

ORBIT TRACKING IN FFAGS AND REVERSE-BEND CYCLOTRONS USING THE CYCLOPS CODE

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FFAG'08 workshop, Manchester, 1st - 5th September 2008

FFAG DESIGN TOOLS

FFAG designs have generally been developed using synchrotron lattice codes - or adaptations of them - perhaps because their designers have mostly come from a synchrotron background.

But synchrotron codes are poorly adapted for use in accelerators with fixed magnetic fields:

- The central orbit is a spiral - rather than a fixed-radius ring - with the equilibrium-orbit (E.O.) radius depending on energy;
- A wide radial region of magnetic field must be characterized.

As a result, special arrangements must be made to deal with momentum-dependent effects accurately.

ORBIT-TRACKING TOOLS

Méot et al¹. have avoided these problems by using ZGOUBI:

- an orbit tracking code originally developed for the study and tuning of mass spectrometers and beam lines.

Here, we report studies made with the cyclotron orbit code CYCLOPS², which tracks particles through magnetic fields specified on a polar grid and determines the equilibrium orbits and their optical properties.

This has the advantages of:

- Being designed for multi-sector machines with wide aperture magnets
- Simultaneous computation of orbit properties at all energies
- Capability of tracking through measured magnetic fields
- Availability of the sister code GOBLIN for accelerated-orbit studies.

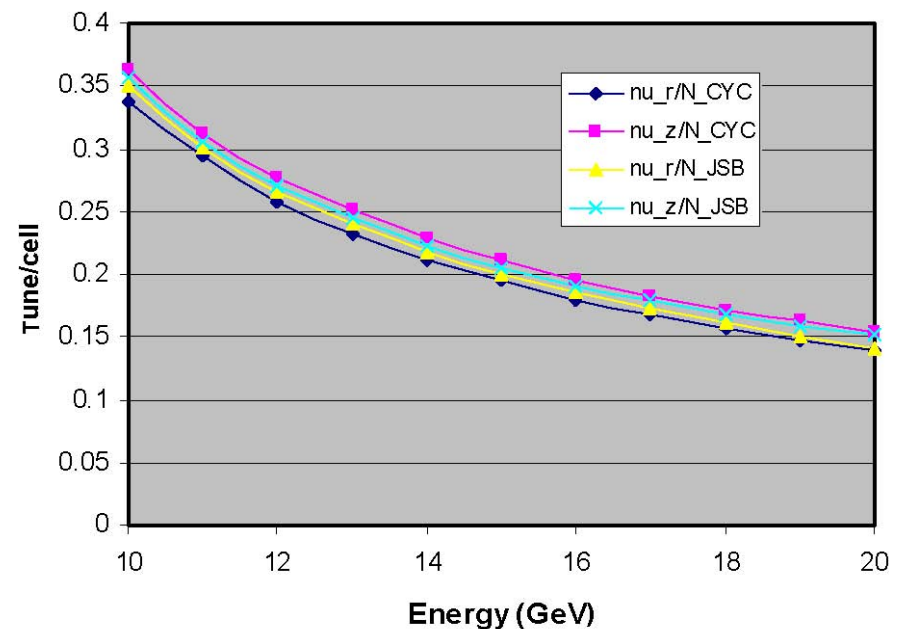
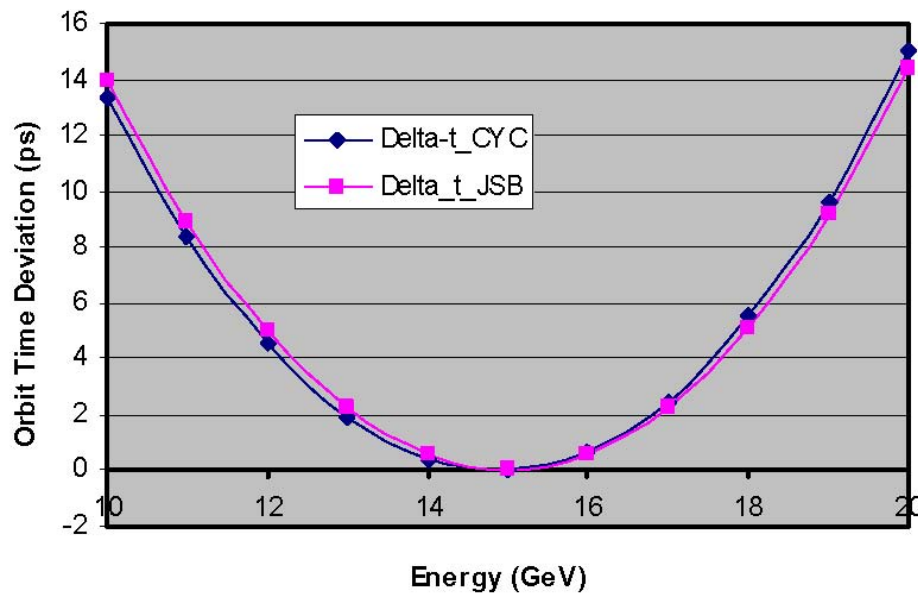
Some initial results were reported³ at Cyclotrons'07.

1. F. Lemuet, F. Méot, G. Rees, Proc. PAC'05, 2693 (2005).
2. M.M. Gordon, Part. Accel. **16**, 39 (1984).
3. M.K. Craddock, Y.-N. Rao, Cyclotrons'07, 370 (2007).

TEST RUNS WITH THE FODO-2 LATTICE

FODO-2 was one of several **linear non-scaling FFAG** lattices developed by **Scott Berg** in 2003-4 for 10-20 GeV muons:

- D is a positive-bending, and F a negative-bending, sector magnet.



