Mathematical Physics Activities

Robin W Tucker

Cockcroft Institute and Lancaster University

2 November 2009

Mathematical Physics Activities

- Cockcroft Institute and University Staff: 2008-2009
    - R W Tucker (Lancaster University Faculty)
    - D A Burton (Lancaster University Faculty)
    - J Gratus (Lancaster University Faculty)
    - S Goto (Lancaster RA)
    - A Hale (Lancaster University RA)
    - V Perlick (Lancaster University RA)
    - D Christie (Lancaster University RA)
    - A Noble (Strathclyde/Lancaster University RA)
    - T Walton (Lancaster University RA)

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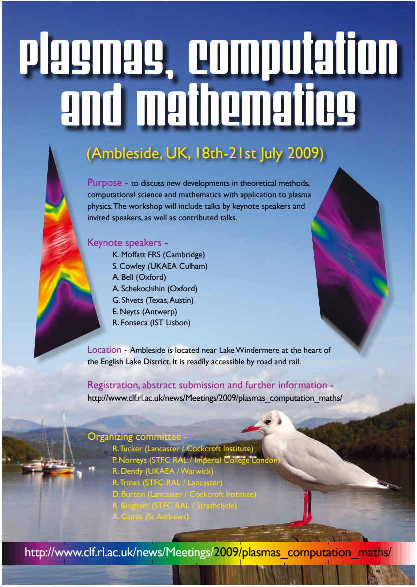
  - D Christie (Lancaster University RA)
  - A Noble (Strathclyde/Lancaster University RA)
  - T Walton (Lancaster University RA)
- External Active Collaborators: 2008-2009
  - Dino Jaroszynski (Strathclyde University, Scotland)
  - Tekin Dereli (Koc University, Turkey)
  - Raoul Trines (RAL, UK)

Mathematical Physics Activities

- Lancaster University Theoretical Accelerator Science PhD Research Students: 2008-2009
    - Alison Hale (PhD Sept 09)
    - Roberto Gallego Torrome
    - Jonathan Smith
    - Cherry Canovan
    - Michael Ferris
    - Haibao Wen

Mathematical Physics Activities



plasmas, computation and mathematics

(Ambleside, UK, 18th-21st July 2009)

Purpose - to discuss new developments in theoretical methods, computational science and mathematics with application to plasma physics. The workshop will include talks by keynote speakers and invited speakers, as well as contributed talks.

Keynote speakers -

- K. Moffatt FRS (Cambridge)
- S. Cowley (UKAEA Culham)
- A. Bell (Oxford)
- A. Schekochihin (Oxford)
- G. Shvets (Texas, Austin)
- E. Neyts (Antwerp)
- R. Fonseca (IST Lisbon)

Location - Ambleside is located near Lake Windermere at the heart of the English Lake District. It is readily accessible by road and rail.

Registration, abstract submission and further information -
http://www.clf.rl.ac.uk/news/Meetings/2009/plasmas_computation_maths/

Organizing committee -

- R. Tucker (Lancaster / Cockcroft Institute)
- P. Norreys (STFC RAL / Imperial College London)
- R. Dendy (UKAEA / Warwick)
- R. Trines (STFC RAL / Lancaster)
- D. Burton (Lancaster / Cockcroft Institute)
- R. Bingham (STFC RAL / Strathclyde)
- A. Cairns (St Andrews)

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Mathematical Physics Activities

Current Projects: 2009

- Assimilation of CLIC concepts and role of synchrotron radiation and wakefields in its operation.
- Analysis of fundamental role of coherent radiation in novel accelerator diagnostic tools for determination of longitudinal bunch charge profile.
- Simulation tools for FEL design and Analysis
- Simulation tools for RF cavity design and collimation structures.
- Behaviour of charged particles in magneto-plasmas and general aspects of fields in polarisable and magnetisable inhomogeneous moving media.
- Modelling wakefields and their role in beam dynamics
- Analysis of particle trapping and relativistic warm plasma waves near breaking
- Construction of effective models of relativistic plasmas

Mathematical Physics Activities

Current Projects: 2009

- Analysis of fundamental role of coherent radiation in novel accelerator diagnostic tools for determination of longitudinal bunch charge profile. (Tucker, Hale, Gratus)
- Simulation tools for FEL design and Analysis. (Tucker, Hale, Goto)
- Simulation tools for RF cavity design and collimation structures. (Tucker, Burton, Smith, Goto)
- Behaviour of charged particles in magneto-plasmas and general aspects of fields in polarisable and magnetisable inhomogeneous moving media. (Tucker, Walton, Goto)
- Modelling wakefields and their role in beam dynamics. (Tucker, Goto, Burton, Gratus)
- Analysis of particle trapping and relativistic warm plasma waves near breaking. (Burton, Noble, Trines)
- Construction of effective models of relativistic inhomogeneous plasmas. (Tucker, Burton, Gratus, Noble)
- Radiation Reaction and Role of "slow vacuum light" in high field electrodynamics. (Tucker, Dereli, Burton)
- Fundamental issues in non-linear electrodynamics. (Tucker, Perlick)

