



SURFACE ROUGHNESS WAKEFIELDS FOR THE ILC POSITRON UNDULATOR BEAM TUBE¹

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ABSTRACT

A simple, and probably pessimistic, estimation of the surface roughness specification for the undulator beam tube can be made using an inductive impedance model. The results show that in the worst possible case a surface roughness of ~300 nm would only increase the energy spread by factor of 10%. This specification, although quite stringent, can be comfortably met by specialist vessel manufacturers. And, due to currently unknown factors such as the exact bunch charge distribution and to induce the lowest possible energy spread in the bunch the smoothest commercially available beam tube should be used, which has a roughness less than 200 nm.

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INTRODUCTION

A number of models exist for surface roughness and have all been shown to be roughly equivalent [1]. The so called ‘inductive impedance model’ [2] shall be used which makes the following assumptions:

- The beam tube is cylindrically symmetric;
- The charge distribution is smooth;
- The surface roughness is assumed to have no resistance;
- The length of the bunch is longer than the length of the roughness features;
- As the impedance is purely inductive only surface roughness effects that influence the energy spread of the beam are considered.
- The height of the surface roughness bumps is about the same as the length of the bump.

The model assumes a number of bumps located on the surface of height δ , the radius of the beam tube is b . For a particular angular frequency, ω , the impedance, $Z(\omega)$, per unit length L , of a hemispherical bump is given by [3]:

$$\frac{Z(\omega)}{L} = -i\omega I = -i\omega \frac{Z_0 \delta^3}{4\pi cb},$$

where I is the inductance, Z_0 is the vacuum impedance and c the speed of light. The impedance is purely inductive with no resistive part. For different shape bumps the impedance has to be multiplied by a form factor that depends upon the shape of the bump. Different shaped bumps have been modelled in Mafia and various form factors ascribed to them [4]. Figure 1 shows and labels some of the different shapes considered and Table 1 gives their form factors.

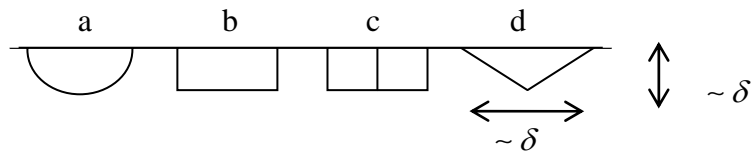


Figure 1: sample bump shapes, from left to right, hemisphere, half cube, rotated half cube and wedge.

Bump Shape	Form Factor, f
Hemisphere (a)	1
Half Cube (b)	2.6
Rotated Half Cube (c)	0.6
Wedge (d)	1.1

Table 1: form factors for various bump shapes from Figure 1.

As well as considering the shape of the surface bumps there is also the proportion of the surface area that is covered by the bumps to consider. For many bumps the total impedance is assumed to be the sum of the impedances of each bump. The total impedance per unit length is then:

$$\frac{Z(w)}{L} = -i\alpha f \omega \frac{Z_0 \delta}{4\pi c b}, \quad (1)$$

where f is the form factor relating to the bump shape and α is the packing factor, giving the fraction of the surface occupied by bumps.

The wake function is then simply the Fourier transform of the impedance. As the impedance is inductive it only has imaginary components and the Fourier Sine integral can be used. For a Gaussian bunch of rms length σ_z the wake potential, $W(s)$, is:

$$W(s) = -\frac{\alpha f Z_0 \delta c}{2\pi^2 b} \int_0^\infty k \exp\left[\frac{k^2 \sigma_z^2}{2}\right] \sin(ks) dk = -\frac{\alpha f Z_0 \delta c}{\pi b \sqrt{2\pi} \sigma_z^3} s \exp\left[\frac{-s^2}{2\sigma_z^2}\right],$$

where $\exp\left[\frac{k^2 \sigma_z^2}{2}\right]$ is the Fourier transform of the longitudinal charge distribution. The induced relative energy spread, σ_E , of a bunch of N electrons of charge e is:

$$\sigma_E = e^2 N \left| \int_{-\infty}^{\infty} \left(\frac{1}{\sigma_z \sqrt{2\pi}} W^2 \exp\left[\frac{-s^2}{2\sigma_z^2}\right] - k_{\parallel}^2 \right) ds \right|^{1/2},$$

where k_{\parallel}^2 is the square of the longitudinal loss factor and is equal to zero for purely inductive impedances. This gives, for the total induced energy spread of a round beam tube L metres long:

$$\sigma_E = \alpha f \frac{e N Z_0 \delta c L}{3^{3/4} 2^{3/2} \pi^{3/2} b \sigma_z^2}.$$

A similar expression has also been derived [5]. The expression is given in terms of the inductance, I , and is:

$$\sigma_E = \frac{e N L c^2 I}{3^{3/4} 2^{1/2} \pi^{1/2} \sigma_z^2}, \quad (2)$$

where the inductance is determined from measurement and simulation. For the same form factor and packing factor the two expressions for σ_E give the same result.

