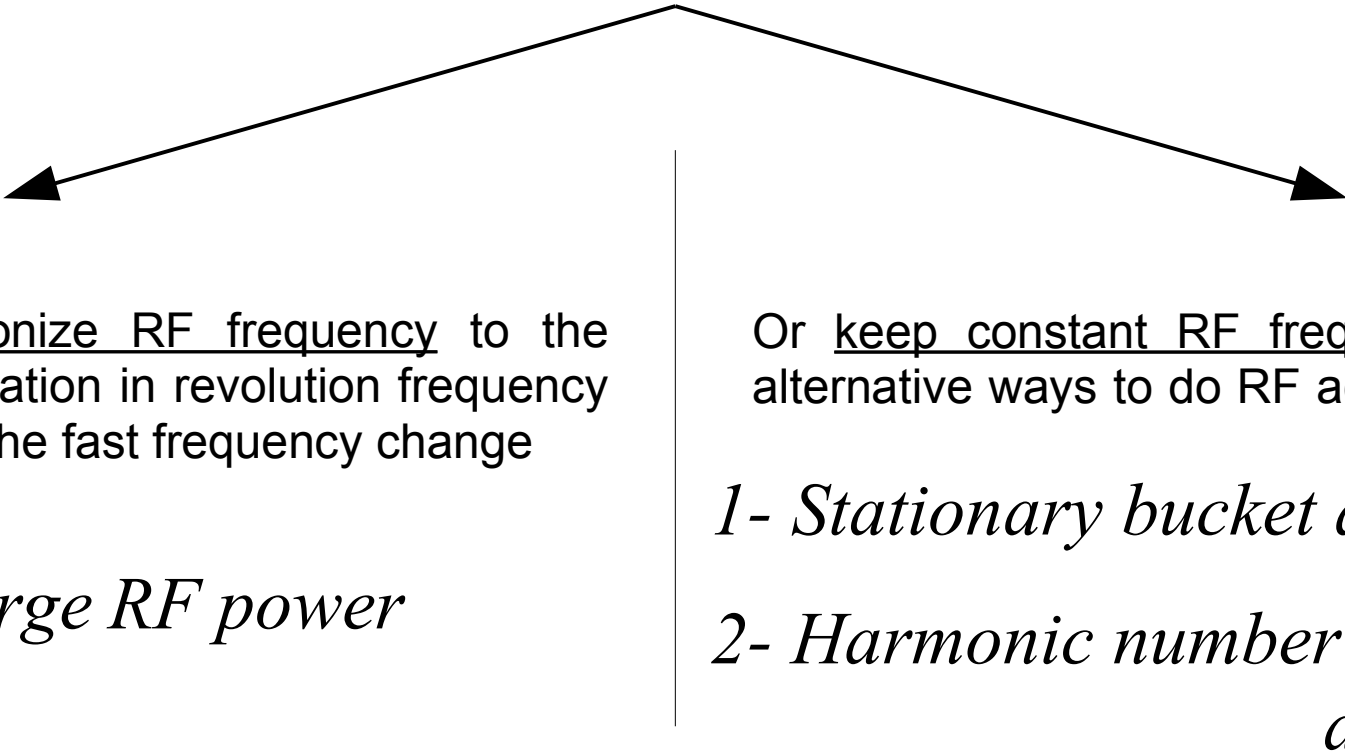


Fast acceleration of relativistic particles in a scaling FFAG ring

Fast acceleration of relativistic particles in a scaling FFAG ring

Relativistic particles ($\beta \approx 1$): $f_{\text{revolution}} \propto P^{-\frac{1}{k+1}}$



Either synchronize RF frequency to the necessary variation in revolution frequency and deal with the fast frequency change

\Rightarrow *Very large RF power*

Or keep constant RF freq. and look for alternative ways to do RF acceleration like:

1- Stationary bucket acceleration

*2- Harmonic number jump
acceleration*

- Fast acceleration of relativistic particles in a scaling FFAG ring -

Part 1 – Stationary bucket acceleration in a scaling FFAG ring

Stationary bucket acceleration principle

$$\cancel{f_{RF}(t) = h \cdot f_{rev.}(t)} \longrightarrow f_{RF} = h \cdot f_{rev.}^s$$

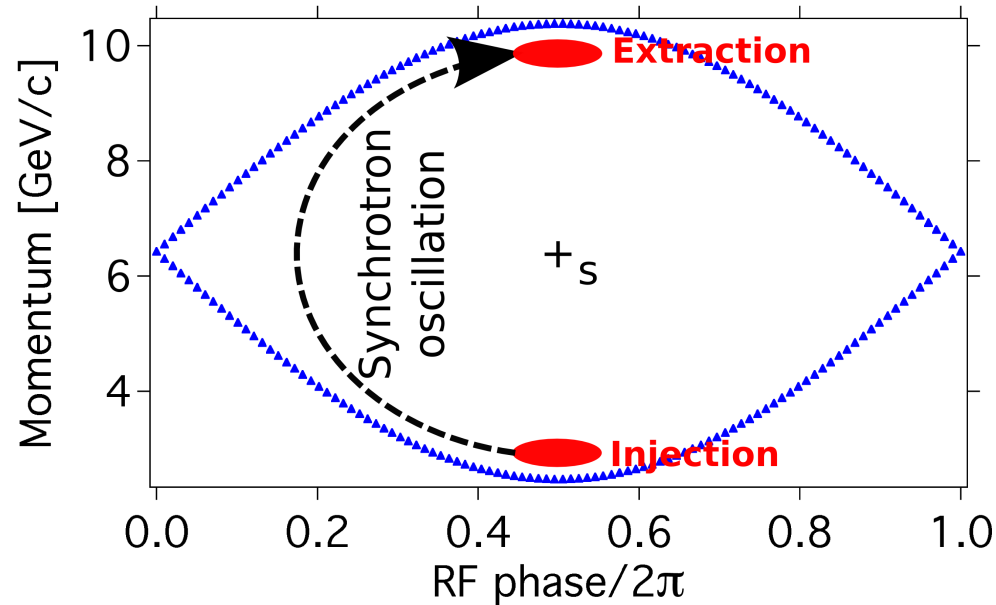


Figure 1 – Stationary bucket acceleration, principle.

