Principles of FFAG modeling

Shinji Machida

ASTeC/STFC/RAL

01 September, 2008

http://www.astec.ac.uk/intbeams/users/machida/doc/ffag/machida20080901.pdf & ppt

Characteristics of FFAG (1)

differences from other accelerators

- Location of magnets does not specify ideal orbits.
  - Often a beam does not go through the magnet centre.
  - Cyclotron has the similar situation.

- Focusing at the edge of magnets often plays an important role.
  - Not only in spiral magnets.

- Field profile of the magnet has a strong nonlinearity.
  - Body field in a scaling machine.
  - Off-axis end field region.

Characteristics of FFAG (2)

sometimes become important

- Paraxial approximation is usually not accurate.
  - In particular, for a large emittance muon beam.

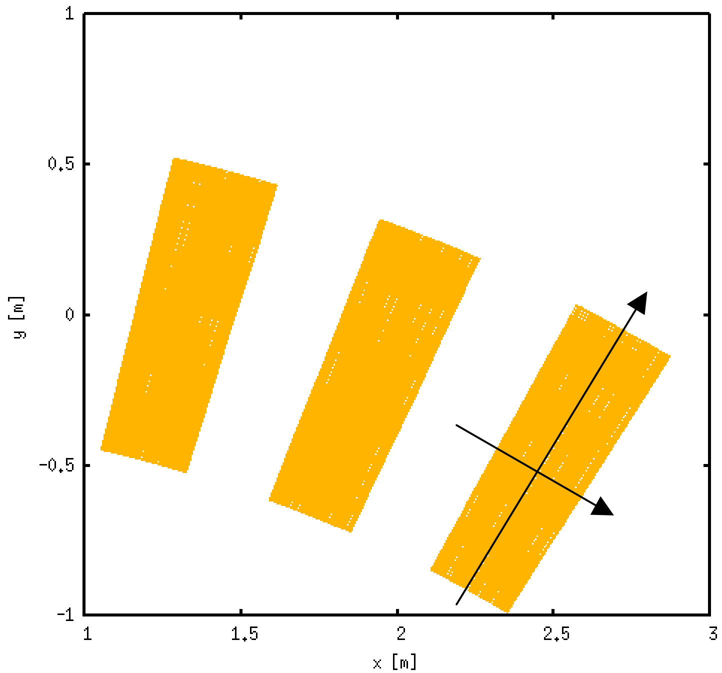
- 6-D tracking is essential.
  - Transverse and longitudinal dynamics are strongly coupled.

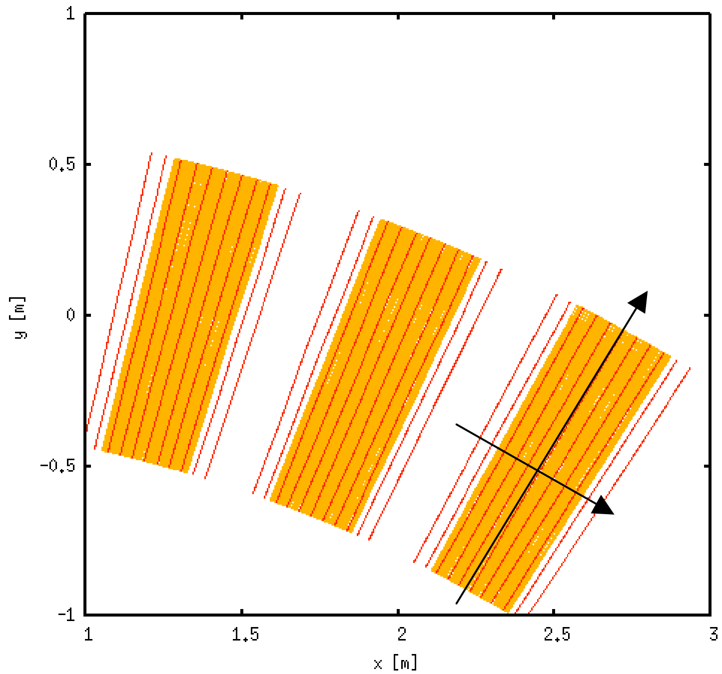
- Intensity dependent effects such as beam loading and space charge should be considered.

Orbit and optics calculation (1)

layout of accelerator

- Place lattice components and attach coordinate system to each component.
- Define magnetic and/or electric fields at thin slices in the coordinate system.

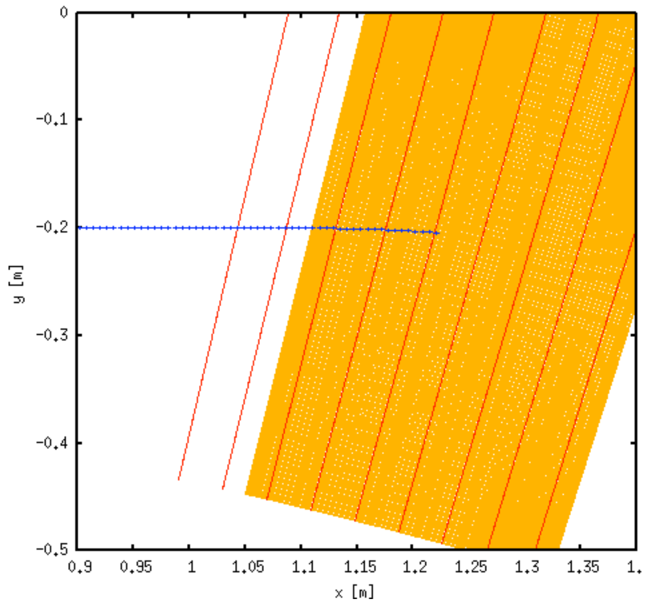




Orbit and optics calculation (2)

integration of the equation of motion

- Integrate the equation of motion to update particle's position.
  - Should not violate symplectic condition.
  - However, Runge-Kutta integration is often sufficient.



