Lagrangian and Hamiltonian Dynamics

Problems for the Tutorial 1 March 2010

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1. Show that the Lorentz force equation for a relativistic particle in a magnetic field

$$\frac{d}{dt}\Big(\frac{m\,\dot{\vec{x}}}{\sqrt{1-\frac{|\dot{\vec{x}}|^2}{c^2}}} \Big) = q\,\dot{\vec{x}} \times \vec{B}$$

implies that the particle's speed is constant, $|\dot{\vec{x}}(t)| = v_0 = \text{constant}$.

2. Recall that, for a relativistic particle in a magnetic field $\vec{B}$, the Lagrangian is of the form

$$L(\vec{x},\dot{\vec{x}}) = -m\,c^2 \sqrt{1-\frac{|\,\dot{\vec{x}}\,|^2}{c^2}} + q\,\vec{A}(\vec{x})\cdot\dot{\vec{x}}$$

where $\vec{A}$ is a vector potential for $\vec{B}$, i.e., $\nabla \times \vec{A} = \vec{B}$. Consider the special case that $\vec{B}$ is a *constant* magnetic field.

   (a) Rewrite the Lagrangian in spherical polar coordinates, with the magnetic field in the z-direction. Choose the vector potential such that the Lagrangian is independent of φ.

   (b) Write, with the Lagrangian from (a), the Euler-Lagrange equations.

   (c) Show that the equations of motion admit circular solutions of the form $r(t) = r_0$, $\vartheta(t) = \pi/2$, $\varphi(t) = \omega t$. Determine the relation between the frequency ω and the radius r_0. How does this result change if we use the non-relativistic Lorentz force equation?