Stationary bucket acceleration principle

Relativistic particle in a scaling FFAG:
FFAG:

$$\eta_s \approx -\frac{1}{k+1}$$

Maximum bucket height ($\Phi_s = \pi$):

$$\Delta P = \sqrt{2e \cdot \frac{\gamma_s E_0 V_{RF}}{\pi h \eta_s}}$$

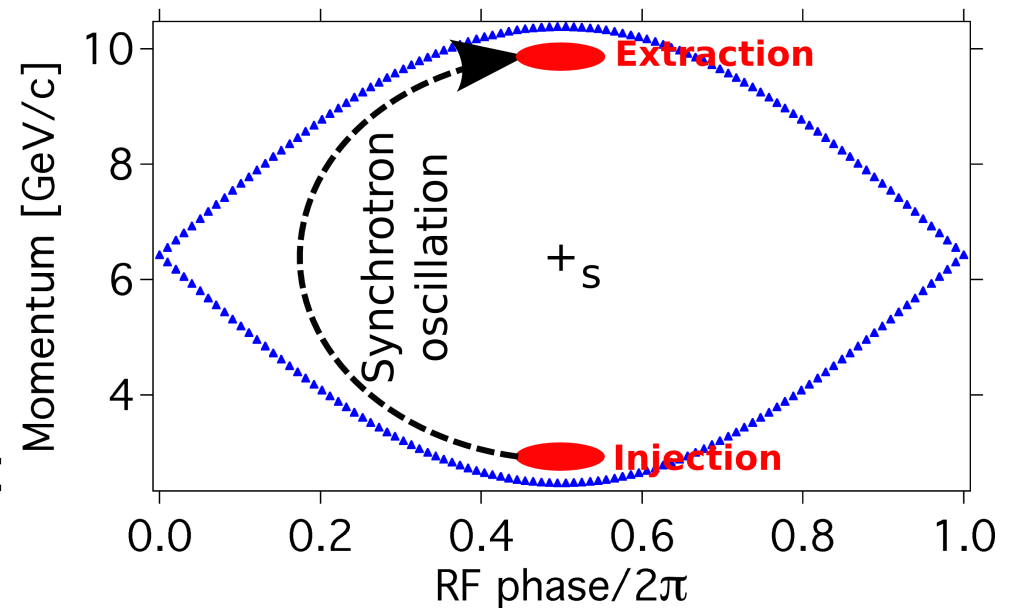


Figure 1 – Stationary bucket acceleration, principle.

3 to 10 GeV muon - stationary bucket acceleration

Particle	Muon
Injection energy	3 GeV
Extraction energy	10 GeV
Injection radius	120 m
Max RF voltage per turn	4.1 GV
RF frequency	200 MHz
Excursion	0.32 m
Lattice type	Radial FDF scaling FFAG
Number of cells	100
k value	450
$\beta_F, \beta_D, \beta_{\text{LongDrift}}$	$0.630^\circ, 0.504^\circ, 0.558^\circ$
B_{max}	2.9T
Q_x	29.7
Q_z	7.9

Table 1 - 3 to 10 GeV muon ring parameters.

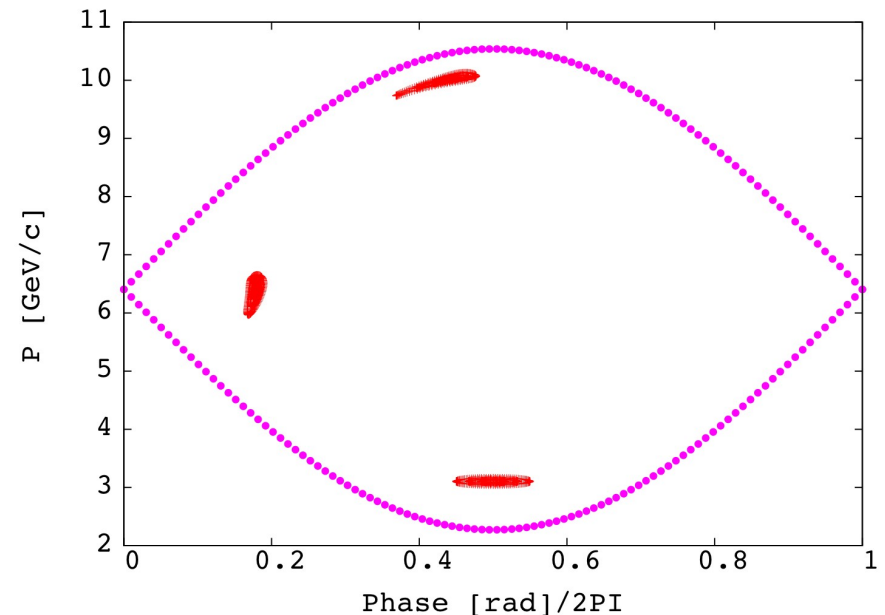


Figure 2 – 2D (longitudinal) tracking; 3 to 10 GeV muon acceleration with 200 MHz RF frequency.

3 to 10 GeV muon - stationary bucket acceleration

Particle	Muon
Injection energy	3 GeV
Extraction energy	10 GeV
Injection radius	120 m
Max RF voltage per turn	1.5 GV
RF frequency	80 MHz
Excursion	0.32 m
Lattice type	Radial FDF scaling FFAG
Number of cells	100
k value	450
$\beta_F, \beta_D, \beta_{LongDrift}$	$0.630^\circ, 0.504^\circ, 0.558^\circ$
B_{max}	2.9T
Q_x	29.7
Q_z	7.9

Table 2 - 3 to 10 GeV muon ring parameters.