- The way I use.
  - Integration is performed with kick-drift.

at a slice of a magnet

$$\Delta p_y = e\left(v_z B_x - v_x B_z\right)\left(\Delta x / v_x\right)$$

$$\Delta p_z = e\left(v_x B_y - v_y B_x\right)\left(\Delta x / v_x\right)$$

$$p_{x,new} = \sqrt{p_t^2 - p_{y,new}^2 - p_{z,new}^2}$$

at a slice of a rf cavity

$$E_{new} = E + eV \sin \omega_{rf} t$$

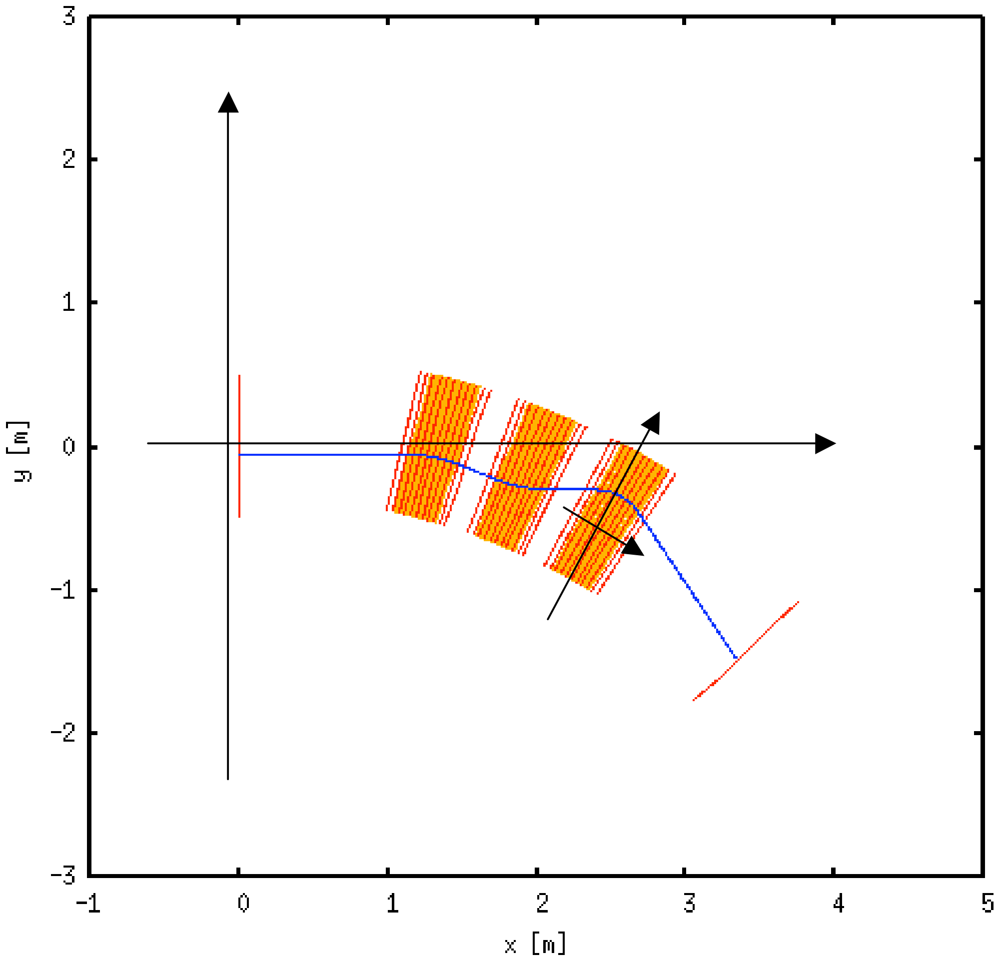
$$p_{x,new} = \sqrt{\left(E_{new}/c\right)^2 - m_0^2 c^2 - p_y^2 - p_z^2}$$

y: horizontal, z: vertical, x: longitudinal

Orbit and optics calculation (3)

local and global coordinates

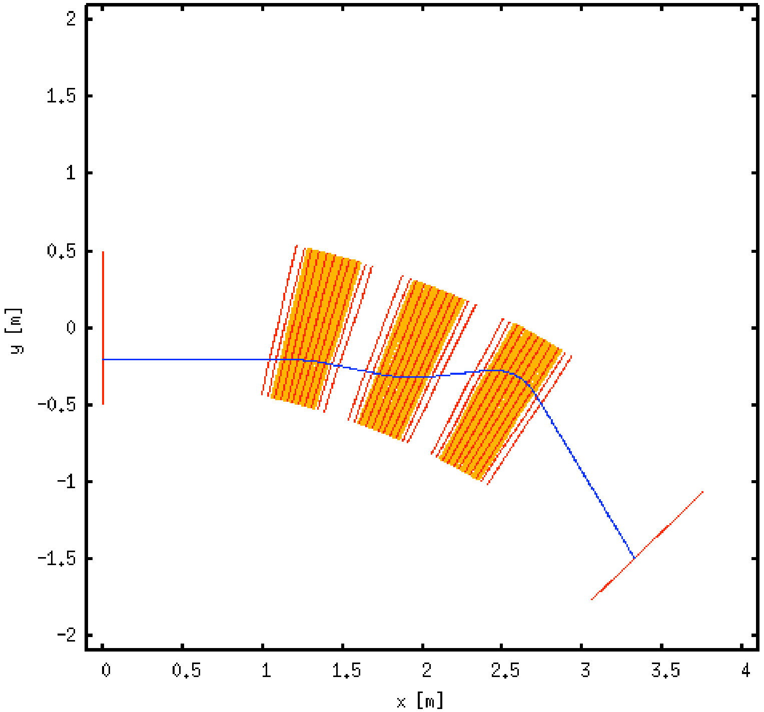
- When a particle reaches field free region, 6-D positions are transferred into the global coordinates.
  - Looking for the next components with a local coordinate attached.
  - Local coordinate is either Cartesian or Cylindrical.
  - Local coordinate is chosen such that the description of field profile becomes simpler and the integration becomes easier.



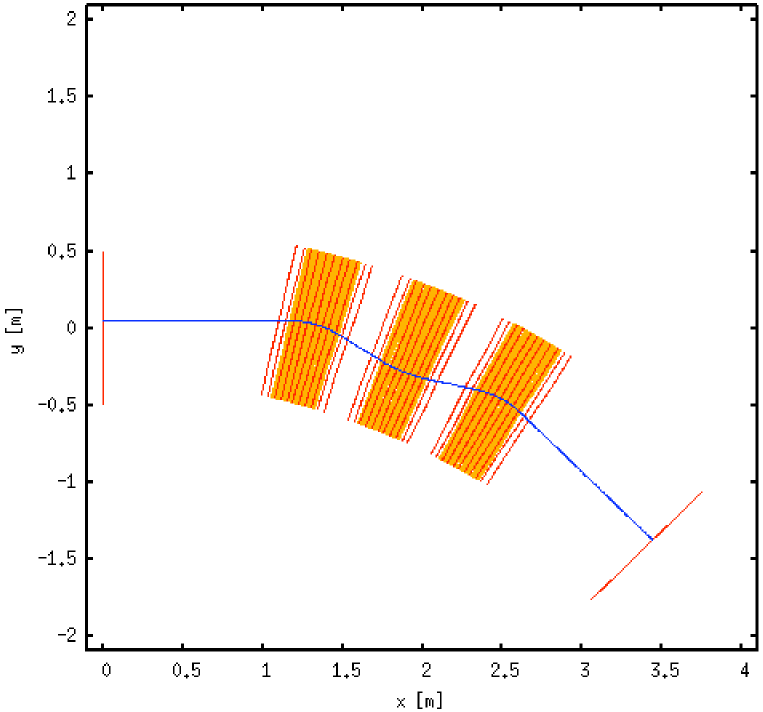
Orbit and optics calculation (4)

boundary condition and iteration

- Make iteration to find an orbit which satisfies the periodic boundary condition.