PARAMETERS

The parameters given in Table 2 shall be used. The ILC “minimum” beam parameters [6] give the greatest effect and so only those are considered. Using these parameters the induced energy spread can be calculated for various different form factors and surface roughness values, these are shown in Figure 2 and Figure 3 for a beam tube radius of 2 mm and 3 mm respectively. In Figure 4 the roughness required to produce an energy spread of 0.005% as a function of beam tube radius is plotted for different form factors.

Parameter	Unit	Value
Energy	GeV	150
Nominal σ_E	%	0.05
Increase in σ_E from Nominal	%	10
Undulator Length	m	200
rms Bunch Length	mm	0.15
Bunch Charge	N	$1 \cdot 10^{10}$
Packing Factor	α	0.5

Table 2: ILC and undulator parameters.

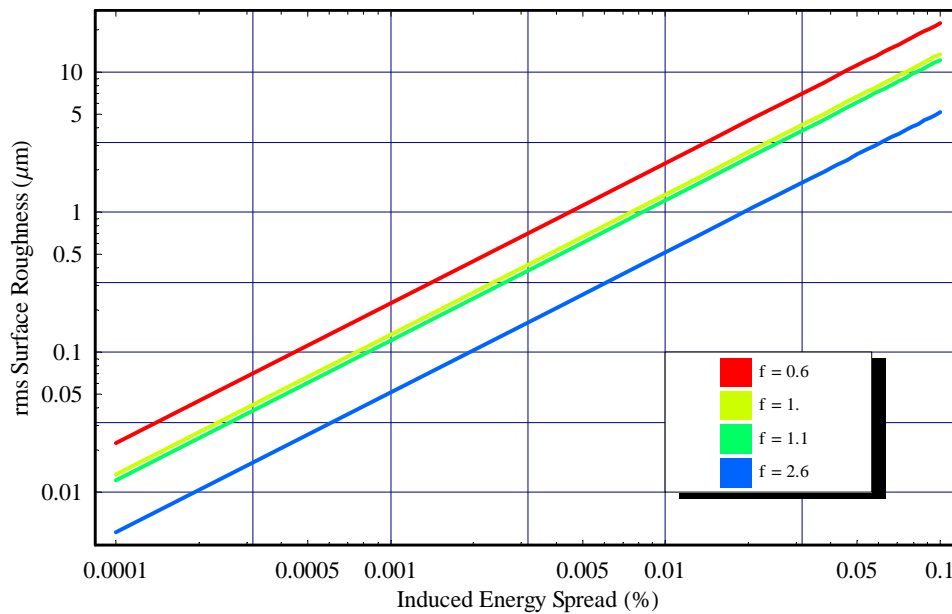


Figure 2: surface roughness v induced energy spread for different form factors and a 2mm radius beam tube