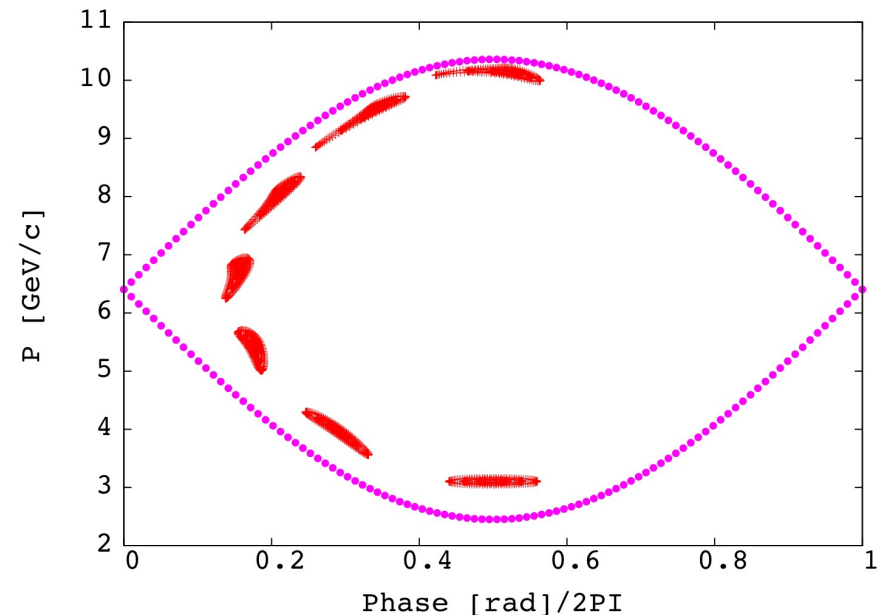


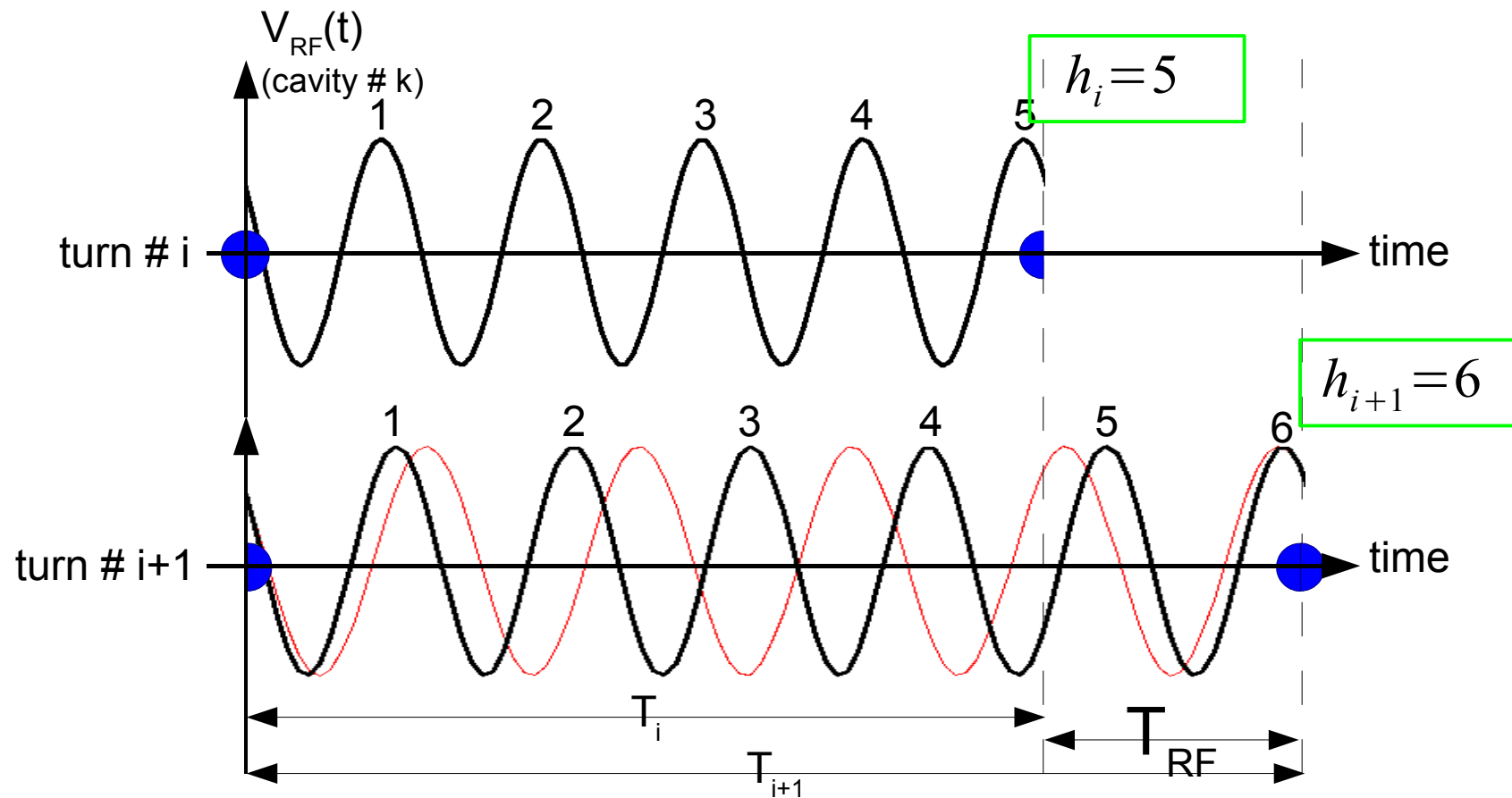
Figure 3 – 2D (longitudinal) tracking; 3 to 10 GeV muon acceleration with 80 MHz RF frequency.

- Fast acceleration of relativistic particles in a scaling FFAG ring -

Part 2 – Harmonic number jump acceleration in a scaling FFAG ring

Harmonic number jump acceleration principle

$$\cancel{f_{RF}(t) = h \cdot f_{rev.}(t)} \longrightarrow f_{RF} = h(i) \cdot f_{rev.}(t)$$



Harmonic number jump acceleration principle

$$T_{i+1} - T_i = \frac{m}{f_{RF}} \quad m \in N \quad (1)$$

We will here only treat the case $m = 1$:

$$T_{i+1} - T_i = \frac{1}{f_{RF}} = \textit{Constant} \quad (2)$$

General considerations

$$T_{i+1} - T_i = \frac{1}{f_{RF}} \quad (2)$$

$$T_{i+1} - T_i = \frac{\Delta C_i}{\beta c} \quad (3)$$

$$\Rightarrow \Delta R_i \simeq \frac{\beta c}{2\pi f_{RF}} = \frac{\beta \lambda_{RF}}{2\pi} \quad (4)$$

Relativistic particles:

$$\textit{average excursion} \simeq \frac{N_{turn}}{2\pi} \cdot \lambda_{RF} \quad (5)$$