before

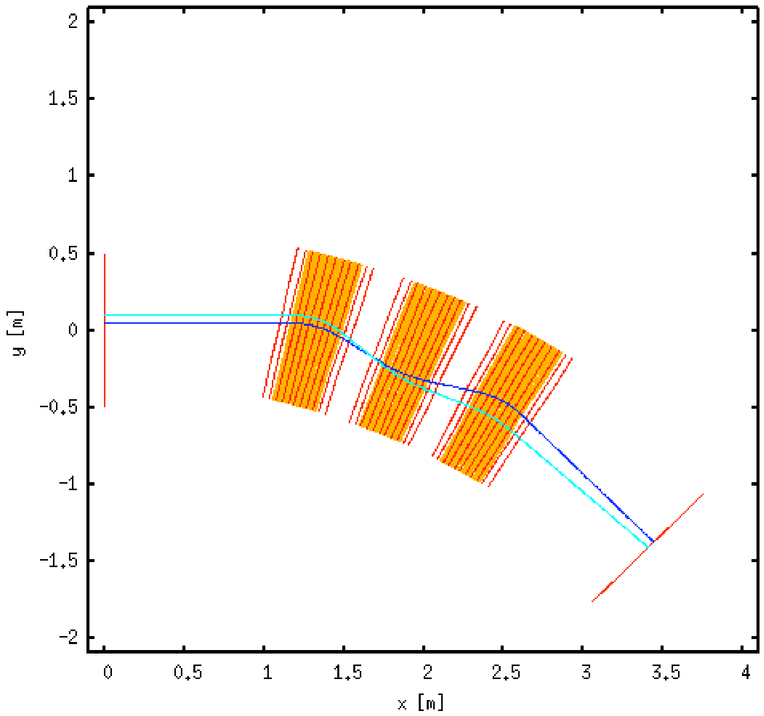


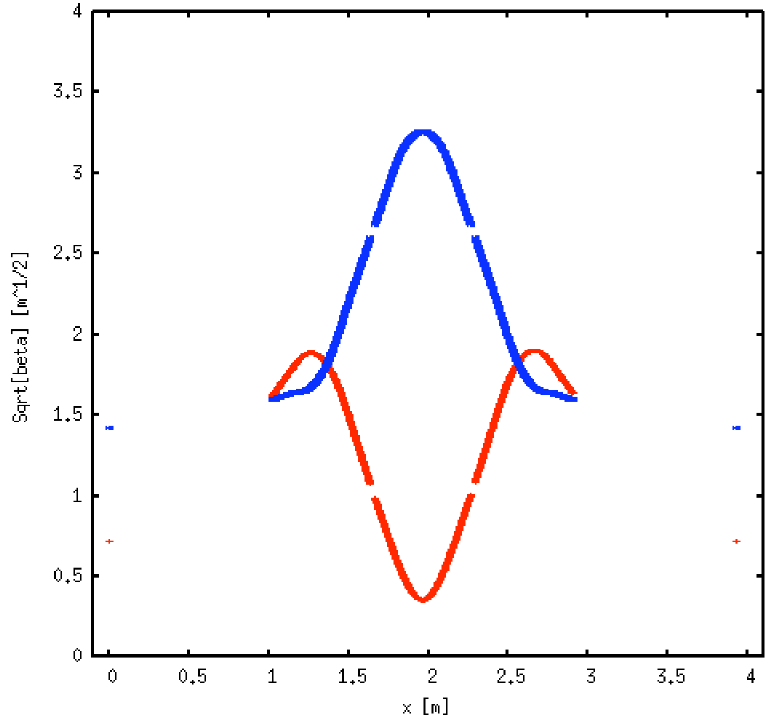
after

Orbit and optics calculation (5)

lattice functions

- Reconstruct a transfer matrix by launching a few particles with slightly shifted initial coordinates.
- Calculate lattice functions by transferring the lattice functions from the initial point.





End fields (1)

quadrupole

- Soft edge model with Enge type fall off.
- Scalar potential in cylindrical coordinates.

$$P_2(r,\theta,z) = \frac{r^2 \sin 2\theta}{2}\left[G_{2,0}(z) + G_{2,2}(z) r^2 + \cdots\right]$$

where

$$G_{2,2k}(z) = (-1)^k \frac{2}{4^k k!(2+k)!} \frac{d^{2k} G_{2,0}(z)}{dz^{2k}}$$

and

$$G_{2,0}(z) = \frac{G_0}{1 + \exp\left(\sum_{i=0}^{5} C_i z^i\right)} \qquad z = \frac{s}{g}$$

s: distance from hard edge.

g: scaling parameter of the order of gap.

C_i: Enge coefficient.

End fields (2)

truncation

- Up to G_{20} and G_{21}
  - Edge focusing
- Up to G_{22} and G_{23}
  - Octupole components of fringe fields
- Up to G_{24} and G_{25}
  - Dodecapole …

- Feed-down of multipoles has large effects when G_{22} and higher order is included.
  - In particular, off-axis orbit.
- No clear criterion up to which term we need to keep.
  - Is it the same problem as fringe field of "multipole" in Zgoubi?

End fields (3)

scaling magnet

- Soft edge model with Enge type fall off applied to r^k magnet.
- Expansion in cylindrical coordinates.

$$B_z(r,\theta,z) = B_0\left(\frac{r}{r_0}\right)^k G(\theta) - \frac{1}{2}\frac{B_0}{r_0{}^2}\left(\frac{r}{r_0}\right)^{k-2} z^2 + \cdots$$

where

$$G\,(z) = \frac{G_0}{1 + \exp\left(\sum_{i=0}^{5} C_i z^i\right)} \qquad\qquad z = \frac{s}{g}$$

s: distance from hard edge.