Mathematical Physics Activities

Recent Research Output 2009-

- Electromagnetic Fields Produced by Moving Sources in a Curved Beam Pipe, S Goto, R W Tucker, J. Math. Phys. 50, 1, 2009
- Differential Form Valued Forms and Distributional Electromagnetic Sources, R W Tucker J. Math. Phys. 50, 3, 033506, 2009
- A Distributional Formulation of Electromagnetic Fields and Sources in the Presence of Media Discontinuities, R W Tucker, To be Published 2009
- Energy Spectra from Electromagnetic Fields generated by Ultra-relativistic Charged Bunches in a Perfectly Conducting Cylindrical Beam Pipe, A Hale, R W Tucker. Submitted for Publication 2009
- Wake Potentials and Impedances of Charged Beams in gradually tapering structures, D A Burton, D Christie, J Smith, R W Tucker. Submitted for Publication 2009
- Longitudinal Wave-breaking Limits in a Unified Geometric Model of Relativistic Warm Plasmas, D A Burton, A Noble. Submitted for Publication 2009
- Maxwell's Equations in a Uniformly Rotating Dielectric Medium and the Wilson-Wilson Experiment, C Canovan, R W Tucker. Submitted for Publication 2009
- Relativistic Dispersion and Inhomogeneous Plasmas, J Gratus, R W Tucker. Submitted for Publication 2009
- Modelling Collisions in a Relativistic Plasma, A Noble, D A Burton. Submitted for Publication 2009

Mathematical Physics Activities

Refereed Proceedings 2009-

- Discontinuous Distributions in Thermal Plasmas, D A Burton, A Noble, H Wen, Il Nuovo Cimento C 32 (1), 1, 2009
- An Intrinsic Approach to Forces in Magneto-Electric Media, R W Tucker, T Walton, Il Nuovo Cimento C 32 (1), 205, 2009

Mathematical Physics Activities

Invited Papers 2009-

- D A Burton, R W Tucker Challenges in Controlling Matter and Light, Central Laser Facility, Rutherford and Appleton Laboratory, Annual Report, 2009.
- R W Tucker, A Distributional Formulation of Electromagnetic Fields and Sources in the Presence of Media Discontinuities, Quantum (Foundations, Groups, Information) Symposium York, 29-30 September 2009. To be published.

Mathematical Physics Activities

High Field Intensity Phenomena Induced by Lasers and Plasmas

- Motivation: Can High Intensity Laser Experiments Explore Fundamental Electrodynamics?
- Explore analytically High Field Intensity Phenomena Induced by Lasers and Plasmas
- Devise Experiments using Field Intensities with Current and near Future Technologies
- Discuss Solutions to Vacuum Born-Infeld Electrodynamics with Bounded Electromagnetic Energy
- Construct a Macroscopic Fluid Model based on Born-Infeld Vacuum Electrodynamics

Mathematical Physics Activities

High Field Intensity Phenomena Induced by Lasers and Plasmas

- Quantum Vacuum Effects
  - Pair-production
  - Delbrueck scattering in Coulomb and Laser Fields
- Significant non-linear Raman and Kerr effects (Alpha-X)
- Approach Limits of Lorentz-Dirac Radiation reaction theory
- Approach Limits of validity of linearisation for Maxwell-Vlasov perturbation techniques
- Approach fundamental issues due to infinite Coulomb fields of point charged particles

Mathematical Physics Activities

Field Intensities with Current and near Future Technologies

- Electron Pair Quantum Vacuum effects set by the critical field strengths

$$|\mathbf{e}| = \frac{m_e^2 c^3}{\hbar\, e} = 1.3 \times 10^{16} \mathrm{V/cm}$$

$$|\mathbf{b}| = \frac{m_e^2\, c^3}{\hbar\, e} = 4.4 \times 10^{13} \mathrm{G} = 4.4 \times 10^{9} \mathrm{T}$$