Single cavity case

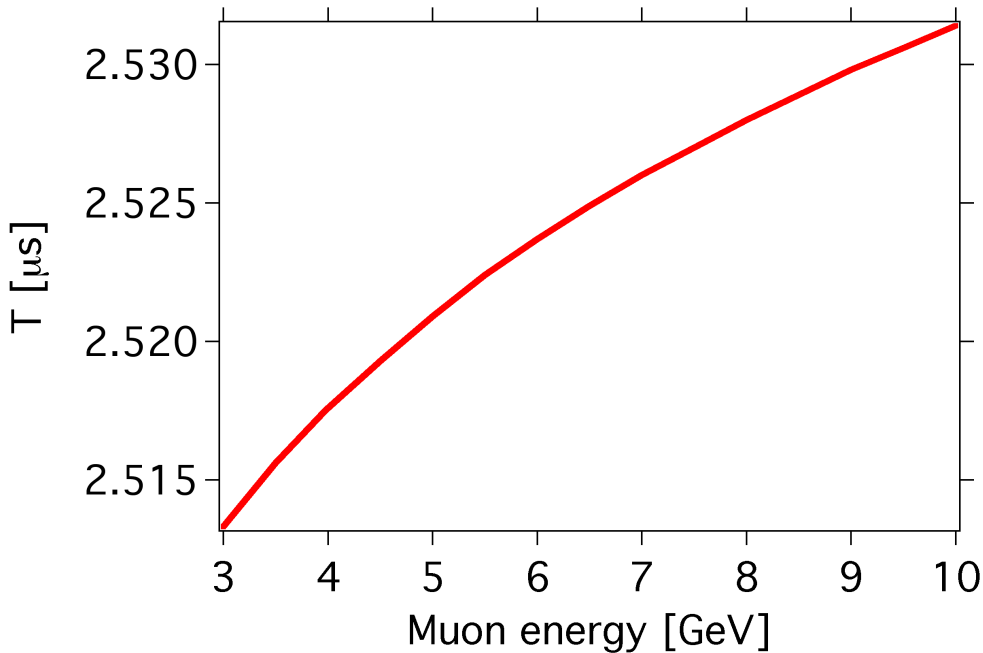


Figure 4— $T_{\text{rev}}(E)$ in the case of 3 to 10 GeV muon accelerated in a 120 m diameter scaling FFAG ring ($k=167$).

$$T(E_{i+1}) - T(E_i) = \frac{1}{f_{RF}} \quad (2')$$

$T(E)$, first order in ΔE :

$$T(E_{i+1}) = T(E_i) + \left[\frac{\Delta T}{\Delta E} \right]_{E_i} \cdot (E_{i+1} - E_i) \quad (6)$$

Single cavity case

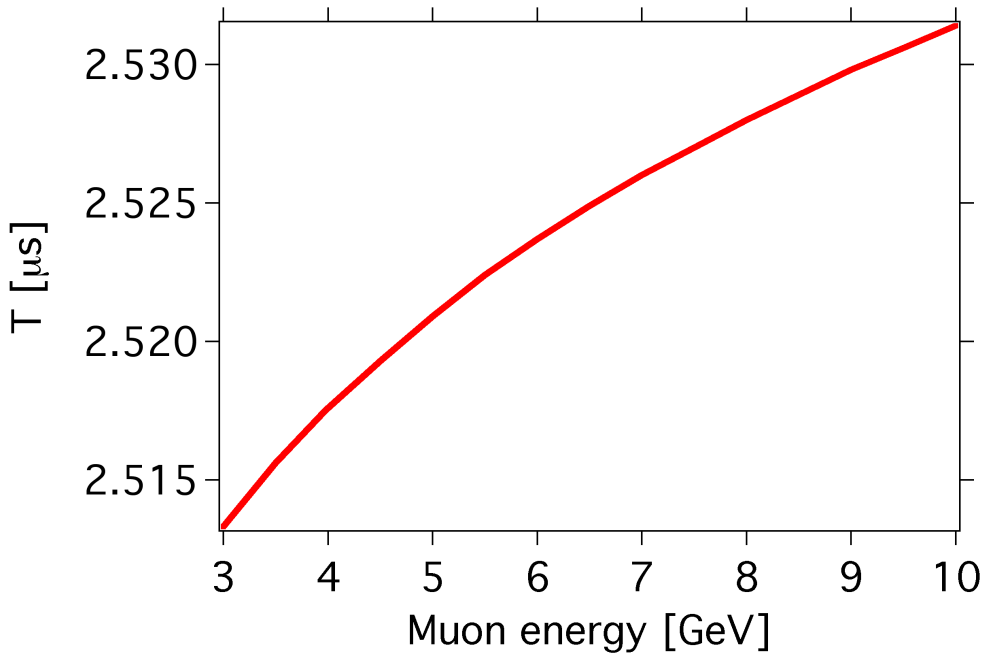


Figure 4— $T_{\text{rev}}(E)$ in the case of 3 to 10 GeV muon accelerated in a 120 m diameter scaling FFAG ring ($k=167$).

$$T(E_{i+1}) - T(E_i) = \frac{1}{f_{RF}} \quad (2')$$

$T(E)$, first order in ΔE :

$$T(E_{i+1}) = T(E_i) + \left[\frac{\Delta T}{\Delta E} \right]_{E_i} \cdot (E_{i+1} - E_i) \quad (6)$$

$$\Rightarrow (E_{i+1} - E_i) = \frac{T(E_{i+1}) - T(E_i)}{\left[\frac{\Delta T}{\Delta E} \right]_{E_i}} \quad (6')$$

$$\Rightarrow \Delta E_i = \frac{1/f_{RF}}{\left[\frac{\Delta T}{\Delta E} \right]_{E_i}} \quad (7)$$

Multi-cavity case

Same energy gain in each cavity: $E_{1,k} = E_{i,0} + \Delta E_i \cdot \frac{k}{N}$ (8)

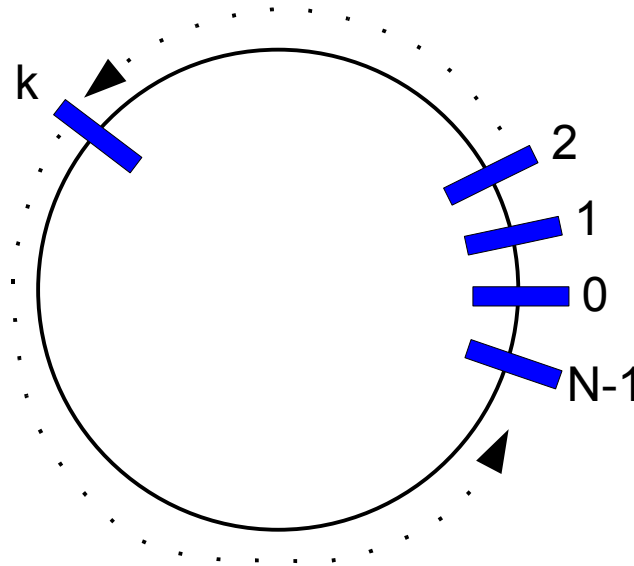


Figure 5 – N cavities
homogeneously distributed around the ring.