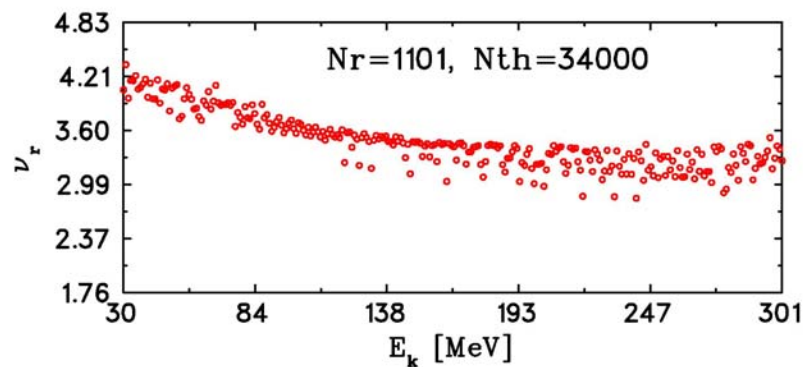
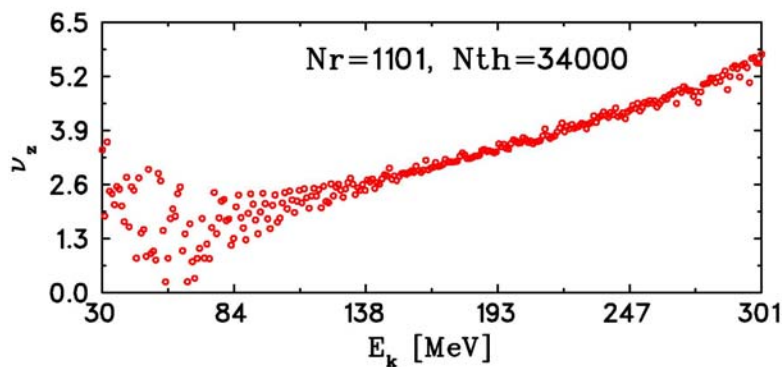
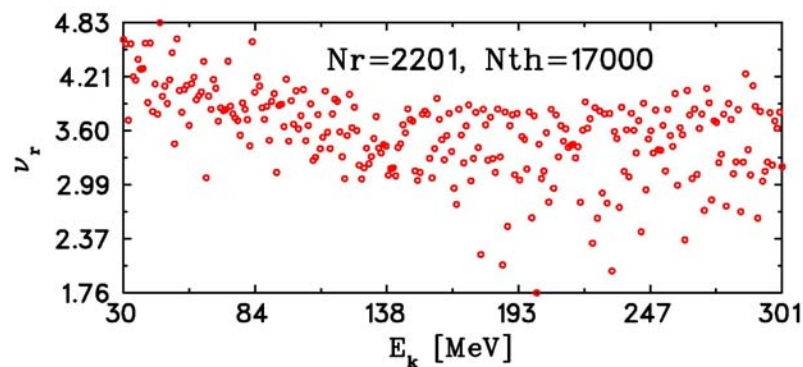
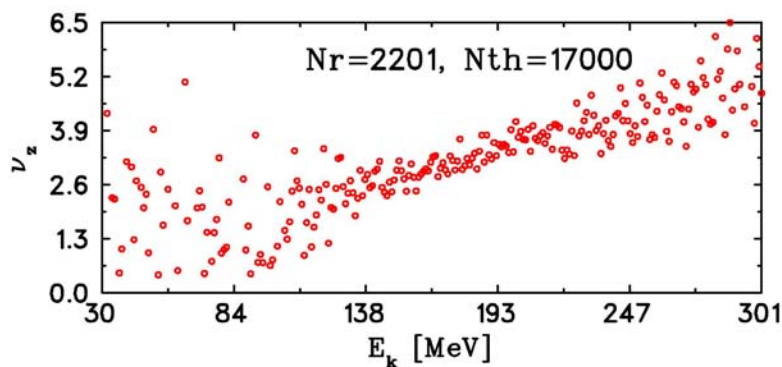
- In order to get **good E.O. solutions** with the **hard magnet edges**, a **very fine field grid** was required (800 θ values, 400 r values).
- The **agreement** between the **CYCLOPS results** (CYC) and **Berg's** (JSB) was **very satisfactory**.

JOHNSTONE-KOSCIELNIAK MEDICAL FFAG (1)

Carol Johnstone and Shane Koscielniak have developed a LNS FFAG, based on a FODO lattice, for cancer therapy with 18-400 MeV/u carbon ions⁴.

This uses **edge-** as well as **gradient-focusing** to **minimize the tune variation**.