- These can be reached in a laser with peak intensity

$$I = \frac{c|\mathbf{e}|^2}{8\pi} = 2.3 \times 10^{29} \mathrm{W/cm}^2$$

or near the nucleus of a Uranium atom

Mathematical Physics Activities

Field Intensities with Current and near Future Optical Laser Technologies

- State-of-art
  - 10 Joule
  - 30 fs pulse
  - 1μm spot
  - $2 \times 10^{22} W/cm^2$
- (2009) Astra-Gemini, etc
  - 10-100 Joule
  - 10-100 fs pulse
  - 5μm spot
  - $2 \times 10^{22} - 10^{23}\ W/cm^2$

Mathematical Physics Activities

Field Intensities with Current and near Future Optical Laser Technologies

- (2009) MPIQ
  - 5 Joule
  - 5 fs pulse
  - 5μm spot
  - $2 \times 10^{22} - 10^{23}\ W/cm^2$
- (2012) ELI
  - 5 Joule
  - 5 fs pulse
  - 5μm spot
  - $2 \times 10^{25} - 10^{26}\ W/cm^2$

Mathematical Physics Activities

Should one explore effective non-linear electrodynamic theories that may describe modifications to classical linear (Maxwell) electrodynamics in high-field regimes before the expected breakdown due to quantum effects?

Mathematical Physics Activities

Non-linear Vacuum Electrodynamics

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} + \dot{\mathbf{B}} = 0$$

$$\nabla \cdot \mathbf{D} = 0$$

$$\nabla \times \mathbf{H} - \dot{\mathbf{D}} = 0$$

$$\mathbf{D} = \chi_{11}(\mathbf{B},\mathbf{E})\,\mathbf{E} + \chi_{12}(\mathbf{B},\mathbf{E})\,\mathbf{B}$$

$$\mathbf{H} = \chi_{21}(\mathbf{B},\mathbf{E})\,\mathbf{E} + \chi_{22}(\mathbf{B},\mathbf{E})\,\mathbf{B}$$

Embed fields in tensors $F = F(\mathbf{B},\mathbf{E})$ and $G = G(\mathbf{D},\mathbf{H})$ and rewrite partial differential equations and constitutive relations as

$$d\,F = 0 \qquad d \star G = 0$$

$$G = \psi(\alpha,\beta)\,F + \chi(\alpha,\beta) \star F$$

Mathematical Physics Activities

In Maxwell Electrodynamics Radiation Reaction is described by the Lorentz-Dirac equation:

For the relativistic motion of a particle of mass m, charge q, 4-acceleration a_α and 4-velocity u^β in an external electromagnetic field $F^{\text{ext}}_{\alpha\beta}$:

$$ma_\alpha = qF^{\text{ext}}_{\alpha\beta}u^\beta + \frac{2}{3}q^2(\eta_{\alpha\beta} + u_\alpha u_\beta)\frac{Da^\beta}{d\tau}$$

Conceptual and practical problems:

- Infinite negative bare mass to compensate for infinite self-energy of the particle.
- Lorentz-Dirac equation is *third order* in time derivatives of the particle's worldline.
    - Self-acceleration in regions with no external field.
    - Runaway solutions.

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Find a new equation!

Mathematical Physics Activities

The General Relativistic (charged thermodynamic) Coupled Field-Fluid System in Non-Linear Electrodynamics

For a charged convective 4-current $J^{U1} = \rho_e\, V$ and general Lagrangian $\mathcal{L}(\alpha, \beta, g)$, dependent on the electromagnetic invariants

$$\alpha = \star(F \wedge \star F), \quad \beta = \star(F \wedge F)$$

and the metric tensor g on spacetime, the coupled field-fluid system for the field F, mass density ρ_m, charge density ρ_e, and 4-velocity V is:

Mathematical Physics Activities

$$V \cdot V = -1, \quad d\,F = 0, \quad d \star G = \rho_e \star \tilde{V}, \quad G = \frac{\partial \mathcal{L}}{\partial \alpha} \star F + \frac{\partial \mathcal{L}}{\partial \beta} F,$$

$$(\rho + p)\mathcal{A} = \rho_e\, F(V) + \Pi_V(F(\eta)) - d\,p - \xi$$

$$(\rho + p)\,\nabla \cdot V = -V(\rho) + \xi(V) + F(\eta, V)$$

where $\mathcal{A} \equiv \nabla_V \tilde{V}$ is the 4-acceleration of the flow, p is thermodynamic pressure, the vector fields $\xi = \xi(F)$, $\eta = \eta(F)$ induce electrodynamic pressures and the matter mass-energy is

$$\rho = \rho_m(1 + \mathcal{E}(\rho_m, p))$$

The thermodynamic temperature T and entropy $\mathcal{S}$ of the fluid are defined via the Gibbs' relation

$$T\,d\mathcal{S} = d\,\mathcal{E} + p\,d\left(\frac{1}{\rho_m}\right)$$

Mathematical Physics Activities

The Maxwell Relativistic (charged thermodynamic) Coupled Field-Fluid System

In Maxwell Electrodynamics

$$\mathcal{L} = \alpha$$

and hence $\eta = \xi = 0$ and the system reduces to:

$$V \cdot V = -1, \quad d\,F = 0, \quad d \star G = \rho_e \star \tilde{V}, \quad G = \epsilon_0\,F$$

$$(\rho + p)\mathcal{A} = \rho_e\,F(V) - d\,p$$

$$(\rho + p)\,\nabla \cdot V = -V(\rho)$$

exhibiting flow under the Lorentz force $\rho_e\,F(V)$ and thermodynamic pressure gradients.