Multi-cavity case

Same energy gain in each cavity: $E_{1,k} = E_{i,0} + \Delta E_i \cdot \frac{k}{N}$ (8)

$$T(E_{i,k}) = T(E_{i,0}) + \left[\frac{\Delta T}{\Delta E} \right]_{E_i} \cdot \Delta E_i \frac{k}{N} \quad (6'')$$

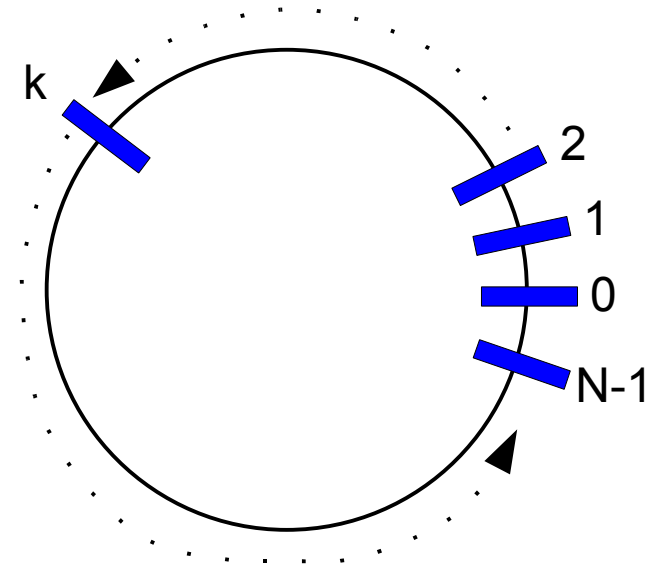


Figure 5 – N cavities homogeneously distributed around the ring.

Multi-cavity case

Same energy gain in each cavity: $E_{1,k} = E_{i,0} + \Delta E_i \cdot \frac{k}{N}$ (8)

$$T(E_{i,k}) = T(E_{i,0}) + \left[\frac{\Delta T}{\Delta E} \right]_{E_i} \cdot \Delta E_i \frac{k}{N} \quad (6'')$$

$$t(k) = t(1) + \sum_{j=0}^{N-1} \frac{T(E_{i,k})}{N} \quad (9)$$

$$\Rightarrow t(k) = t(1) + T(E_{i,0}) \cdot \frac{k}{N} + \left[\frac{\Delta T}{\Delta E} \right]_{E_i} \cdot \Delta E_i \cdot \frac{(k+1)k}{2N^2} \quad (9')$$

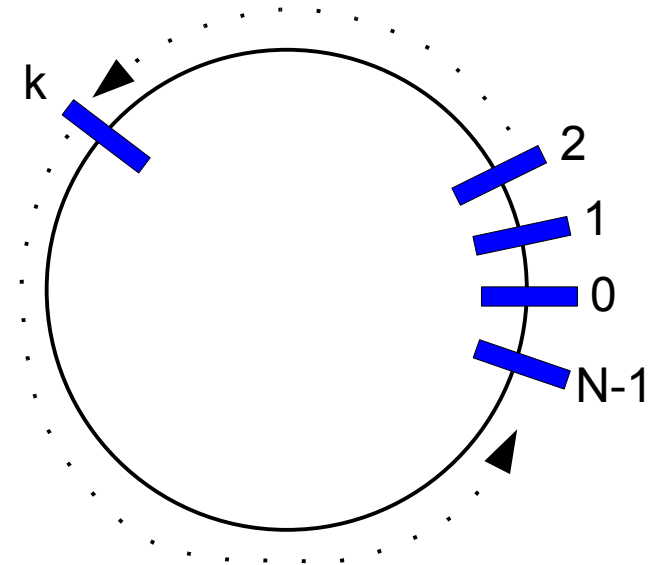


Figure 5 – N cavities homogeneously distributed around the ring.

$$t(k+N) - t(k) = T(E_{i,0}) + \Delta E_i \left[\frac{\Delta T}{\Delta E} \right]_{E_i} \cdot \left(\frac{1}{2} + \frac{2k+1}{2N} \right) \quad (10)$$

Multi-cavity case

If one would like to arrive on turn $i+1$ at the same RF phase at the turn i :

$$f_{RF}(i, k) = \frac{h_i}{t(k+N) - t(k)} \quad (11)$$

$$\Rightarrow f_{RF}(i, k) = \frac{h_i}{T(E_i) + \Delta E_i \left[\frac{\Delta T}{\Delta E} \right]_{E_i} \cdot \left(\frac{1}{2} + \frac{2k+1}{2N} \right)} \quad (10')$$

If one take: $\Delta E_i = \frac{T(E_{i,0})}{h_i \left[\frac{\Delta T}{\Delta E} \right]_{E_i}}$, assuming then that: $\frac{T(E_{i,0})}{h_i} = \text{Constant} = \hat{f}$

$$f_{RF}(i, k) = \frac{\hat{f}}{1 + \frac{1}{h_i} \cdot \left(\frac{2k+1}{2N} - \frac{1}{2} \right)} = \hat{f} \cdot \left(1 - \frac{1}{h_i} \cdot \left(\frac{2k+1}{2N} - \frac{1}{2} \right) \right) + o\left(\frac{1}{h_i}\right) \quad (12)$$

Multi-cavity case

Assuming: $h_0 \gg 1$ and $h_0 \gg \Delta_{0..i} h$ one can get:

$$f_{RF}(k) \simeq \hat{f} \cdot \left(1 - \frac{1}{h_0} \cdot \left(\frac{2k+1}{2N} - \frac{1}{2} \right) \right) \quad (13)$$