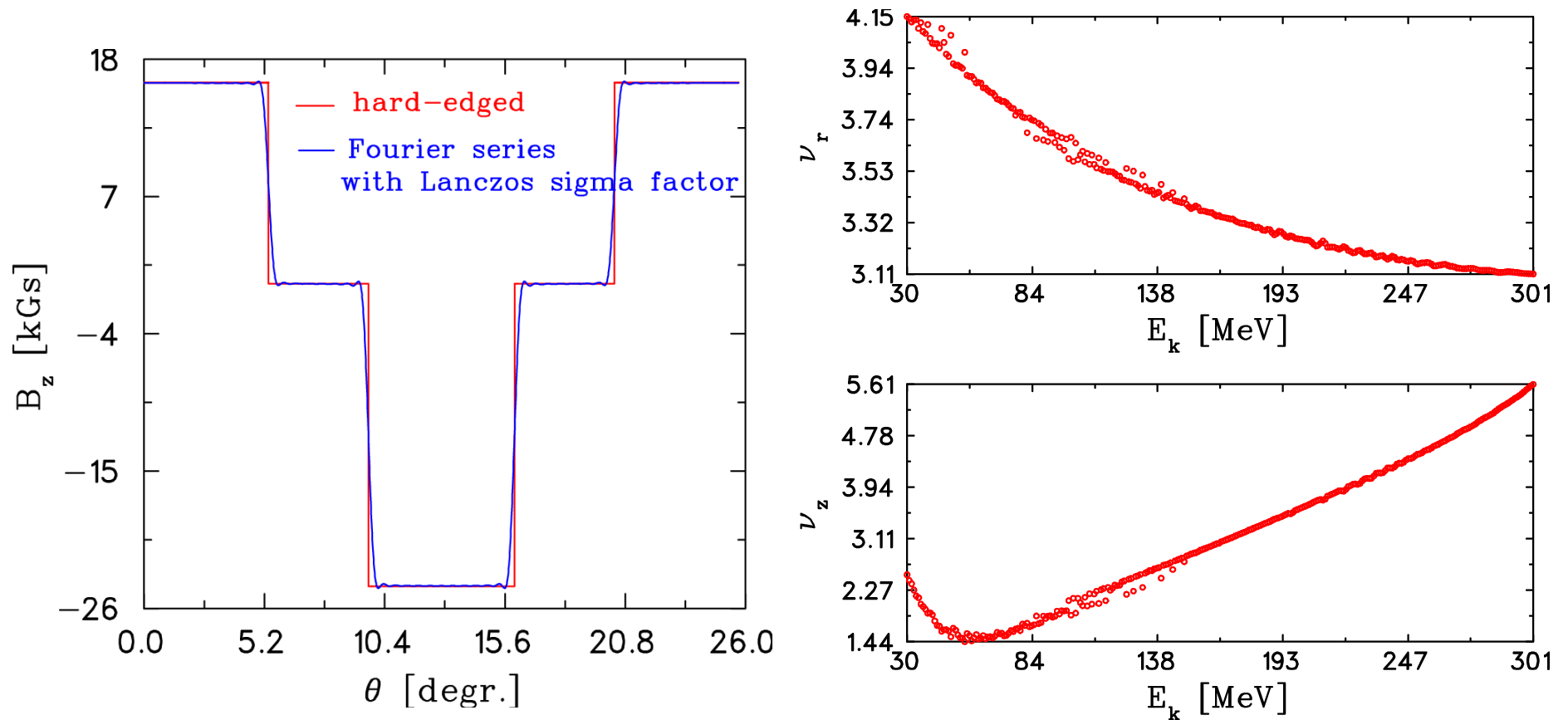
But **non-radial hard magnet edges** are tricky to model with a polar grid - and lead to **noisy results** from CYCLOPS - even with 37 million grid points!



4. C. Johnstone, S.R. Koscielniak, *Proc. PAC'07*, 2951-3 (2007).

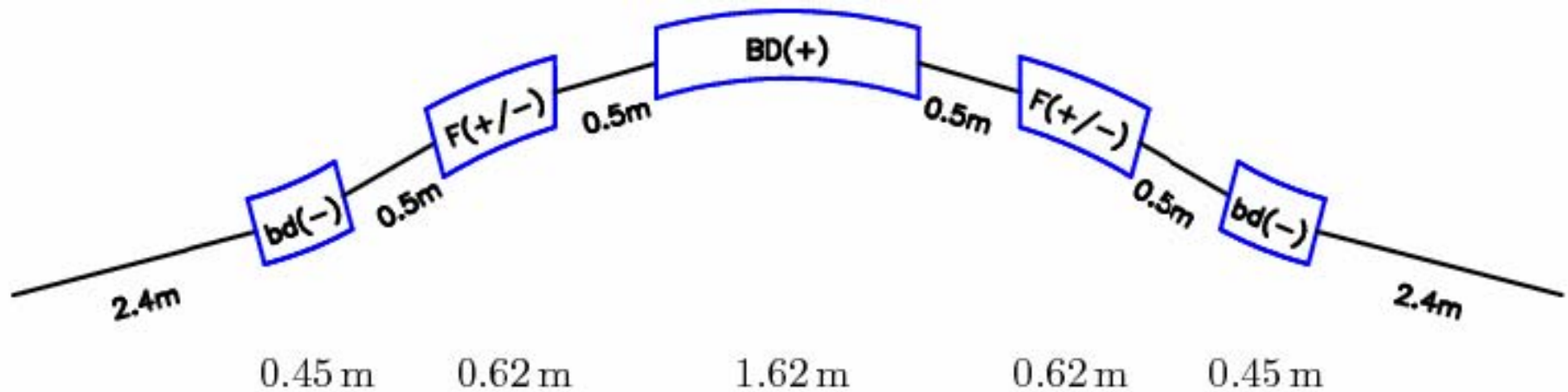
JOHNSTONE-KOSCIELNIAK MEDICAL FFAG (2)

To **smooth the field's hard edges** our initial technique was to fit $B(\theta)$ at each radius by a Fourier series, combined with Lanczos σ -factors to **eliminate ringing** - simple and quite effective:



REES'S ISOCHRONOUS IFFAG

G.H. Rees^{1,5} has designed several FFAGs using novel 5-magnet "pumpkin" cells, in which variations in field gradient and sign enable each magnet's function to vary with radius - providing great flexibility.