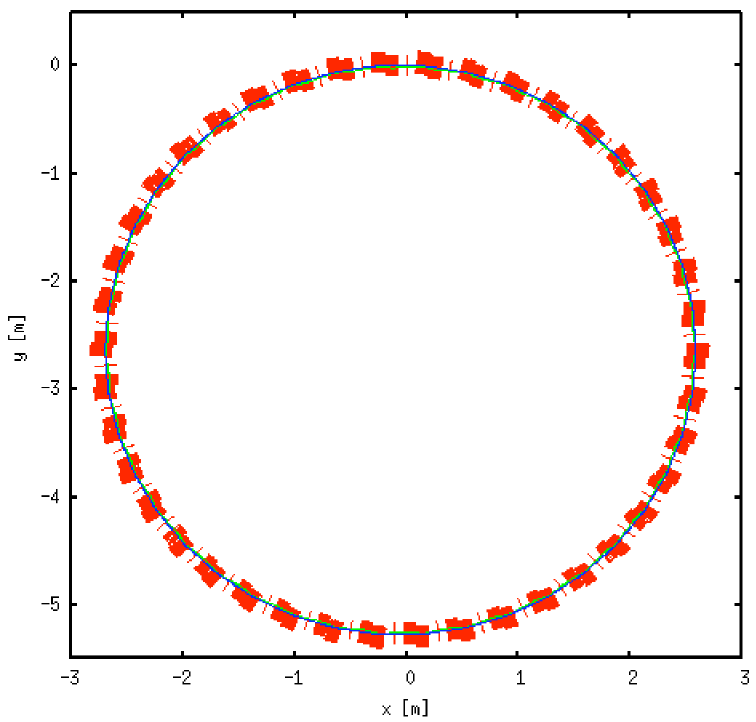
g: scaling parameter of the order of gap.

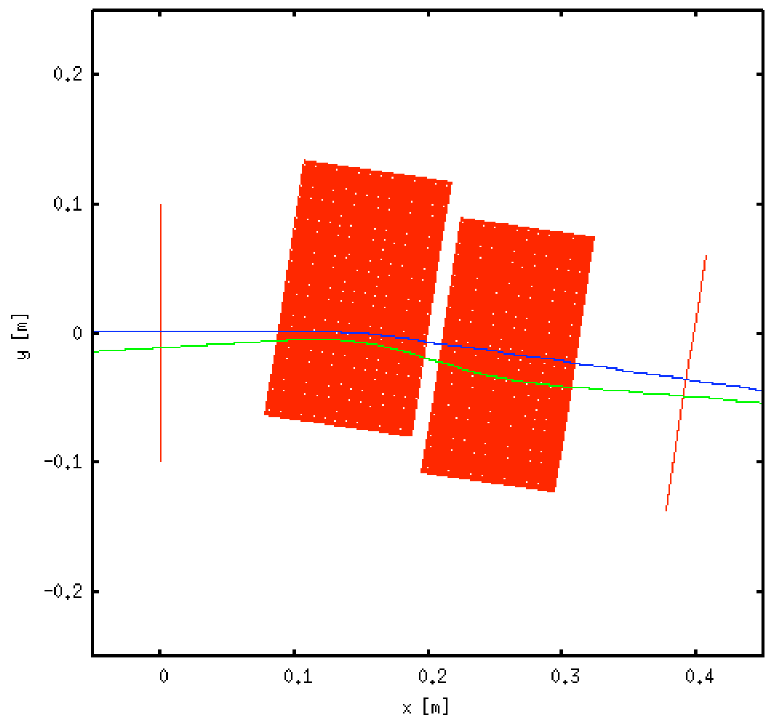
C_i: Enge coefficient.

EMMA modeling (1)

layout of a machine and orbit

- Place magnets and rf cavities.
- Find closed orbit by iteration for several momenta.
  - One cell or a whole ring (periodic condition).



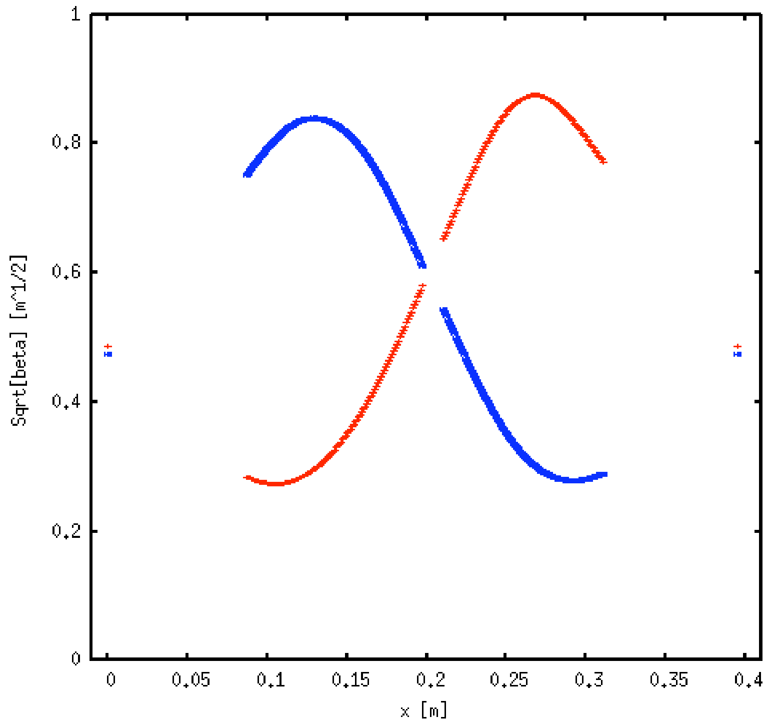


EMMA modeling (2)

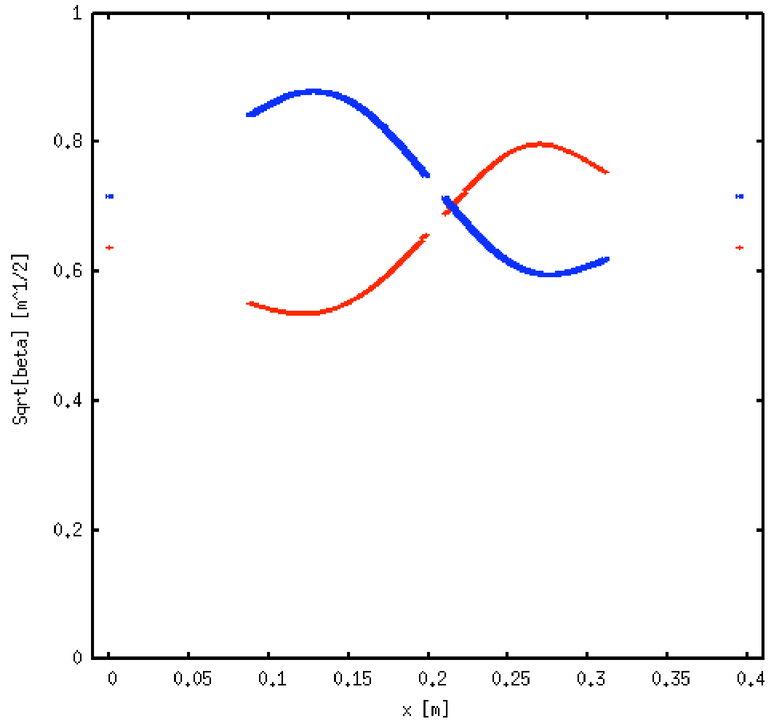
optics calculation

- Calculate lattice function at the initial position and phase advance.
- Calculate lattice function all over the machine.