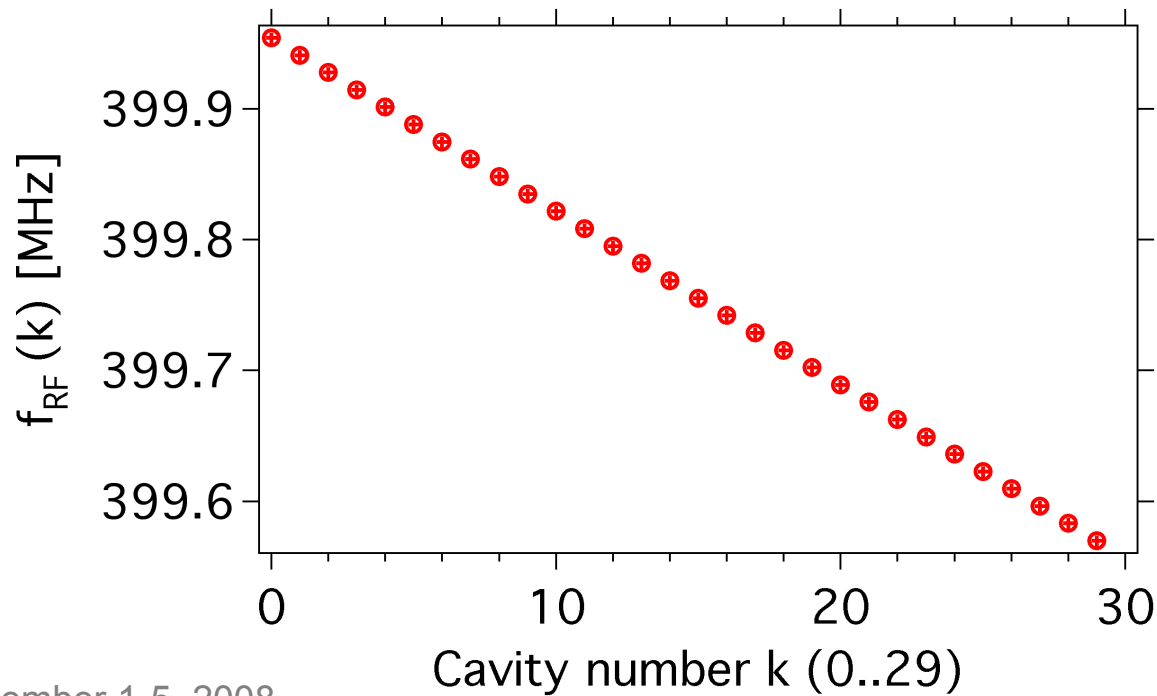


Figure 6 – Each cavity works at a constant frequency, but frequency are different from one cavity to the next.

Acceleration with variable RF voltage

30 cavities case:

Particle	Muon
Injection energy	3 GeV
Extraction energy	10 GeV
Injection radius	120 m
Variable RF voltage	1.8 to 5.2 GV/turn
RF frequency	400 MHz
Lattice type	Double-beam F0D0 scaling
Number of cells	72
k value	167
packing factor	0.55
B_{\max}	2.9T
Q_x	20.4
Q_z	5.5

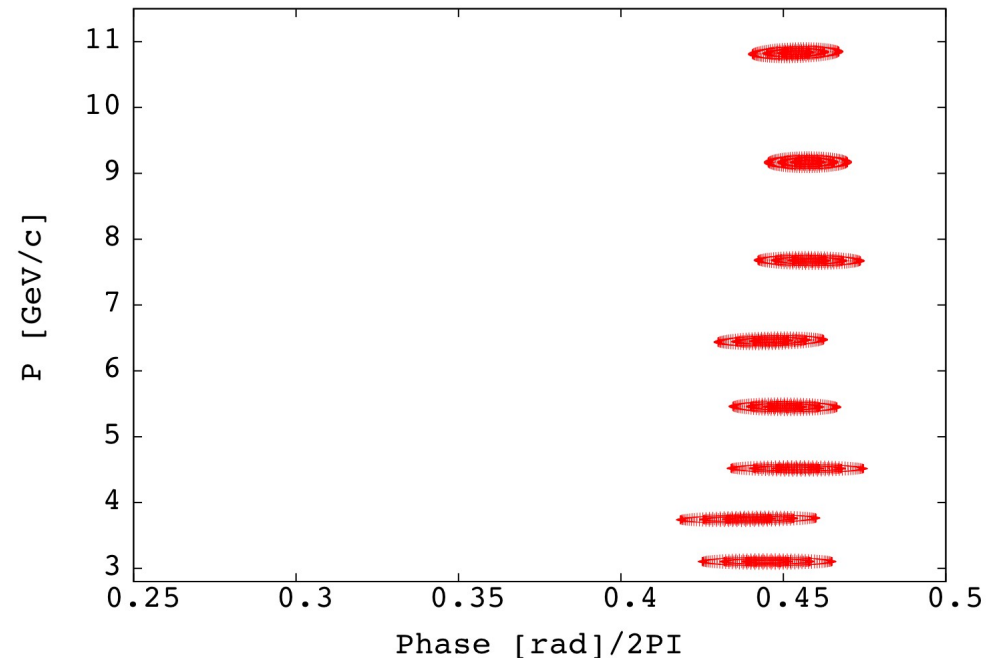


Table 3- Example of parameters for a 3 to 10 GeV muon ring.

Figure 7 - 2D (longitudinal) tracking; 3 to 10 GeV acceleration (7 turns) with variable RF voltage

Acceleration with **constant** RF voltage

30 cavities case:

Particle	Muon
Injection energy	3 GeV
Extraction energy	10 GeV
Injection radius	120 m
Constant RF voltage	1.9 GV/turn
RF frequency	400 MHz
Lattice type	Double-beam F0D0 scaling
Number of cells	72
k value	167
packing factor	0.55
B_{\max}	2.9T
Q_x	20.4
Q_z	5.5

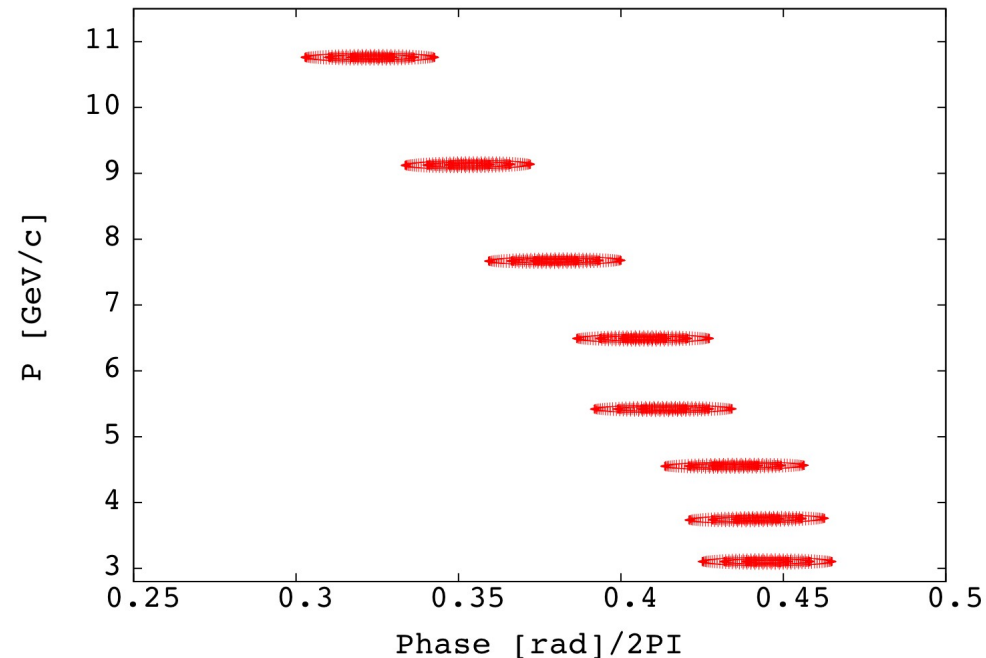


Table 4- Example of parameters for a 3 to 10 GeV muon ring.