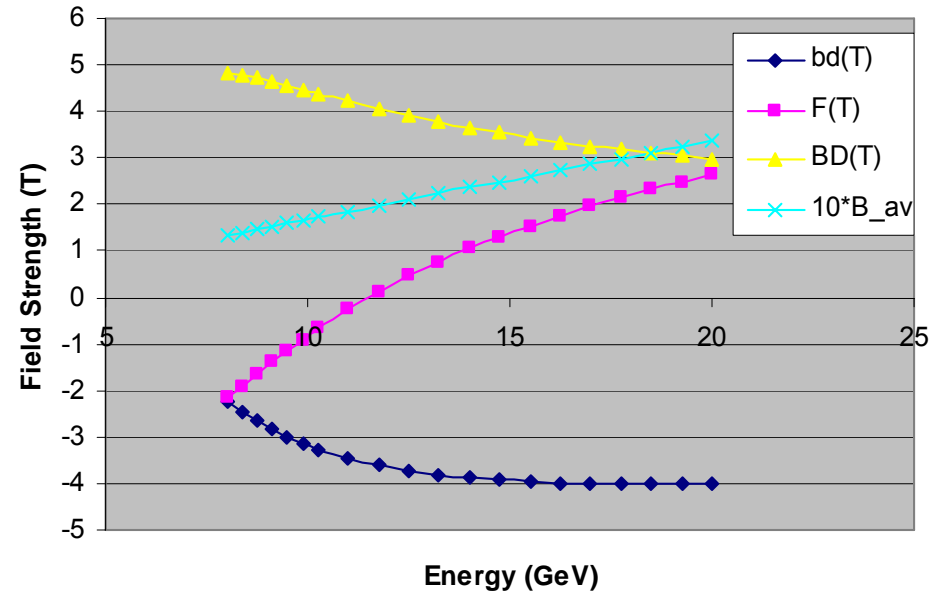


- The example shown is an **isochronous** design (**IFFAG**) for accelerating **muons from 8-20 GeV** in 16 turns.
- This is remarkable in achieving both isochronism and vertical focusing at highly relativistic energies ($77 \leq \gamma \leq 190$) without invoking spiral magnet edge focusing [recall isochronous $\Delta v_z^2 = -(r/B_{av})(dB_{av}/dr) = -\beta^2 \gamma^2$].
- Highest energy spiral-sector isochronous cyclotron design had $\gamma \leq 15$.

IFFAG FIELDS & TUNES

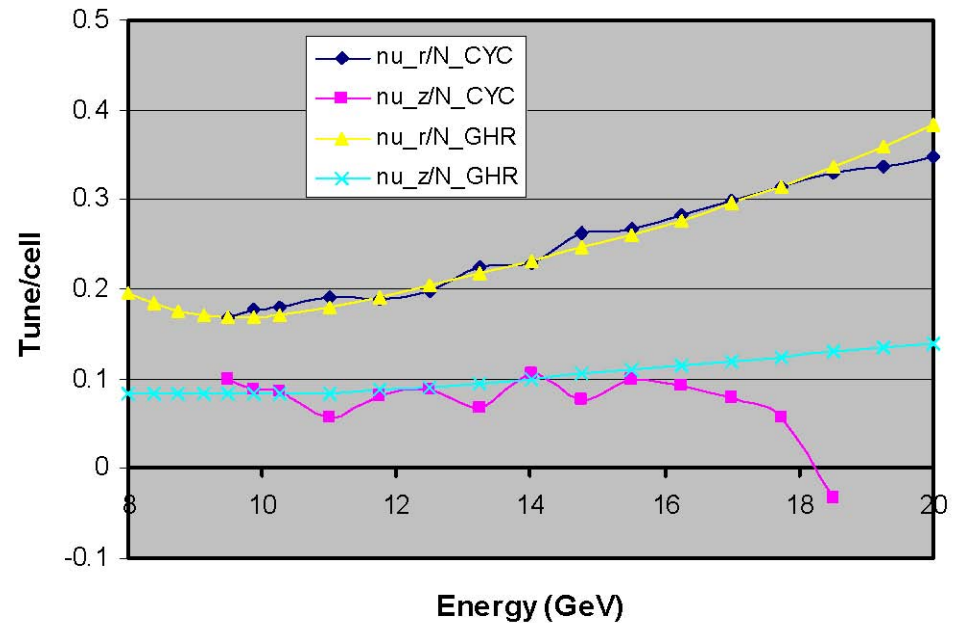
For the **fields**, note how:

- **F** reverses sign at ~ 11 GeV
- **bd** focusing vanishes at high E
- **BD** focusing vanishes at low E
- B_{av} rises linearly with E .



For the **tunes** our initial results³ (\blacklozenge \blacksquare) were in general agreement with Rees's (\blacktriangle \times) - except above 17 GeV - but rather noisy.

As before, a **finer mesh** is needed to map the non-radial hard edges adequately - requiring more grid points than *CYCLOPS* can handle.