at 10.5 MeV/c



at 20.5 MeV/c



EMMA modeling (3)
acceleration

- Use the following parameters
  - a=1/6, b=1/4
  - $\varepsilon_{n,rms} = 3 \times 10^{-6} [\pi \, \mathrm{mrad}]$
  - $\sigma_t = 12.5/4 [\mathrm{ps}]$
  - $\sigma_{dp/p} = 0.005$
  - Gaussian with a cut at 2 sigma
  - Acceleration with 10 turns

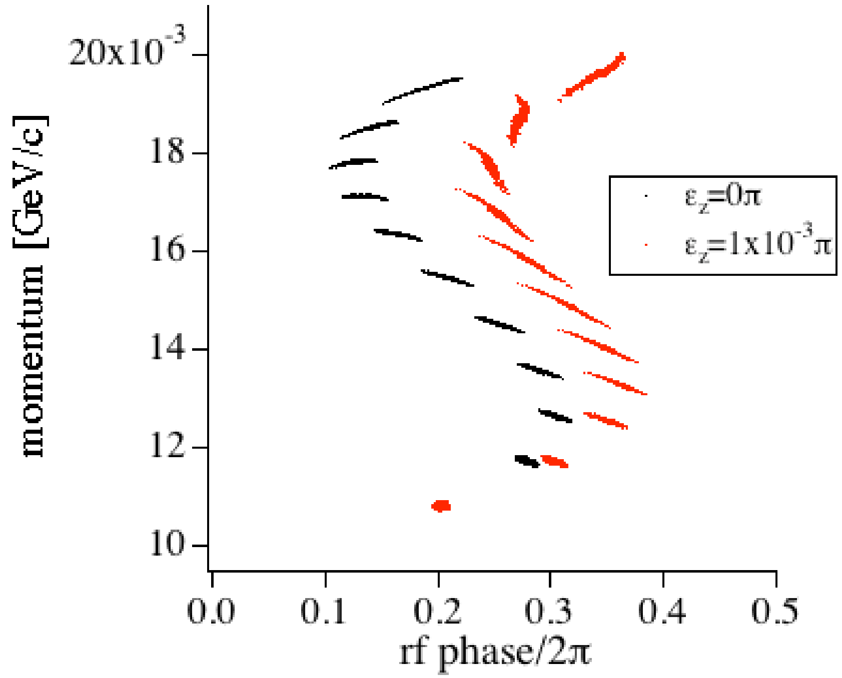


Figure shows *dp/p* of a few %.

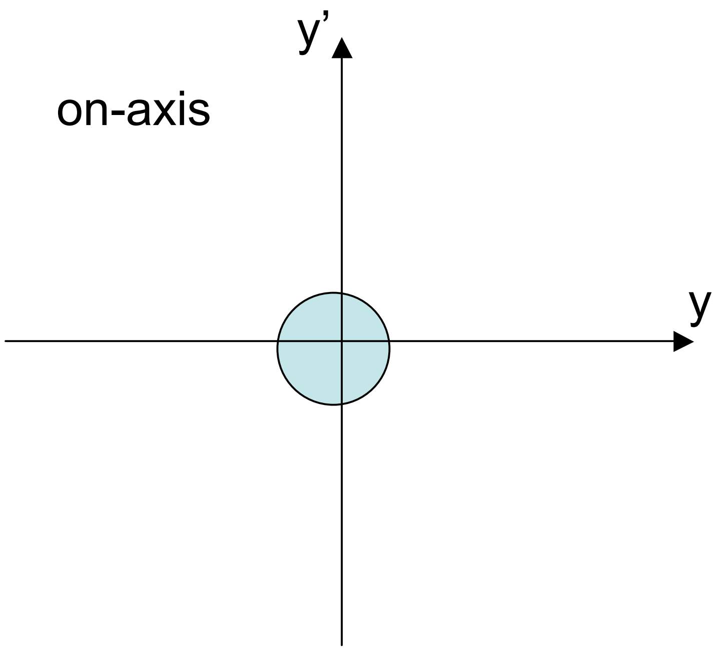
black: on-axis

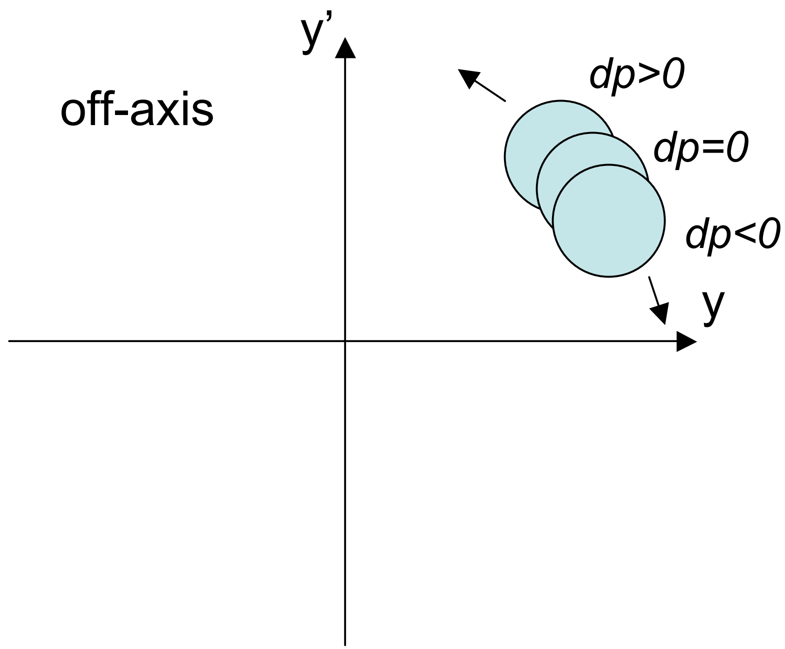
red: off-axis in vertical with 1 π mm rad (normalized)

EMMA modeling (4)

6-D beam behavior with finite chromaticity

- As long as a beam is injected on-axis, tune spread due to finite chromaticity does not appear.
- Once a beam is injected off-axis, smearing in phase space occurs.

