Scaling FFAG for muon acceleration

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FFAG synchrotron : synchronous acceleration
FFAG cyclotron : isochronous acceleration
(MURA, Symon, Meads)
Neutrino factory
ISS scenario: Chris Prior nufact07

Neutrino Factory Overview

- Proton driver
  - primary beam on production target
- Target, capture, decay
  - create \( \pi \), decay into \( \mu \)
- Bunching, phase rotation
  - reduce \( \Delta E \) of bunch
- Cooling
  - reduce transverse emittance
- Acceleration
  - from \( \sim 130 \text{ MeV} \) to 20–50 GeV
- Decay Ring
  - store for \( \sim 500 \) turns
  - long \( \nu \) production straight
ns-FFAG chains

**Advantages**

- small aperture
- const. rf frequency (high frequency & field)

**Problems (issues)**

- time of flight (path length) for large amplitude: cascade rings
Emittance mismatch

Emittance degradation in passing through a succession of ns-FFAGs

A: Zero emittance beam in 10-20 GeV FFAG
B: Finite transverse amplitude beam in 10-20 GeV FFAG
C: 30,000 $\pi$ mm.mrad beam in two FFAGs
D: Correction with second harmonic RF
Scaling FFAG

**Advantages**

- Zero-chromaticity
  - *No resonance crossing*
- Constant momentum compaction
  - *Large longitudinal (and also transverse) acceptance*
  - *Good for muon acceleration with FFAG chains*

**Issues**

- Large orbit excursion needs;
  - *Frequency modulation with ordinary RF resonance acceleration.*
  - *Cf. RF acceleration with high frequency (200MHz) RF cavity which is needed to the present IDS-NF.*
Acceleration in Scaling FFAG

- **Variable(tuned) RF frequency**
  - Difficult for muon acceleration
    - *Needed energy gain is fairly large:* >~MV/m

- **Fixed(constant) RF frequency**
  - Stationary RF bucket acceleration
  - Harmonic Number Jump (HNJ) acceleration
Fixed RF frequency(1)

- **Stationary RF bucket acceleration**
  - Constant & small enough phase slip  ---  Large energy gain
  - relativistic beam
  - constant Momentum Compaction
  - Adequate for scaling FFAG

\[ \eta = \frac{1}{\gamma^2} - \alpha \approx -\alpha = -\frac{1}{k+1} \]

Diagram:
- Voltage vs. time
  - slow & constant
Stationary bucket acceleration

Scaling FFAG muon ring

- Energy: 10-20 GeV
- DFD lattice: r~200 m
- RF frequency: 3 MHz
- Energy gain: 1 MV/m

Recent new results for 3-10 GeV ring: T. Planche’s presentation in this workshop
Harmonic Number Jump (HNJ) acceleration

**HNJ-acceleration (microtron)** (Kolomenski, Fujisawa, Ruggiero)

- Difference of revolution period between n-th and (n-1)-th turn equals m(integer) times rf period.

\[ T_n - T_{n-1} = Trf \times m \]

- \( T_n \): revolution period for n-turn
- \( Trf \): rf period
- \( m \): integer (\(<0\): before, \(>0\): after transition)
HNJ Acceleration

Revolution period for n-th turn

\[
\frac{T_n}{T_1} = \left( \frac{C_n}{C_1} / \frac{v_n}{v_1} \right)
\]

C: circumference, v: particle velocity

\[
\frac{C_n}{C_1} = \frac{h_n}{h_1}, \quad p_n = p_1 \left( \frac{h_n}{h_1} \right)^{k+1}, h_n = h_1 + n \times m
\]

Orbit excursion vs. RF wave length (see T. Planche’s presentation)

\[
\Delta R_1^{N_t} = N_t \frac{\beta \lambda_{RF}}{2\pi}
\]

Nt : number of turns (for muon, \( \beta = 1 \))

— Nt < 6 turns if \( \lambda_{RF} > \Delta R \).

— Smaller \( \Delta R \) requires smaller \( \lambda_{RF} \).

cf. \( \lambda_{RF} = 1.5m \) (f=200MHz) \( \rightarrow \Delta R > 1m \)
HNJ in scaling FFAG

$k=2, \gamma=1-10$

\[ \text{nor. harmonic number} \]

\[ \text{radius} \]

\[ \text{energy gain} \]

\[ \text{gamma} \]
$k = 100, \gamma = 50-100$

![Graph showing relationship between radius and energy gain with harmonic number as a parameter.](image)
5-10GeV Scaling FFAG
radial sector - FDF lattice

Ring parameters
- $r=200\text{m}$
- $N=70\text{cells}$
- $B_{\text{max}} \sim F: 1.6\text{T}, D: 1.5\text{T}$
- $k=150$
- Orbit excursion — $62\text{cm}$
- Beam size (half) at 10GeV
  — $H: 7.4\text{cm}+1.2\text{cm}=8.6\text{cm}, V: 6.1\text{cm} \ @\text{s.s}$.
  — $H: 7.4\text{cm}+1.2\text{cm}=8.6\text{cm}, V: 6.1\text{cm} \ @\text{F-magnet}$
Multi rf system for HNJ acceleration