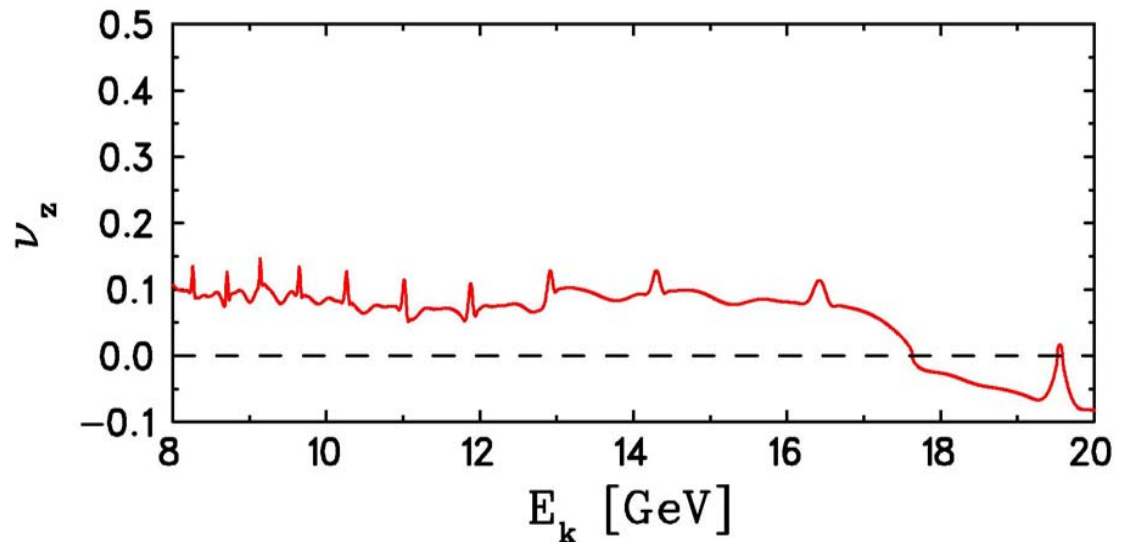
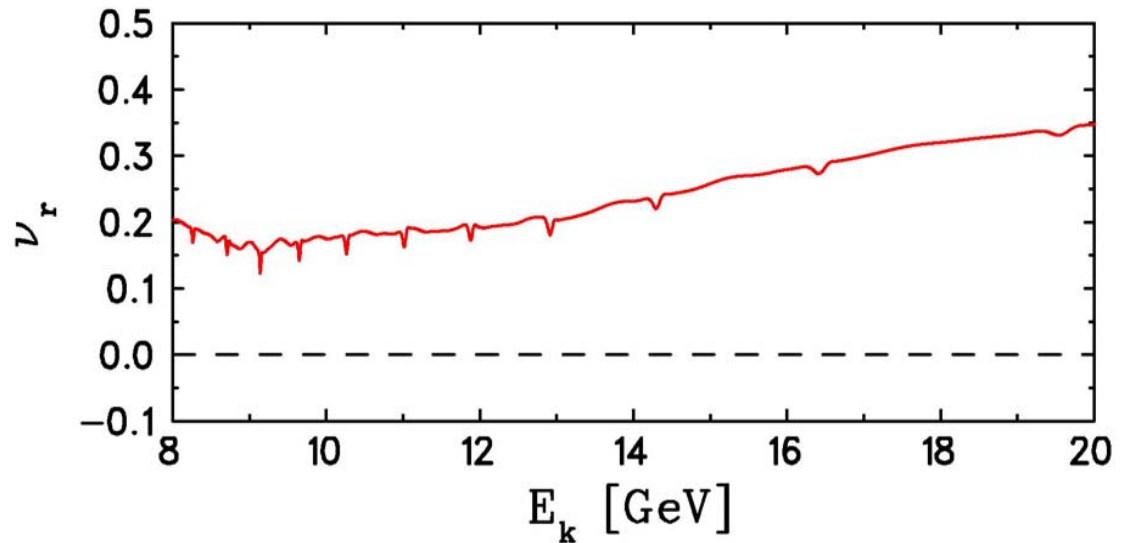


IFFAG WITH SOFTENED MAGNET EDGES

Smoothing the hard-edge data by **Fourier-fitting them azimuthally** gives less noisy results, but leaves regular spikes - probably due to the remaining radial steps in the data.

A scheme to **smooth in both r and θ** by replacing the hard edges in the raw data by sloping ones a few grid steps wide is under way. This should eliminate the spikes.

Note that the results for v_z continue to disagree with Rees's above 17 GeV.



RADIAL-SECTOR CYCLOTRONS WITH REVERSE BENDS

The IFFAG is essentially an isochronous ring cyclotron with an unusually complicated magnet arrangement - 5 magnets/cell rather than 1.

An isochronous cyclotron's top energy is limited by vertical focusing:

$$v_z^2 \approx -\beta^2 \gamma^2 + F^2(1 + 2\tan^2 \varepsilon)$$

where ε is spiral angle and the magnetic "flutter" (mean square deviation)

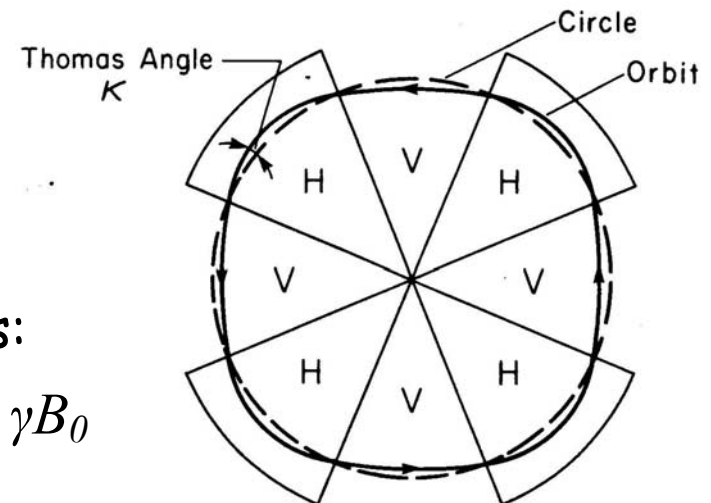
$$F^2 \equiv \left\langle \left(\frac{B(\theta)}{B_{av}} - 1 \right)^2 \right\rangle.$$

How high an energy could a radial-sector cyclotron reach by just converting the field-free "valley" to a reverse bend to maximize F^2 ?

To begin with, we assume:

- N radial sectors
- Hard-edge magnets
- No drift spaces
- Equal and opposite hill and valley fields:

$$B_h = -B_v = B(r) = \gamma B_0$$



Denoting the **angular fraction of a sector taken up by a hill** as h , and **ignoring scalloping effects** on the orbit length and average field around it:

$$B_{av} = 2(h - 1/2)B .$$

The **flutter is determined entirely by h** , and so is the **same at all energies**:

$$F^2 = 1/4(h - 1/2)^{-2} - 1 .$$

For the **axial focusing to remain positive up to some maximum energy γ_m** , but no further, the tune formula tells us that:

$$h - 1/2 = 1/2\gamma_m .$$

If the maximum magnetic field available, B_m , is applied at maximum energy γ_m , then the **"central field" B_c** and **"cyclotron radius" R_c** are given by:

$$B_c = B_m / \gamma_m^2$$

$$R_c = (m_0 c / e) \gamma_m^2 / B_m ;$$

i.e. **the ring radius required increases as the square of the desired energy.**

The recipe for hill and valley field strength is:

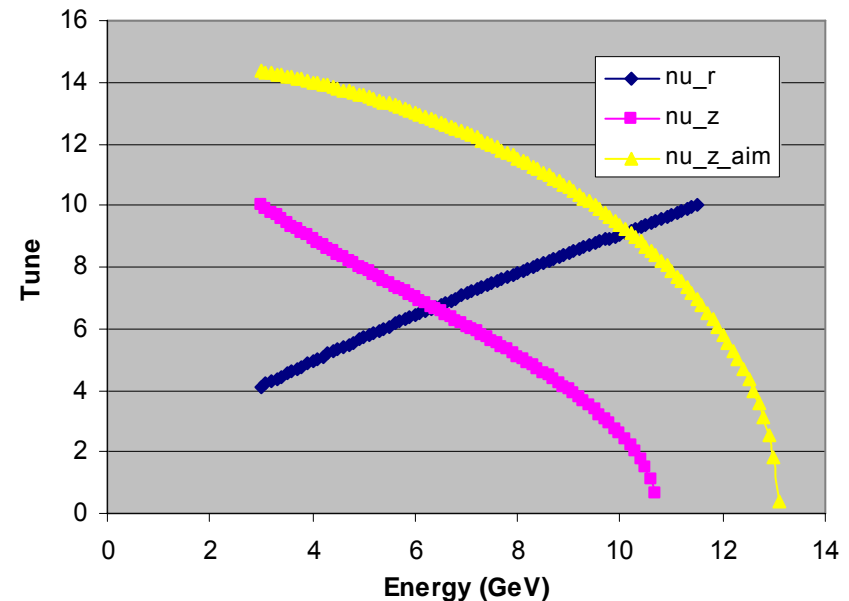
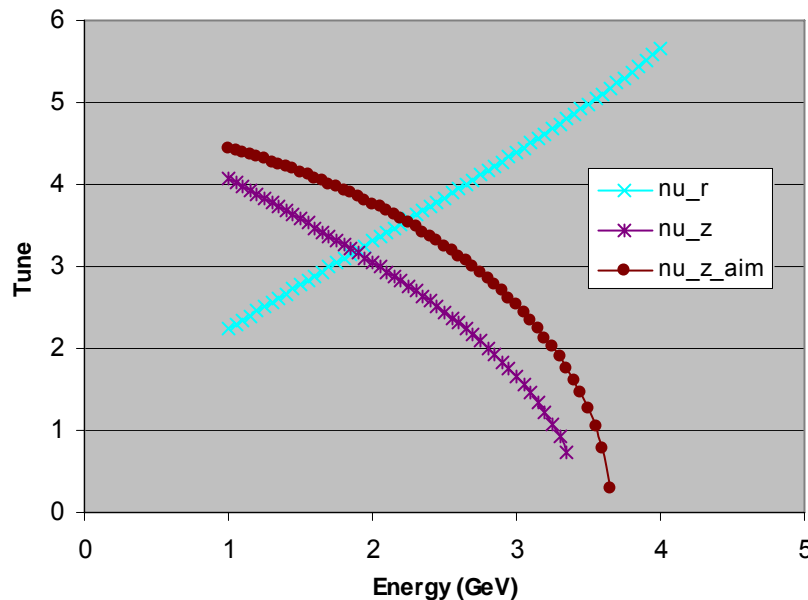
$$B(r) = (B_m / \gamma_m) / \sqrt{\{1 - (r/R_c)^2\}} .$$

Symon's circumference factor, the ratio of the actual circumference to that obtainable with uniform B_m and no reverse bends: $C = \gamma_m$.

REVERSE-BEND CYCLOTRONS - SIMULATIONS (1)

We studied two cases with specifications similar to old spiral-cyclotron proposals^{6,7}, (with $N \approx 3\gamma_m$ so $v_r \approx \gamma$ stays well below the $N/2$ resonance):

E (GeV)	γ_m	N	h	F^2	$B_m(T)$	$B_c(T)$	$R_c(m)$
1 – 4	5	15	0.6	24	5	0.2	15.65
3 - 13	15	45	0.533	224	5	0.022	140.83