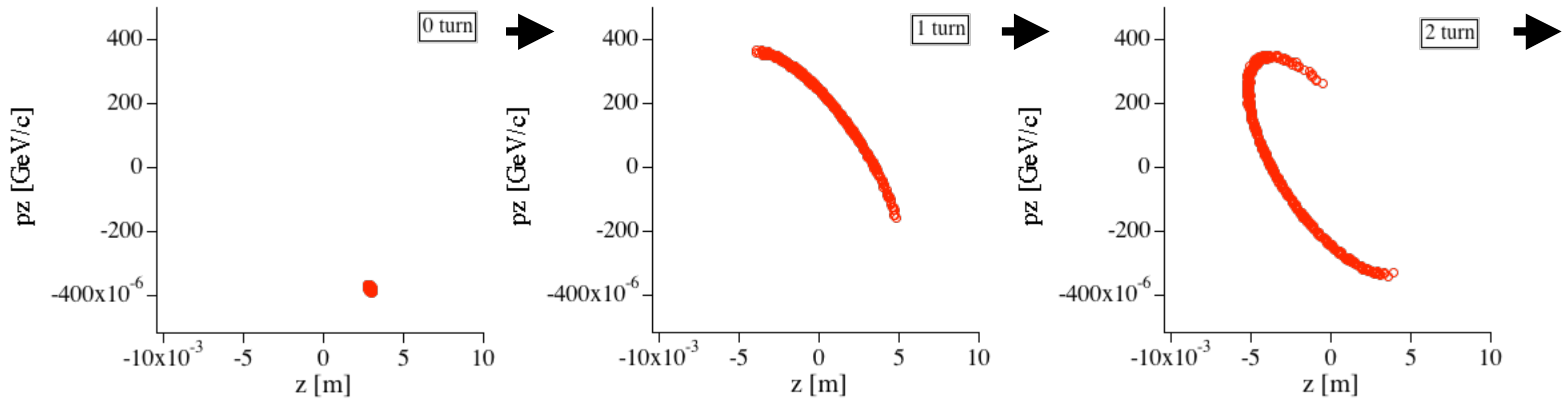


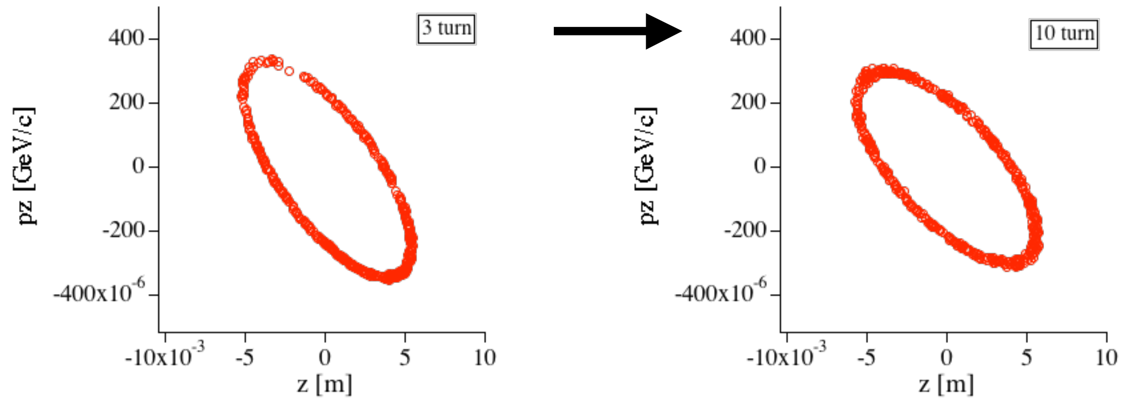


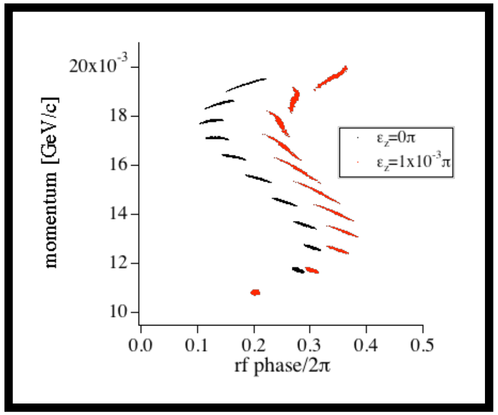
EMMA modeling (5)

phase space evolution (z,pz)

- A beam is initially displaced in vertical direction with 1 π mm rad.







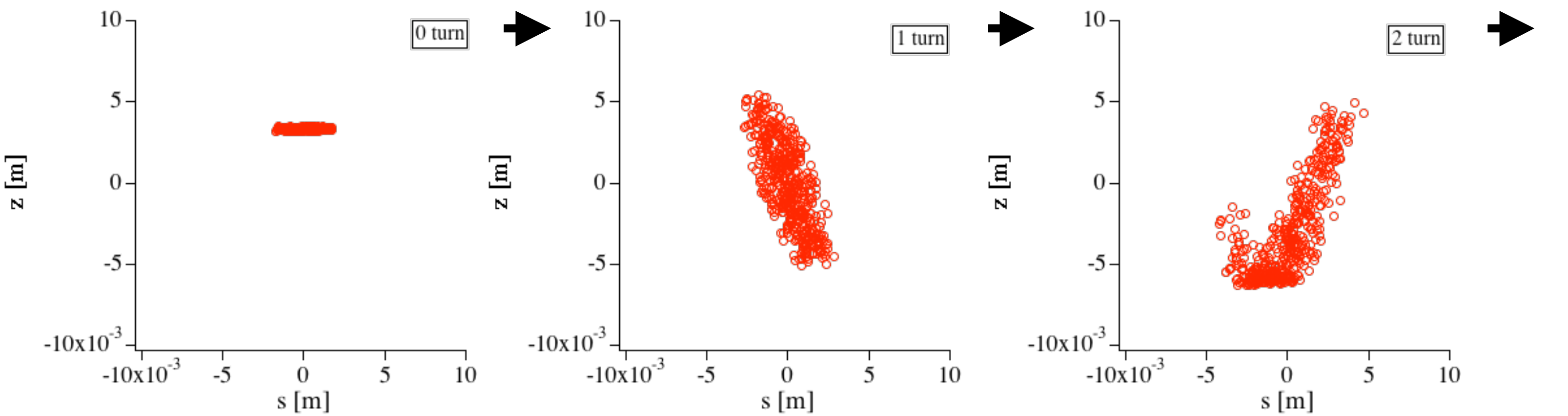
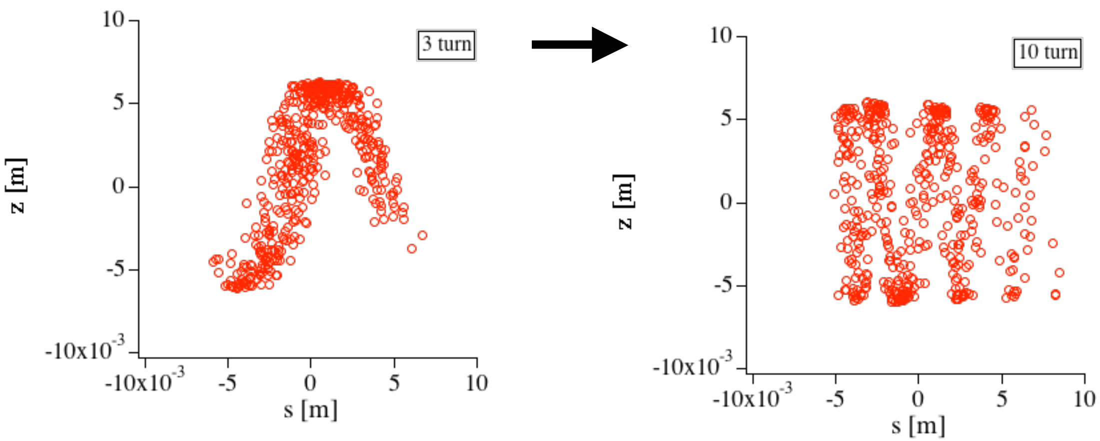
longitudinal