Acceleration of Mu: need ~0.5MV/turn ----> multi rf system

Frequency of each rf cavity for HNJ acceleration

- monotonic change
  - $f_0 > f_1 > f_2 > f_3 > ...$
  - *Details: see T. Planche’s talk*

Question?
Radial FFAG 5-10GeV

- rf parameters
  - $h=1200$
  - $f=400\text{MHz}$
  - $\phi_s=2\pi/3$

- 1-cell cavity 15MV/cavity(15MV/m)
Scaling FFAG with HNJ

Scaling FFAG with HNJ for low energy (3-10GeV) ring

- $N_t = 6-7$ turns with $200(400)$MHz RF cavity: $\Delta R \sim 1$ m
  - Good matching $\Rightarrow$ Phase Rotation & non-scaling FFAG

Energy gain/turn

- $1$ GeV/turn, $5$ MV/m $\Rightarrow$ 200 RF sections/ring

Orbit shift is almost constant.

- Orbit shift $\sim 15$ cm ($n \sim 7$ turns)
  - Good for injection/extraction (only septa, no kicker)

Question

- Acceleration of both Mu(+) and Mu(-) : If not possible, no hope for scaling FFAG to be a muon accelerator in neutrino factory.
Scaling FODO

two beam accelerator

Scaling FFAG ring with FODO lattice has opposite directional orbits for particles with same charge state. In other words, it has same orbits for particles with opposite charge state.

Thus, HNJ acceleration for both Mu(+) and Mu(-) with mluti-rf cavity becomes possible.
two beams FODO

In addition to the singlet conditions,

$$\beta_F = \beta_D$$

and assume

$$\frac{\theta_F}{\theta_D} = \left(\frac{r_0}{r_3}\right)^{k+1}$$

- $Q_h^2 \sim 2k$, $Q_v^2 \sim \Phi^2N^2/kS^2 \sim 2N^2/k$
- $k=100$, $N=100$ \hspace{1em} $Q_h \sim 14$, $Q_v \sim 14$
- $r=100m$, $\Delta r=0.7m$
Two-beam FFAGs for muon acceleration

<table>
<thead>
<tr>
<th>Ring parameters</th>
<th>Parameter value</th>
</tr>
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<tbody>
<tr>
<td>Lattice type</td>
<td>Double-beam FODO scaling FFAG</td>
</tr>
<tr>
<td>Mean radius</td>
<td>120 m</td>
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<tr>
<td>Number of cells</td>
<td>72</td>
</tr>
<tr>
<td>Maximum B field</td>
<td>2.9 T</td>
</tr>
<tr>
<td>Circumference factor</td>
<td>10.3</td>
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<tr>
<td>Packing factor</td>
<td>0.55</td>
</tr>
<tr>
<td>k value</td>
<td>167</td>
</tr>
<tr>
<td>Qx (horizontal tune)</td>
<td>20.43</td>
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<tr>
<td>Qz (vertical tune)</td>
<td>~5.5</td>
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<tr>
<td>RF freq.</td>
<td>300 MHz</td>
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<tr>
<td>Injection Energy</td>
<td>3.2 GeV</td>
</tr>
<tr>
<td>Extraction Energy</td>
<td>10 GeV</td>
</tr>
<tr>
<td>RF pick voltage</td>
<td>2.1 GV/turn</td>
</tr>
<tr>
<td>Number of turns</td>
<td>5</td>
</tr>
<tr>
<td>Excursion</td>
<td>0.82 m</td>
</tr>
</tbody>
</table>

3.1 T

200MHz, 400MHz
Longitudinal tracking(1)

RF voltage varied along radius : 2.1 GV/turn

before

after

injection beam emittance
Longitudinal tracking(2)

Constant RF voltage along radius : 2.1 GV/turn

before

injection beam emittance

after
How increase $N_t$ in HNJ?

$$\Delta R_1^{N_t} \quad N_t \frac{\lambda_{RF}}{2\pi}$$

- RF cavity with large opening aperture
  - $A_p > \lambda_{RF}$  cf. Crab cavity?
- Low dispersion straight section
  - Locally depressed orbit excursion without breaking scaling law.

Summary

Two-beam singlet Scaling FFAG seems to be suitable for acceleration of Mu(+) and Mu(-) with HNJ at same condition.

- Beam orbit capability
  - *Two beams, Large acceptance, RF frequency (200-400MHz)*

- No larger deterioration in longitudinal motion

- RF acceleration capability
  - *Multi-cavity with different frequency*
  - *Constant (not radially varied) RF voltage*

- Good for FFAGs chain.

Fixed-Field Fixed-Frequency
Alternating Gradient Synchrotron

$F^3$AG Synchrotron
Summary (cont.)

**Issues**
- Proper acceleration field matching
- Beam loading
- Long-trans. coupling
- **Increase of number of turns**
  — More relaxed RF field

**We are still learning from Scaling FFAG.**