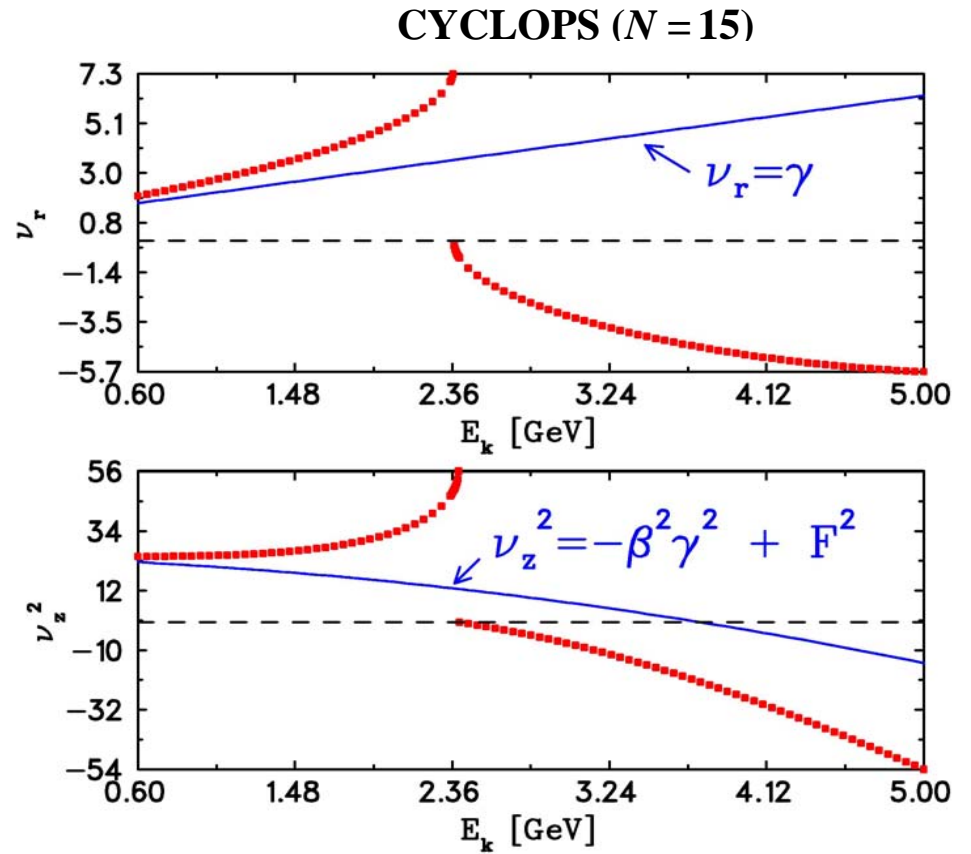
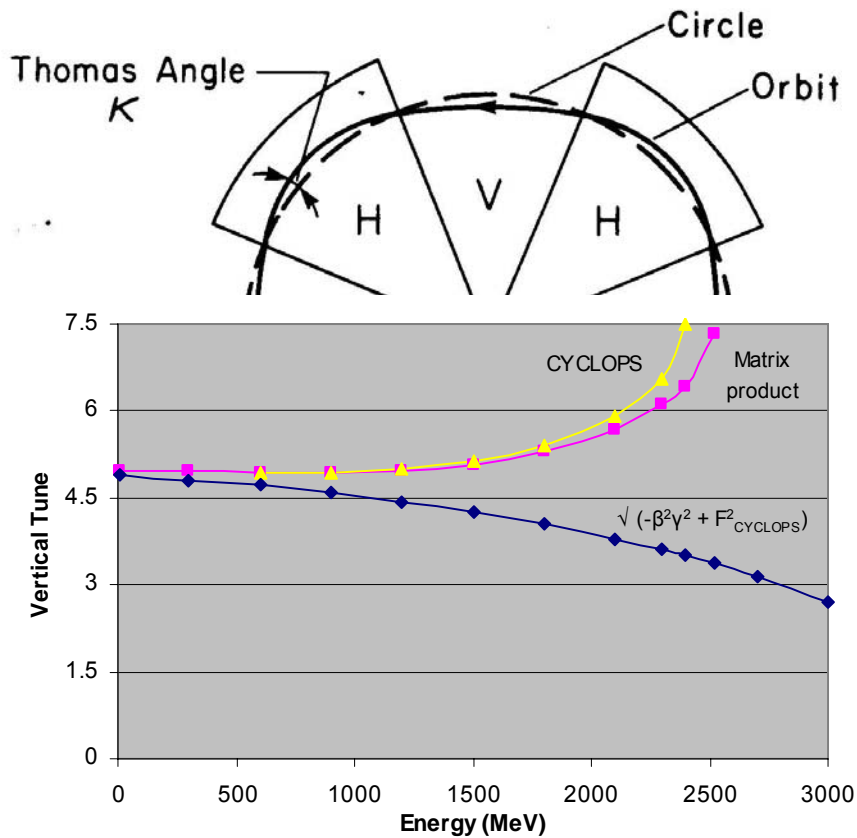
In both cases ν_z drops to 0 below the desired E_{max} - reflecting a **drop in flutter** due to the **orbits not following arcs of constant B**.

6. M.K. Craddock, C.J. Kost, J.R. Richardson, Proc.CYC'78, IEEE Trans. NS-26, 2065 (1979).

7. J.I.M. Botman, M.K.Craddock *et al.*, Proc. PAC'83, IEEE Trans. NS-30, 2007 (1983).

REVERSE-BEND CYCLOTRONS - SIMULATIONS (2)

The flutter and focusing can be restored by **shaping the field contours to follow the scalloped orbits**:

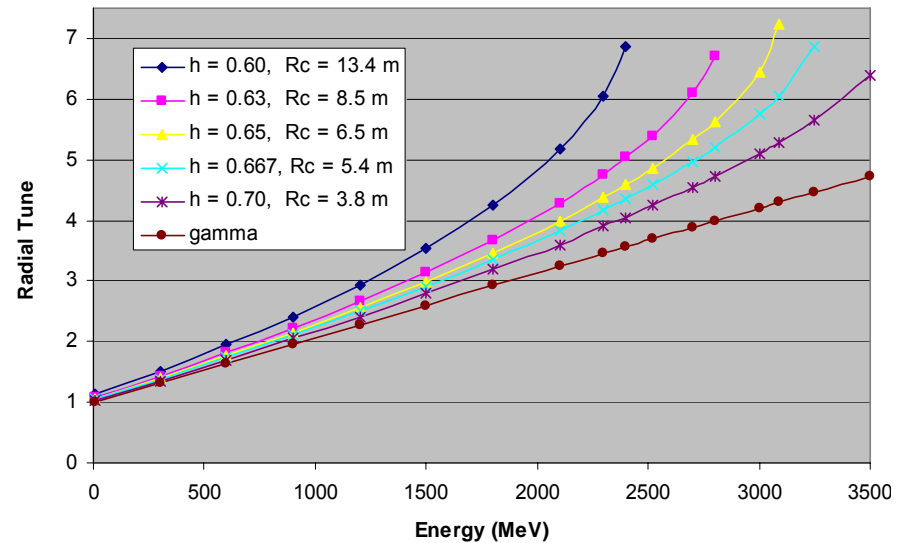
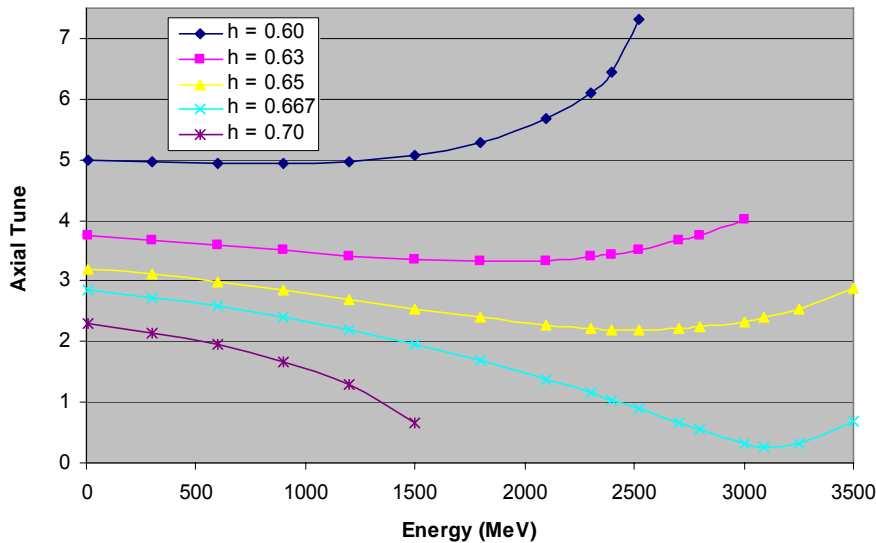