Mathematical Physics Activities

The General Relativistic Electrically Neutral Pressureless Cold Fluid System

If $p = 0$, $\rho_e = 0$, $\rho = \rho_m$ the system becomes

$$V \cdot V = -1$$

$$d\,F = 0, \qquad d\star G = 0$$

$$G = \frac{\partial \mathcal{L}}{\partial \alpha} \star F + \frac{\partial \mathcal{L}}{\partial \beta}\, F$$

$$\rho_m \mathcal{A} = \Pi_V\left(F(\eta) - \xi\right)$$

$$\rho_m \nabla \cdot V = -V(\rho_m) + F(\eta, V) + \xi(V)$$

Thus for fields F such that either η or ξ are non-zero, electrodynamic pressures may arise even in the absence of matter-induced U1 electric currents.

Mathematical Physics Activities

Born-Infeld electrodynamics

Such pressures can arise from solutions in Born-Infeld electrodynamics where

$$\mathcal{L}(\alpha,\beta,g)=\frac{\epsilon_0}{\kappa^2}(1-\sqrt{\Delta(\alpha,\beta)})$$

with

$$\Delta(\alpha,\beta)=1-\kappa^2\,\alpha-\frac{\kappa^4}{4}\,\beta^2$$

and κ is a new fundamental (dimension-full) coupling in nature.

Mathematical Physics Activities

Born-Infeld plane waves in a Static Magnetic Field.

These exact solutions describe plane propagating waves superposed on a uniform static magnetic field transverse to the direction of propagation in vacuo.

$$F = \mathcal{E}(z - vt)\, d(z - vt) \wedge dx - B\, dy \wedge dz$$

describing the fields:

$$\mathbf{e} = -\frac{v}{c}\mathcal{E}(z - vt)\,\hat{\mathbf{x}}, \quad \mathbf{b} = \frac{\mathcal{E}(z - vt)}{c}\,\hat{\mathbf{y}} - \frac{B}{c}\,\hat{\mathbf{x}}$$

$$v = \frac{c}{\sqrt{1 + c^2\kappa^2 B^2}}$$

with constant B and arbitrary longitudinal profile $\mathcal{E}$: Switching on a static magnetic field B slows down the propagating electromagnetic field to a phase speed $v < c$ *in vacuo*.

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If one sets $\kappa \simeq \frac{\epsilon_0\, r_0^2}{e}$ in terms of the classical radius of the electron: $r_0 = \frac{e^2}{4\pi\epsilon_0\, m_e c^2} \simeq 2.8 \times 10^{-15} m$, the Born-Infeld electron model bounds $\kappa < 10^{-22}$.

The light transit time difference between when the static B field is switched on and off in the magnet region of length L_0 is

$$\tau = \frac{L_0}{2}\kappa|B|$$

Then for L_0 in metres and $|B|$ in Tesla

$$\tau < \frac{L_0}{2}\,|B|\,10^{-6}\ \text{ps}$$

where $1\text{ps} = 10^{-12}$ sec.

The retardation is cumulative!

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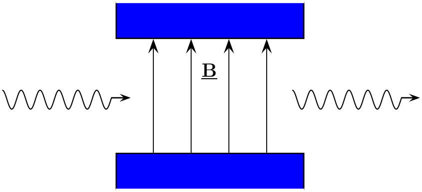

Figure 1: In Maxwell Electrodynamics a plane electromagnetic wave propagates unchanged in an external vacuum static magnetic field with speed c. In Born-Infeld Electrodynamics its speed is reduced to $v = \frac{c}{\sqrt{1+c^2\kappa^2|B|^2}}$

The light transit time difference between when the field is switched on and off in the magnet region of length L_0 is

$$\tau = \frac{L_0}{2}\kappa|B|$$

The Born-Infeld electron model bounds κ. Then for L_0 in metres and $|B|$ in Tesla

$$\tau < \frac{L_0}{2}|B|10^{-6}\text{ps}$$

where 1ps= 10^{-12} sec.