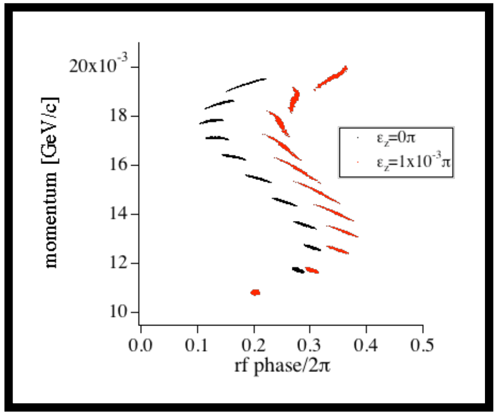
- In 3 turns, a probe beam becomes a ring.
  - Chromaticity is large at low momentum.

EMMA modeling (6)

configuration space evolution (s-longitudinal,z-vertical)

- A beam is initially displaced in vertical direction with 1 π mm rad.



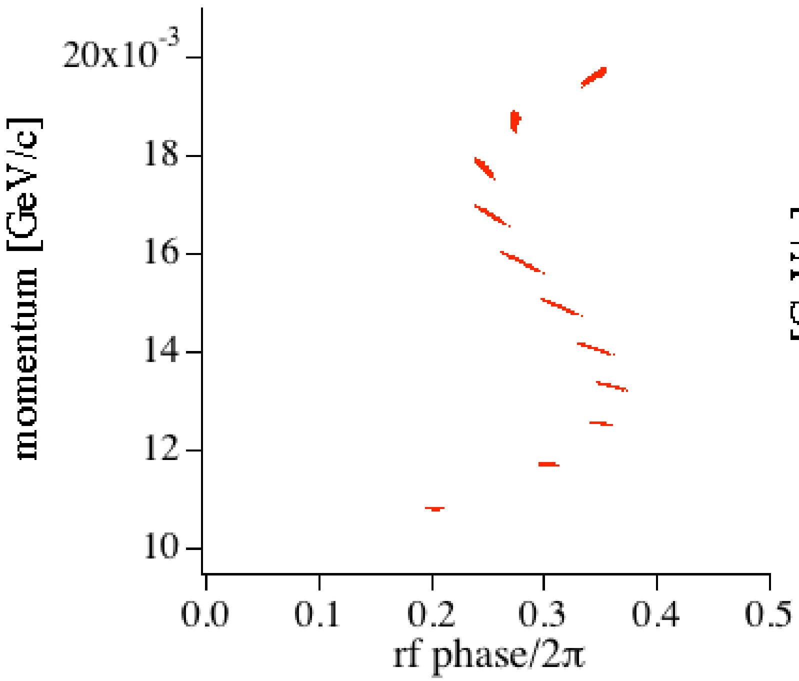
longitudinal

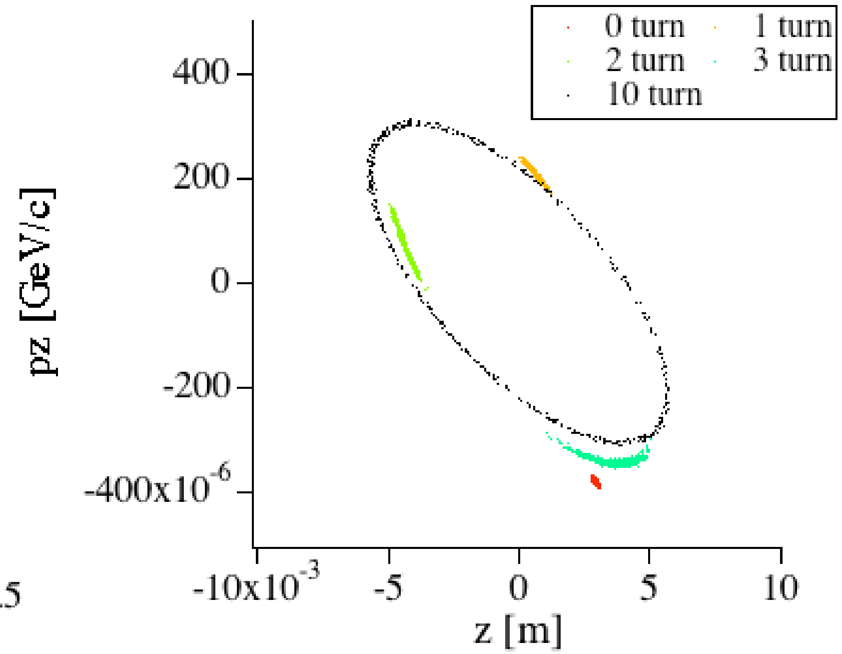
- Correlation among longitudinal position, (momentum,) and vertical position is manifest.

EMMA modeling (7)

when initial dp/p is one tenth (0.0005)

- A beam is initially displaced in vertical direction with 1 π mm rad.





- A probe beam becomes a ring after 10 turns.

Summary

- Lattice components and design orbit are independent. It has to be taken into account in FFAG modeling.
- 6-D tracking reveals interesting (and sometime annoying) characteristics of a linear nonscaling FFAG.
  - Phase slip due to large transverse amplitude.
  - Amplitude growth due to resonance crossing.
  - Smearing of a probe beam in transverse phase space due to finite chromaticity.

  (Zgoubi, PTC, and S-code are available.)

- Beam loading and space charge effects should be taken into account.