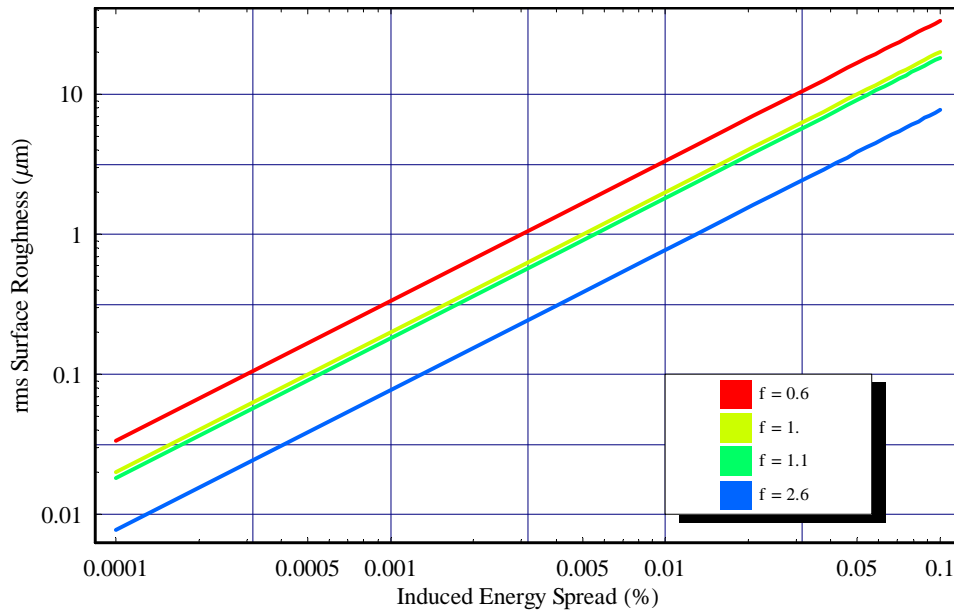


Figure 3: surface roughness v induced energy spread for different form factors and a 3mm radius beam tube

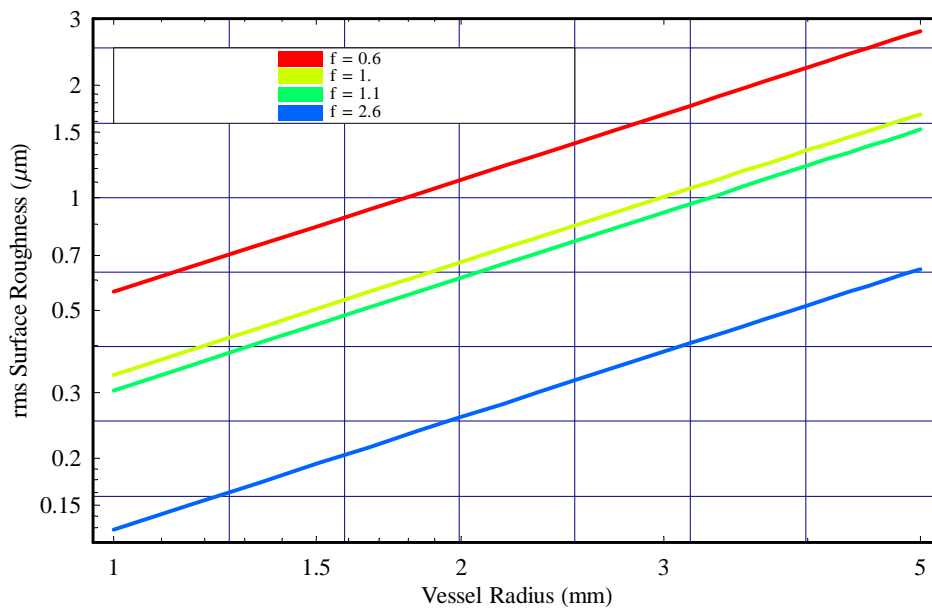


Figure 4: surface roughness required to produce an energy spread of $0.5 \cdot 10^{-4}$ for different beam tube radii and form factors.

CONCLUSIONS & DISCUSSION

The requirements on the surface roughness seem quite demanding. According to the model, to keep the increase in energy spread below 10% of the nominal value the surface roughness must be less than $\sim 300\text{nm}$, for the worst form factor. The surface roughness tolerance is not strongly dependant on the radius of the beam tube, it decreases as the inverse of the radius. Other wakefield effects often scale as an inverse power of the radius.

The inductance of high quality smooth pipes from the VALEX Corporation [7] was measured in [5]. A 2.5 mm radius beam tube, gave an inductance of $\sim 0.28 \text{ pH m}^{-1}$. The average roughness of the VALEX tube was measured to be $\sim 125 \text{ nm}$.

From Equation (2) the impedance budget for the ILC undulator is 33 pH m^{-1} and if a similar tube to [5] were used then the inductance per meter would be $\sim 0.35 \text{ pH}$. This indicates that the theory is pessimistic compared to measured inductances of real beam tubes. One of the reasons for this is that the assumption that the height of the bumps is approximately the same as the length of the bumps is not confirmed by the measurements [5].

If it is assumed that the measured inductance is correct for many metres of pipe then the corresponding form factor is ~ 0.06 and the height of the bumps required to keep the increase in energy spread less than 10% of the nominal is $\sim 12 \text{ microns}$.

However even given all these factors the conclusions of [5] state that there is an active program underway to improve upon the $\sim 125 \text{ nm}$ surface roughness of the VALEX beam tubes. There are other effects that have not been included, such as the true distribution of the bunch charge, which may have features that are approaching the scale of the surface roughness. Also misalignments of the beam tubes will have some effect. Given these unknowns it would seem prudent to use the smoothest beam tube possible to help mitigate any future deleterious effects of the surface roughness.

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