Figure 8 - 2D (longitudinal) tracking; 3 to 10 GeV acceleration (7 turns) with constant RF voltage

Acceleration with **constant** RF voltage

2D tracking, 0.24 eV.s beam acceleration:

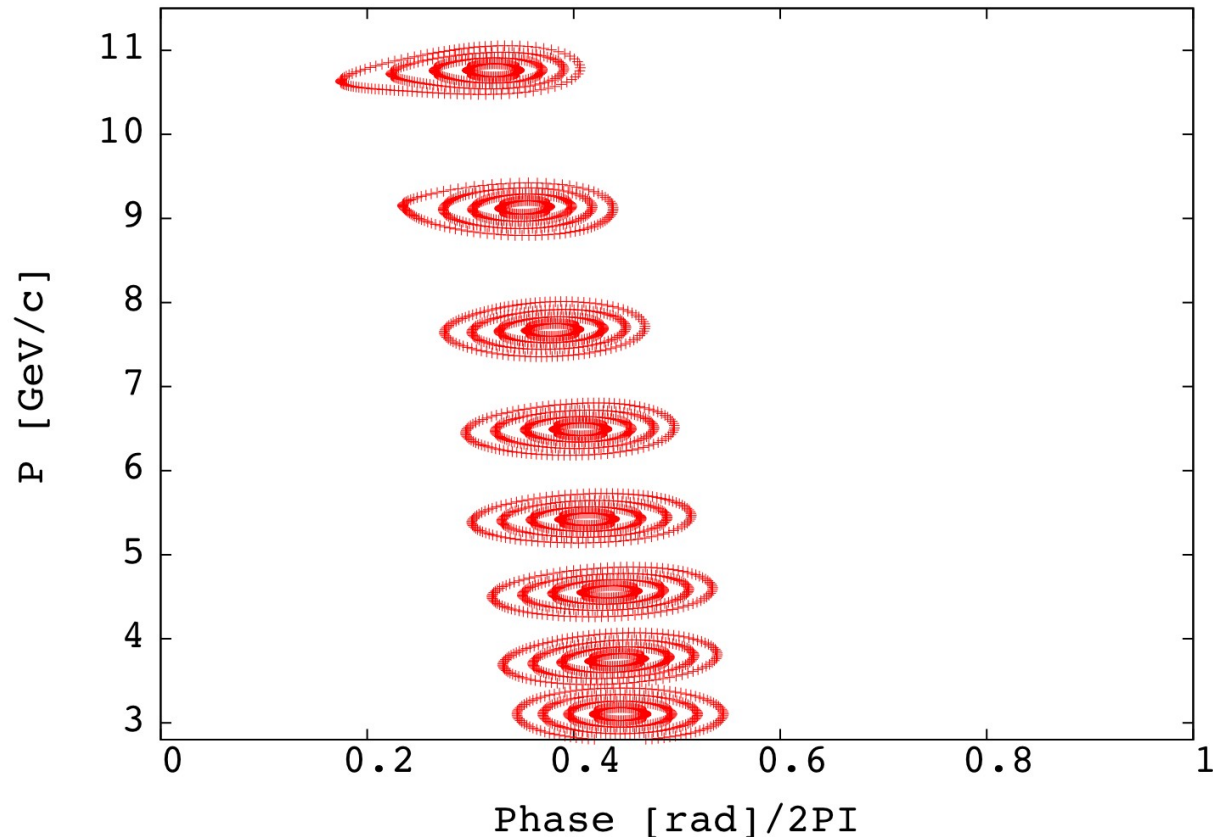


Figure 9 - 2D (longitudinal) tracking; 3 to 10 GeV acceleration (7 turns) with constant RF voltage; **longitudinal emittance = 0.24 eV.s.**

Acceleration with **constant** RF voltage

4D tracking, 0.24 eV.s beam acceleration (36 cavities assumed):

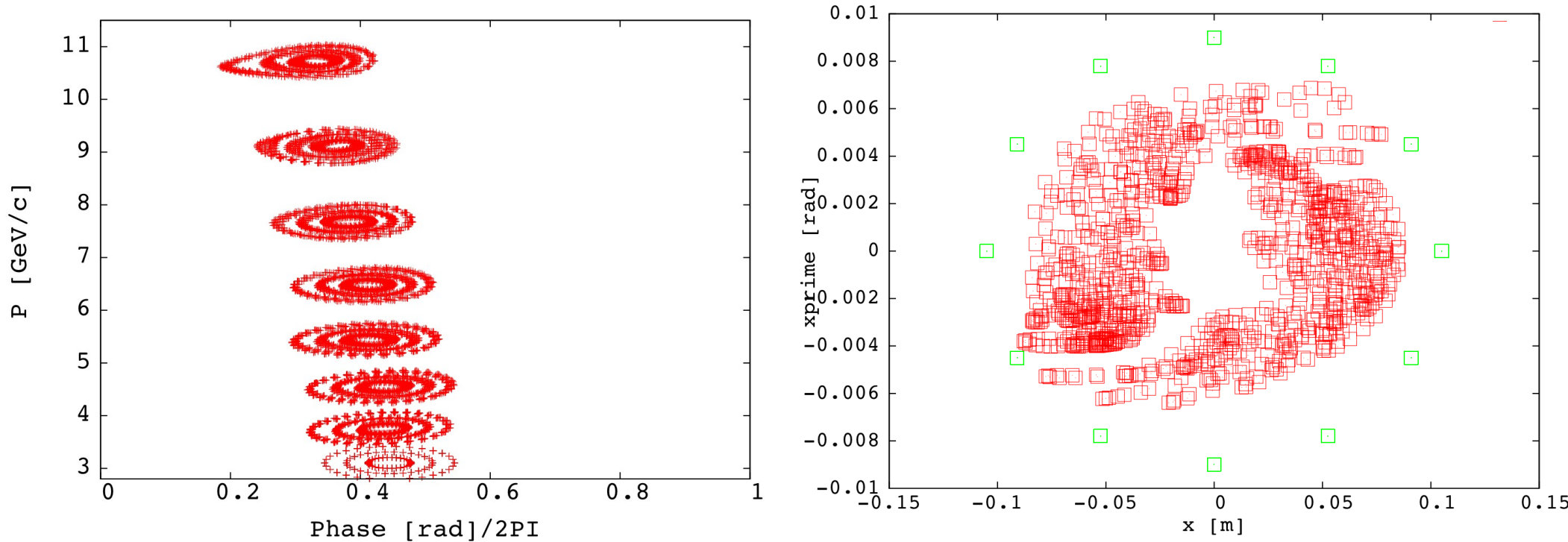


Figure 10 - 4D tracking (FD double beam lattice, RK integration in hard-edge model); 3 to 10 GeV acceleration (7 turns) with constant RF voltage; longitudinal emittance = **0.24 eV.s**; normalised horizontal emittance = **30.000π mm.mrad**.