  (only S-code?)

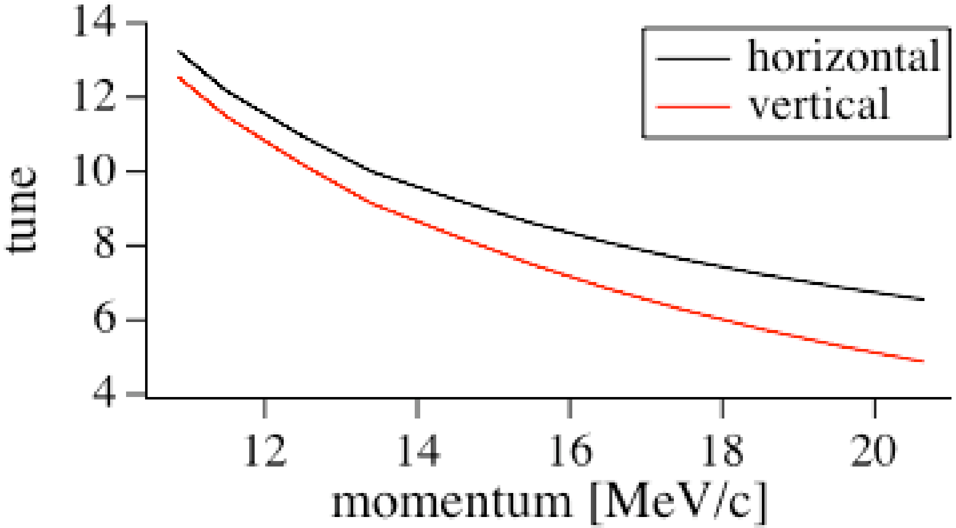
Backup slides

Problem to be discussed (1)

chromaticity

- Nonscaling FFAG has finite chromaticity.
- There is no synchrotron oscillations.

- Phase advance depends on momentum and its difference will be accumulated.



- *From the figure above, average chromaticity is roughly 10.*

$$\xi = \frac{dQ}{dp/p} = \frac{7}{10/15} \sim 10$$

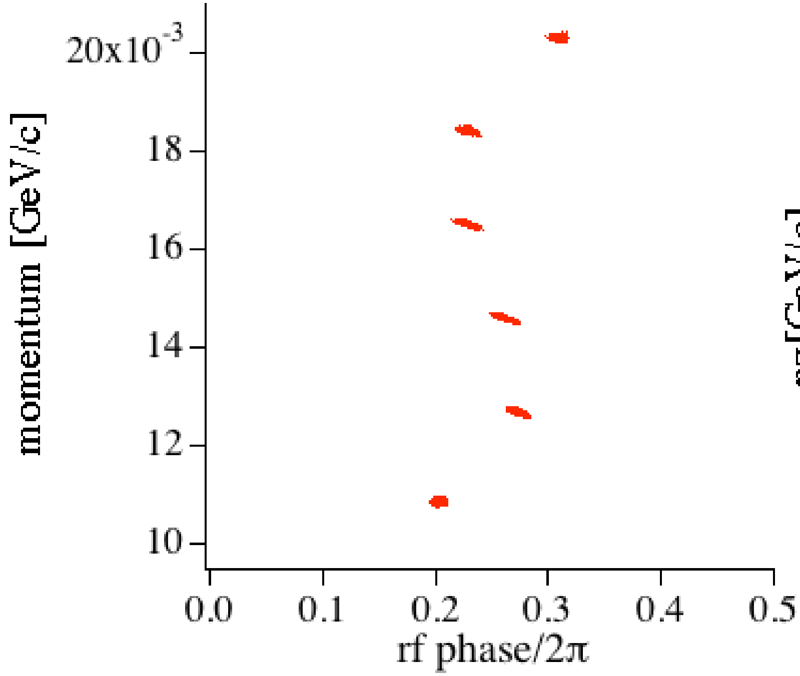
- *Take two particles with momentum difference of 1%, phase difference (/2π) after 10 turns becomes 1.*

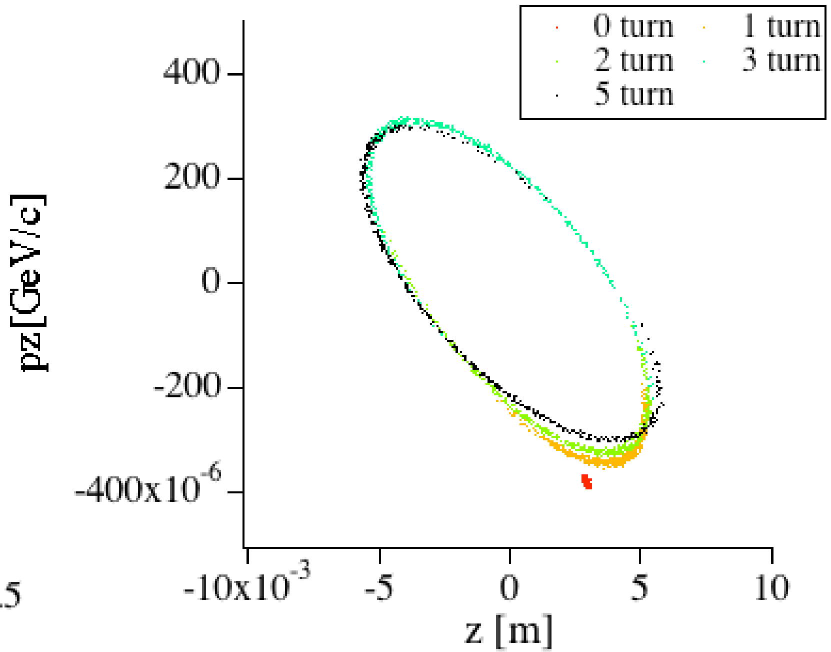
$$\Delta Q = \int_0^T dQ = (\xi \cdot dp/p) \cdot T = 10 \cdot 0.01 \cdot 10 \sim 1$$

Simulation results (5)

when voltage is twice as much

- A beam is initially displaced in vertical direction with 1 π mm rad.





- A probe beam almost becomes a ring after 5 turns.

Simulation results (6)
summary

- A probe beam injected off-axis quickly smears out and becomes a ring with a radius of the off-set amplitude.
- A beam with one tenth of *dp/p* or twice as much voltage does not make difference.

- This is a process which can not be restored at extraction.

c.f. In a synchrotron, it is observed as an "echo" because all the particles go back to a point after one synchrotron oscillation.

$$\Delta Q = \int_0^T dQ = \int_0^T \xi \cdot \left(dp/p \right)_0 \cos 2\pi Q_s dt = 0$$

when $T=nT_s$ (synchrotron period).