COCKCROFT INSTITUTE

Post-graduate lectures, autumn 2009.

Course on Conventional Magnets for Accelerators (N.Marks).

Tutorial questions for distribution on 30/11/09 and completion, return and discussion on 7/12/09:

1.
i) Write down the expression for the two dimensional scalar potential of an upright octupole magnet using cylindrical co-ordinates (r, θ).

ii) From this, establish the equation for the pole shape of an ideal octupole magnet in Cartesian co-ordinates (x, y) and obtain the expressions for the components of magnetic induction, B_X B_Y.

iii) Derive an expression for the Ampere-turns per pole required to excite an octupole with infinite permeability yoke, inscribed radius R, and octupole field coefficient G_O.

iv) If the octupole magnet is fully symmetrical, what are the first three 'allowed' error harmonics that can be present in this magnet?

SOLUTION:

i) Octupole: therefore $n = 4$;

Upright (not skewed) magnet, therefore $J_4 = 0$;

Scalar potential $\phi = K_4 r^4 \sin 4\theta$;

ii)) Ideal pole shape is line of scalar potential

ie $r^4 \sin 4\theta = \pm C$ (where C is some constant) (1)

As: $\sin 4\theta = 4 (\sin \theta \cos^3 \theta - \sin^3 \theta \cos \theta)$;

$\phi = 4 K_4 r^4(\sin \theta \cos^3 \theta - \sin^3 \theta \cos \theta$

and ideal pole is: $y x^3 - y^3 x = \pm C/4$;

Determine Constant C in terms of inscribed pole radius R:

At: $\theta = \pi/8$, $r = R$;

Substituting in (1): $R^4 = C$

Hence, ideal pole is: $y x^3 - y^3 x = \pm R^4/4$

Magnetic Induction: $\mathbf{B} = -\nabla \phi$;

Hence: $B_x = -d\phi/dx$; $B_y = -d\phi/dy$

Hence: $B_x = -4 K_4 (y^3 - 3 x^2 y)$;
$B_y = -4 K_4 (3 xy^2 - x^3)$;

(we have said nothing about excitation, so K_4 cannot be evaluated).

iii) On the y=0 axis in a right octupole:

$$B_y = G_O x^3;$$

At very large positive x, pole position is given by

$$y = R^4/(4 x^3)$$

For a small parallel gap (g) with field (B), Amp turns are given by:

$$nI = Bg/ \mu_0;$$

So for Octupole:

$$nI = (G_O x^3) [R^4/(4 x^3)]/ \mu_0$$

$$nI = G_O R^4/(4 \mu_0);$$

iv) harmonic number n = 12, 40, 56.

2. A 'C' cored dipole magnet has the following parameters:

flux density at the beam:	1.4	T;
pole full gap height :	35	mm;
pole width:	180	mm;
length of magnetic circuit (in steel):	0.5	m;

i) calculate the Ampere-turns required in the coils assuming infinite permeability in the yoke;

ii) if a coil is to run with a current density of 3.5 A/mm^2, what cross section of coil is required in each of two coils around the pole, above and below the gap.

iii) assuming that the fringe flux in the cross-section on each side of the pole has an **equivalent total magnetic width** equal to one gap height , what is the expected flux density in the pole root;

iv) what width of yoke is required to limit the flux density in top, bottom and back-legs to 1.2 T;

v) if the average permeability in the yoke at this flux density is 2,000, what increase is needed in the coil Ampere-turns to provide the correct flux-density at the beam?

vi) draw a rough diagram of the magnet cross-section – **this will be used for a practical two-dimensional f.e.a. modelling exercise in spring 2010.**

SOLUTIONS:

i) For infinite permeability in a dipole $nI = Bg/\mu_0$

hence: nI = 38,993 At;

ii) total conductor area = 11.14 x 10^3;

each coil = 5.57 x 10^3;

iii) total flux/m length = 1.4 x (0.18 + 2 x 0.035) T/m;

= 0.35 T/m

pole width = 0.18 mm;

hence flux density in pole root = 1.944 T;

(too high – pole must be broadened at root)

iv)	total flux/m	= 0.35 T/m;
	required flux density in yoke	= 1.2 T;
	therefore yoke width must be	= 0.35 / 1.2 m;
		= 0.292 m;
v)	reluctance of yoke	= yoke length / permeability;
		= 0.5 / 2,000 m
		= 0.25 mm
	Gap	= 35 mm
	So reluctance is	0.71 % of gap (small!);
	So At have to be increased by	0.71 %.