Summary on 3 to 10 GeV acceleration with a scaling FFAG

With **stationary bucket** acceleration:

- Possible but with
RF frequency $< \sim 100$ MHz.

With **harmonic number jump** acceleration:

No need to change RF voltage during acceleration!

- Excursion issue limits the number of turn.
More optimisation must/could be done.
- Constant frequency operation is possible
but each cavity has to be run at a different frequency.
Need for a double beam lattice.

- Fast acceleration of relativistic particles in a scaling FFAG ring -

Thank you for your attention.

- Fast acceleration of relativistic particles in a scaling FFAG ring -

Annexe

- Fast acceleration of relativistic particles in a scaling FFAG ring -

$$t(k+N) - t(k) = T(E_{i,0}) + \Delta E_i \left[\frac{\Delta T}{\Delta E} \right]_{E_i} \cdot \left(\frac{1}{2} + \frac{2k+1}{2N} \right) \quad (9)$$

Revolution time turn # i:

$$\Delta t_i = t(N) - t(0) = T(E_{i,0}) + \Delta E_i \left[\frac{\Delta T}{\Delta E} \right]_{E_i} \cdot \left(\frac{1}{2} + \frac{1}{2N} \right) \quad (An.1)$$

Revolution time turn # i+1:

$$\Delta t_{i+1} = t(2N) - t(N) = T(E_{i,0}) + \Delta E_i \left[\frac{\Delta T}{\Delta E} \right]_{E_i} \cdot \left(\frac{1}{2} + \frac{2N+1}{2N} \right) \quad (An.2)$$

$$\Rightarrow \Delta t_{i+1} = \Delta t_i + \Delta E_i \left[\frac{\Delta T}{\Delta E} \right]_{E_i} \quad (An.3) \quad \text{like in the single cavity case}$$

- Fast acceleration of relativistic particles in a scaling FFAG ring -

Cavity working at different frequencies: μ^+ and μ^- acceleration?

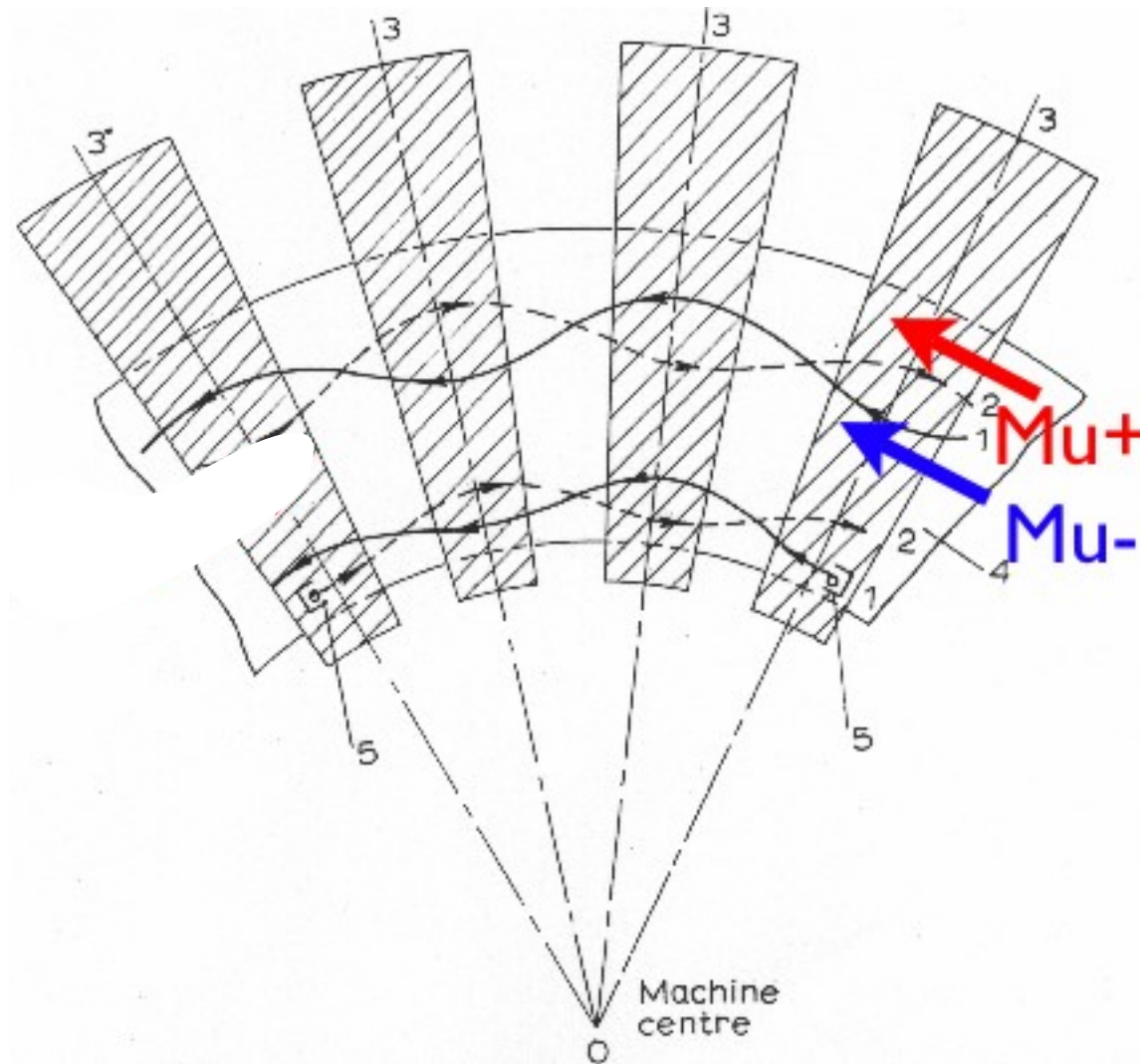


Figure 11 – Scaling FFAG F0D0 double beam lattice. Particle and antiparticle can circulate in the same direction.