But the focusing is now too strong!

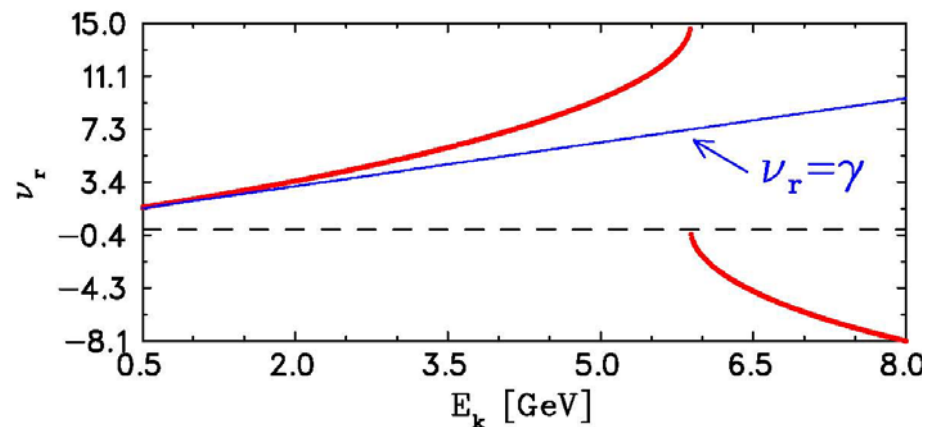
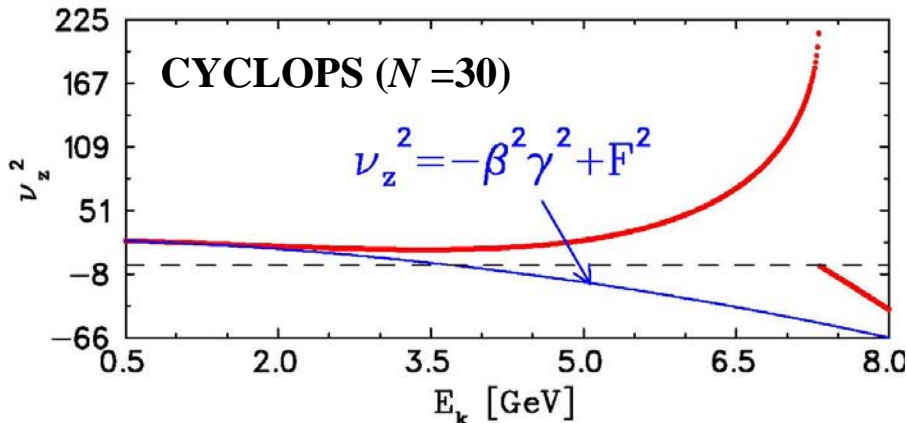
- the $N/2$ resonances ($\nu = 7.5$) are reached at only $\gamma \approx 3.5$;
- ν_z from CYCLOPS tracking and matrix multiplication agree fairly well;
- the tunes are driven well above the low-energy cyclotron formulae.

REVERSE-BEND CYCLOTRONS - SIMULATIONS (3)

The top energy can be raised either by widening the hills
 - reducing the radius and both tunes (say $h = 0.65$, $E = 3$ GeV, $R_c = 6.5$ m),



or by increasing the number of sectors (say to $N = 30$, with $h = 0.6$)
 - a more effective way of repelling $\nu_r = N/2$ ($E = 6$ GeV, but $R_c = 14.9$ m).



SUMMARY

The **cyclotron E.O. code CYCLOPS** has been applied to several FFAGs:

- LNS **FODO-2** for 10-20 GeV muons
- LNS edge-focusing FODO **medical FFAG** for 18-400 MeV/u C ions
- NLNS **isochronous pumplet IFFAG** for 8-20 GeV muons.

CYCLOPS is:

- **designed for energy-dependent E.O.s in wide-aperture magnets**
- **ideal for finding E.O. properties in measured magnetic fields**
- **uncomfortable with hard-edge magnets - data softening needed!**

Using **reverse bends in radial-sector ring cyclotrons** has also been studied:

- **$N/2$ resonance makes simple tune formulae useless at higher energies;**
- **higher energies achievable if the field contours follow the orbits;**
- **with 5-T magnets & 0 drifts: 3 GeV for $R_c = 6.5$ m, 6 GeV for $R_c = 15$ m.**