonstrate features of linear, non-scaling FFAG

• Possible upgrade to non-linear in future

• Main parameters:
  - electrons, 10-20MeV
  - linear magnets, cw RF
  - 42 cells, doublet lattice

• In addition
  - very flexible
  - injection into full acceptance
  - lots of diagnostics
  - need flexible (10-20 MeV) injector
  - small
  - not too expensive!
EMMA - THE FIRST NON-SCALING FFAG

EMMA is a 10-20 MeV electron LNS-FFAG model for a 10-20 GeV muon accelerator for a neutrino factory - currently undergoing beam commissioning at Daresbury, UK.
Trbojevic, Keil and Sessler have proposed a LNS-FFAG gantry composed of either superconducting or permanent ferrite magnets in a close-packed triplet lattice and perhaps weighing only 1.5 tons.

The acceptance is large enough to transmit $C^{6+}$ ions of 150-400 MeV/u at a single excitation, and protons of 90-250 MeV at another.
A pair of nested 8-cell-FDF rings form a multi-ion cancer treatment facility. The inner ring (orbit radii 2.75-3.39 m) takes protons from 30-250 MeV, and acts as injector to the outer ring (5.5-6.9 m) for carbon ions (68-400 MeV/u).
Tune Stabilized NLNS-FFAGs (2)

Tune drop-off with energy is avoided by:

- employing the “edge focusing” that occurs for non-perpendicular magnet entry/exit
- allowing a non-linear $B(r)$ field variation

Nearly flat tunes are obtained, with large dynamic apertures.
Tune Stabilized NLNS-FFAGs (3)

4-T superconducting magnet designs have been prepared
Isochronous Lattices: nonscaling nonlinear FFAGs

• First ring: 18 MeV – 150 MeV isochronous H- FFAG – this energy range was chosen to make a “standalone” therapy machine to drive “low-energy” fixed rooms and SC high-energy ring

3 m outer machine radius, 1m straights, 1.07-1.87 m injection to extraction orbits

Physical layout of 18-150 MeV 4 sector isochronous NC ring

Tune per cell with just quad + sext field profile (top) and then adding octupole (middle) and ring tune (bottom)
Mathematica® initial parameters: field profile up to sextupole only

A 30-250 MeV H- CW FFAG for ADSR

tune splitting horz/vert will drive horiz above and vertical below integer, but radial dependence will remain well reproduced, B<9T @extraction
A 250 - 1000 MeV Proton CW FFAG for ADSR

*Mathematica®* initial parameters: field profile up to octupole

2.0 m straight, 1.7 m aperture

Ring tune and circumference $B < 2.4$ T
G.H. Rees has designed several FFAGs using novel 5-magnet “pumplet” cells, in which variations in field gradient and sign enable each magnet’s function to vary with radius – providing great flexibility – even allowing well-matched insertions!

- an isochronous “IFFAG” for muons (8-20 GeV, N = 123, C = 1255 m, 16 turns, as illustrated - or with insertions, N = 4 x (20 arc + 10 str.), C = 905 m)
- an IFFAG muon booster (3.2-8 GeV, 8 turns)
- an IFFAG electron model (11-20 MeV, N = 45, C = 29.3 m)
- an ν Factory proton driver (3-10 GeV, N = 66, C = 801 m, 50 Hz, 4 MW)
- a νF driver electron model (3.0-5.45 MeV, N = 27, C = 23.8 m)
REES’S ISOCRONOUS IFFAG

G.H. Rees’s IFFAG\(^1\) is remarkable in achieving both isochronism and vertical focusing at highly relativistic energies \((77 \leq \gamma \leq 190)\) without invoking spiral magnet edge focusing.

[Recall that isochronous \(B(r)\) gives \(\Delta v_z^2 = -(r/B)(dB/dr) = -\beta^2 \gamma^2\) – and that the highest energy spiral-sector isochronous cyclotron design had \(\gamma \leq 15\).]

The field profiles for the \(bd\), \(F\) and \(BD\) magnets (right) show how:

- \(F\) reverses sign at \(~11\) GeV
- \(bd\) focusing vanishes at high \(E\)
- \(BD\) focusing vanishes at low \(E\)
- \(B_{av}\) rises linearly with \(E\)
- The vertical defocusing associated with rising \(B_{av}\) is offset by strong AG focusing.

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Tracking studies have been carried out with the ZGOUBI and CYCLOPS codes and generally confirm the tunes predicted by Rees’s matrix transport code.

The agreement is very good for $\nu_r$, but for $\nu_z$ there’s a discrepancy. The tunes found by ZGOUBI (-■-) and CYCLOPS (-▲-) agree with each other, but oscillate around Rees’s values (-■-).
Rees (2005) has successfully incorporated long-drift insertions in an FFAG.
0.18 GeV H$^-$ Linac

0.18 GeV H$^-$ Achromat

3 GeV, 50 Hz, $h=5$, RCS
(1 at 50 Hz or 2 at 25 Hz)

10 GeV, 50 Hz, $N=5$, NFFAGI
with $10^{13}$ protons per bunch

4 MW, 10 GeV, Proton Driver
FURTHER READING

  [link](http://prola.aps.org/abstract/PR/v103/i6/p1837_1)

  [link](http://www-bd.fnal.gov/icfabd/Newsletter43.pdf)

  [link](http://www.worldscinet.com/rast/01/preserved-docs/0101/S17936268080000058.pdf) or  
  [link](http://trshare.triumf.ca/~craddock/RAST-cycFFAG.pdf)